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Answers Key:

5 Marks

1. $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$
$\Rightarrow \sin\left(\sin^{-1}\frac{1}{5}\right)\cos\left(\cos^{-1}x\right)+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$
$[\sin(A+B)=\sin A\cos B+\cos A\sin B]$
$\Rightarrow \frac{1}{5}\times x+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$
$\Rightarrow \frac{x}{5}+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$
Now let $\sin^{-1}\frac{1}{5}=y$
Then, $\sin y=\frac{1}{5}\Rightarrow\cos y=\sqrt{1-\left(\frac{1}{5}\right)^2}=\frac{2\sqrt{6}}{5}\Rightarrow y=\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)$
$\therefore \sin^{-1}\frac{1}{5}=\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)\ldots.(2)$
Let $\cos^{-1}x=z$
Then $\cos z=x\Rightarrow\sin z=\sqrt{1-x^2}\Rightarrow z=\sin^{-1}\left(\sqrt{1-x^2}\right)$
$\therefore \cos^{-1}x=\sin^{-1}\left(\sqrt{1-x^2}\right)$
From 1, 2 and 3 we have
$\frac{x}{5}+\cos\left(\frac{\cos^{-1}(2\sqrt{6})}{5}\right)\cdot\sin\left(\sin^{-1}\sqrt{1-x^2}\right)=1$
$\Rightarrow \frac{x}{5}+\frac{2\sqrt{6}}{5}\cdot\sqrt{1-x^2}=1$
$\Rightarrow x+2\sqrt{6}\sqrt{1-x^2}=5$
$=2\sqrt{6}\sqrt{1-x^2}=5-x$
On squaring both sides, we get:
$(4)(6)\left(1-x^2\right)=25+x^2-10x$
$\Rightarrow 24-24x^2=25+x^2-10x$
$\Rightarrow 25x^2-10x+1=0$
$\Rightarrow (5x-1)^2=0$
$\Rightarrow (5x-1)=0$
$\Rightarrow x=\frac{1}{5}$
Hence, the value of x is $\frac{1}{5}$

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2. $\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\left(\frac{3}{4}\right)$
$\therefore \cos\left[\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{2}{3}\right)\right]$
$=\cos\left[\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right]$
$=\cos\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$
$=\cos\left[\cos^{-1}\left(\frac{6}{5\sqrt{13}}\right)\right]$
$=\frac{6}{5\sqrt{13}}=RHS$
3. $L.H.S.=tan^{-1}\left(\frac{1}{2}\right)+tan^{-1}\left(\frac{1}{5}\right)+tan^{-1}\left(\frac{1}{8}\right)$
$=tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}\times\frac{1}{5}}\right)+tan^{-1}\left(\frac{1}{8}\right)$
$=tan^{-1}\left(\frac{\frac{7}{10}}{\frac{9}{10}}\right)+tan^{-1}\left(\frac{1}{8}\right)$
$=tan^{-1}\left(\frac{7}{9}\right)+tan^{-1}\left(\frac{1}{8}\right)=tan^{-1}\left(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}\times\frac{1}{8}}\right)$
$=tan^{-1}\left(\frac{\frac{65}{72}}{\frac{65}{72}}\right)$
$=tan^{-1}(1)$
$=\frac{\pi}{4}=R.H.S.$
4. $2\tan^{-1}\frac{1}{2}=2\tan^{-1}\frac{2\cdot\frac{1}{2}}{1-\frac{1}{4}}$
$=\tan^{-1}\frac{4}{3}$
$LHS=\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}$
$=\tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}}$
$=\tan^{-1}\frac{31}{17}=RHS$

2 Marks

5. $\left[tan^{-1}\left(-\sqrt{3}\right)+tan^{-1}(1)\right]=-\frac{\pi}{12}$
Alternative Method :
$tan^{-1}\left(-\sqrt{3}\right)+tan^{-1}(1)$
$=tan^{-1}\left[-tan\left(\frac{\pi}{3}\right)\right]+tan^{-1}\left(tan\frac{\pi}{4}\right)$
$=-\frac{\pi}{3}+\frac{\pi}{4}=-\frac{\pi}{12}\ \left[\because tan^{-1}(tan\theta)=\theta\forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$
6. $sin^{-1}\left[sin\left(\frac{3\pi}{5}\right)\right]=\frac{2\pi}{5}$
Alternative Method :
$sin^{-1}\left[sin\left(\frac{3\pi}{5}\right)\right]=sin^{-1}\left[sin\left(\pi-\frac{3\pi}{5}\right)\right]$
$\left(\because\ \frac{3\pi}{5}\notin\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\right)$
$sin^{-1}\left(sin\frac{2\pi}{5}\right)=\frac{2\pi}{5}$
7. $tan^{-1}(1)+cos^{-1}\left(-\frac{1}{2}\right)=\frac{11\pi}{12}$
Alternative Method :

$tan^{-1}(1)+cos^{-1}\left(-\frac{1}{2}\right)$
$=tan^{-1}\left(tan\frac{\pi}{4}\right)+cos^{-1}\left(-cos\frac{\pi}{3}\right)$
$=tan^{-1}\left(tan\frac{\pi}{4}\right)+cos^{-1}\left[cos\left(\pi-\frac{\pi}{3}\right)\right]$
$\left[\because tan^{-1}(tan\theta)=\theta\forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\&cos^{-1}(cos\quad\theta)=\theta\forall\theta\in(0,\pi)\right]$
$=\frac{\pi}{4}+\frac{2\pi}{3}=\frac{11\pi}{12}$

8. $tan^{-1}\left[2sin\left(2cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]=\frac{\pi}{3}$

Alternative Method :

$tan^{-1}\left[2sin\left(2cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$
$=tan^{-1}\left[2sin\left(2cos^{-1}\left(cos\frac{\pi}{6}\right)\right)\right]$
$=tan^{-1}\left[2sin\left(2\left(\frac{\pi}{6}\right)\right)\right]=tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]$
$\Rightarrow tan^{-1}\left(\sqrt{3}\right)=\frac{\pi}{3}$

9. $tan^{-1}\left(tan\frac{9\pi}{8}\right)=\frac{\pi}{8}$

Alternative Method :

$tan^{-1}\left(tan\frac{9\pi}{8}\right)=tan^{-1}\left[tan\left(\pi+\frac{\pi}{8}\right)\right]$
$=tan^{-1}\left[tan\frac{\pi}{8}\right]$
$\left[\because tan^{-1}(tan\theta)=\theta\forall\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right]$
$=\frac{\pi}{8}$

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