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Answers Key:

5 Marks

1. Let $x_1, x_2 \in [0, \frac{\pi}{2}]$, such that $x_1 \neq x_2$
   Then, $\sin x_1 \neq \sin x_2$ and $\cos x_1 \neq \cos x_2$
   $\cos x_1 \neq \cos x_2$ for any two distinct values x in $\cos x_1 \neq \cos x_2$
   since function cannot give same value.
   This is also true for cosine function] Hence, f and g must be one-one.
   $[\because a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)]$
   But $(f+g)^*0) = \sin 0 + \cos 0 = 1$
   and $(f+g)\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1$
   $\therefore$ f + g is not one-one.

3 Marks

2. Since f is one-one, three elements of {1, 2, 3} must be taken to 3 different elements of the co-domain {1, 2, 3} under f. Hence, f has to be onto.
3. Let $x_1, x_2 \in R$.
   Now $f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3$
   $\Rightarrow x_1^3 - x_2^3 = 0$
   $\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2 = 0$
   $\Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$.
   $\therefore f$ is one-one $\Rightarrow f$ injective.
4. R = {(a, b): $a \leq b^3$}
   It is observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \in$ R as $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \frac{1}{8}$
   $\therefore$ R is not reflexive.
   Now,
   (1, 2) $\in$ R (as $1 < 2^3 = 8$)
   But,
   (2, 1) $\notin$ R (as $2 > 1^3 = 1$)
   $\therefore$ R is not symmetric.
   We have R as and $(3, \frac{3}{2}), (\frac{3}{2}, \frac{6}{5}) \in$ R as $3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$
   But $(3, \frac{6}{5}) \notin$ R as $3 > \left(\frac{6}{5}\right)^3$
   $\therefore$ R is not transitive.
   Hence, R is neither reflexive, nor symmetric, nor transitive.
5. Since $A \subset A \forall A \in P(X)$
   $\therefore$ R is reflexive.
   For $A, B, C \in P(X)$,
   ARB and BRC $\Rightarrow A \subset B$ and $B \subset C \Rightarrow A \subset C \Rightarrow$ ARC.
   $\therefore$ R is transitive.
   However, R is not symmetric
   $[\because A \subset B \not\Rightarrow B \subset A]$
   $\Rightarrow ARB \not\Rightarrow BRA$
   Hence R is not an equivalence relation.
6. Given, R = Set of real numbers and R = {(a, b):a <b}
   Symmetric Let (a, b) $\in$ R $\Rightarrow$ a < b, then b < a
   $\Rightarrow$ (b, a) $\notin$ R.
   e.g. 4 < 5 but 5 $\not<$ 4
   So, R is not symmetric.
   Transitive Let (a, b), (b, c) $\in$ R, then
   (a, b) $\in$ R $\Rightarrow$ a < b ...(i)
   (b, c) $\in$ R $\Rightarrow$ b < c ....(ii)
   From Eqs. (i) and (ii), we get
   a < c $\Rightarrow$ (a, c) $\in$ R
   So, R is transitive

2 Marks

7. The given function is f: X $\rightarrow$ Y and relation on X is R={(a, b): f(a) = f (b)}
   Reflexive Since, for every x $\in$ X, we have
   f'(x) = f(x)
   $\Rightarrow$ (xx) $\in$ R, $\forall \in$ X Therefore, R is reflexive.
   Symmetric Let (x, y) $\in$ R
   Then, f(x)=f(y)
   $\Rightarrow$ f(y)=f(x)
   $\Rightarrow$ (y, x) $\in$ R

Thus, (x, y) ∈ R ⇒ (y, x)∈ R, ∀x, y∈ X
Therefore, R is symmetric.
Transitive Let x, y, z∈ X such that
(x, y) ∈ R and (y, z) ∈ R
Given a relation 5 in $N \times N$, defined as
(a, b) S(c, d), if a+d=b+c.
Reflexive Let (a, b) be any arbitrary element of $N \times N$
i.e. $(a, b) \in N \times N$, where $a, b \in N$
Now, as a+b=b+a
[∴ addition is commutative]
Therefore \quad(a, b) S(a, b)
So, S is reflexive.
Symmetric Let$(a, b), (c, d) \in N \times N$, such that (a, b)
S(c, d). Then, a+d=b+c
$\Rightarrow b + c = a + d \Rightarrow c + b = d + a$
$\Rightarrow (c, d)S(a, b)$
So, S is symmetric.
Transitive Let $(a, b), (c, d), (e, f) \in N \times N$ such that (a, b) S(c, d) and (c, d) S(e, f).
Then, a+d=b+c and c+f=d+e
On adding the above equations, we get
a+d+c+f=b+c+d+e
$\Rightarrow a + f = b + e \Rightarrow (a, b)S(e, f)$
So, S is transitive.
Thus, S is reflexive, symmetric and transitive. Hence, S is an equivalence

8. Let y ∈ R (codomain) be any real number and let
f(x) = y.
$\therefore\ y = 3 - 4x \Rightarrow 4x = 3 - y$
$\Rightarrow\ x = \frac{3-y}{4} \in R$, for each y[write x in terms of y]
So, for any real number y ∈ R (codomain), there exists
$\frac{3-y}{4}$ (domain), such that
$f\left(\frac{3-y}{4}\right) = 3 - 4\left(\frac{3-y}{4}\right) = 3 - 3 + y = y$
Hence, f is onto.
9. R is reflexive, since (1, 1), (2, 2) and (3, 3) lie in R. Also, R is not symmetric, as (1, 2) ∈ R but (2, 1) ∉ R. Similarly, R is not transitive, as (1, 2) ∈ R and (2, 3) ∈ R but (1, 3) ∉ R.
10. We have gof (2) = g (f (2)) = g(3) = 7, gof (3) = g(f (3)) = g(4) = 7, gof (4) = g(f (4)) = g(5) = 11 and gof (5) = g(5) = 11.
11. Relation R = {(a, a^3) : a is a prime number less than 5}
R = {(2, 8), (3, 27)},
∴ range = {8, 27}.
12. Here, A = {1, 2, 3, 4}, then
[(1,3)] = {(x, y) ∈ A x A: (x, y) R (1,3)}
= {(x, y) ∈ A x A: x+3=y+1}
= {(x, y) ∈ A x A: y=x+2)
= {(x, y) ∈ A x A:y-x=2}
= {(1,3), (2, 4)}

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