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Answers Key:

Case Study Questions

1. We have, R = {(l_1,l_2) :l_1 is parallel to l_2}
(i) If l_1 is parallel to l_2, then l_2 is parallel to l_1.
So, if $(l_1, l_2) \in R$, then $(l_2, l_1) \in R$
∴ R is symmetric.
(ii) If l_1 is parallel to l_2 and l_2 is parallel to l_3, then l_1 is parallel to l_3.
So, if $(l_1, l_2) \in R, (l_2, l_3) \in R$, then $(l_1,l_3) \in R$
∴ R is transitive.
(iii) Let equation of line parallel to y = 3x + 2 be y = mx + c, where m is the slope of line. Since, y = 3x + 2 and y = mx + c are parallel. Slope of (y =3x + 2) = Slope of(y = mx + c)
⇒ 3 = m i.e. m = 3
Hence, the required line is
y = 3x + c, where $c \in R$
Or
We have, S = {(I_1, I_2) : l_1 is perpendicular to l_2}
For Symmetric If I_1, is perpendicular to I_2, then l_2 is perpendicular to l_1.
So, if $(l_1,l_2) \in S$, then $(l_2, l_1) \in S$
∴ S is symmetric.
For Transitive If I_1, is perpendicular to l_2, and l_2, is perpendicular to l_3, then I_1, is not perpendicular to l_3, it is parallel to I_3.
So, if $(l_1, l_2) \in S, (l_2, l_3) \in S$, then $(l_1, l_3) \notin S$.
∴ S is not transitive.

5 Marks

2. It is given that A = {-1, 0, 1, 2}, B = {-4, -2, 0, 2}.
Also, it is given that f, g : A → B are defined by $f(x) = x^2 - x, x \in A$ and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$
It is observed that:
$f(-1) = (1^2) - (-1) = 1 + 1 = 2$
$g(-1) = 2\left|(-1) - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 3 - 1 = 2$
⇒ f(-1) = g(-1)
f(0) = (0)^2 - 0 = 0
$g(0) = 2\left|0 - \frac{1}{2}\right| - 1 = 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$
⇒ f(0) = g(0)
f(1) = (1)^2 - 1 = 1 - 1 = 0
$g(1) = 2\left|a - \frac{1}{2}\right| - 1 = 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$
⇒ f(1) = g(1)
f(2) = (2)^2 - 2 = 4 - 2 = 2
$g(2) = 2\left|2 - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 3 - 1 = 2$
⇒ f(2) = g(2)
f(a) = g(a) ∀ a ∈ A
Hence, the functions f and g are equal.

3. Given, $f(x) = \frac{4x}{3x+4}$ and $f : R - \left\{-\frac{4}{3}\right\} \to R$
For one-one Let $f(x_1) = f(x_2)$, for some $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$
$\Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$
$\Rightarrow (x_1)(3x_2 + 4) = (x_2)(3x_1 + 4)$
$\Rightarrow 3x_1x_2 + 4x_1 = 3x_1x_2 + 4x_2$
$\Rightarrow 4x_1 = 4x_2 \Rightarrow x_1 = x_2$
$\Rightarrow f(x)$ is one-one function.
For onto Let $y = \frac{4x}{3x+4}$
$\Rightarrow 3xy + 4y = 4x \Rightarrow 4x - 3xy = 4y$
$\Rightarrow x(4 - 3y) = 4y \Rightarrow x = \frac{4y}{4-3y}$
Clearly, x will not define, if 4-3 y=0
$\Rightarrow \quad y = \frac{4}{3}$
Therefore Range of $f(x) = R - \left\{\frac{4}{3}\right\}$
Therefore Range of $f(x) \neq$ Codomain of f(x) So, f(x) is not an onto function.

3 Marks

4. Let y be an arbitrary element of range f. Then $y = 4x^2 + 12x + 15$, for some x in N, which implies that $y = (2x + 3)^2 + 6$. This gives $x = \frac{((\sqrt{y-6})-3)}{2}$, as $y \geq 6$
Let us define $g : S \to N$ by $g(y) = \frac{((\sqrt{y-6})-3)}{2}$
Now gof (x) = g(f (x)) = $g(4x^2 + 12x + 15) = g((2x + 3)^2 + 6)$
$= \frac{\left(\left(\sqrt{(2x+3)^2+6-6}\right)-3\right)}{2} = \frac{(2x+3-3)}{2} = x$

and $fog(y) = f\left(\frac{((\sqrt{y-6})-3)}{2}\right) = \left(\frac{2((\sqrt{y-6})-3)}{2} + 3\right)^2 + 6$

$= ((\sqrt{y-6}) - 3 + 3))^2 + 6 = (\sqrt{y-6})^2 + 6 = y - 6 + 6 = y$

Hence, gof = I_N and fog = I_S. This implies that f is invertible with $f^{-1} = g$.

5. A = {1, 2, 3, 4, 5}

is even R = {ab |a-b| is even}

It is clear that for any element a ∈A, we have|a-a| = 0 (which is even).

∴ R is reflexive.

Let (a, b) ∈ R.

⇒ |a-b| is even

⇒ |-(a-b)| = |b-a| is also even

⇒ (b,a) ∈ R

∴ R is symmetric.

Now, let (a, b) ∈ R and (b, c) ∈ R.

⇒ |a-b| is even and |b -c| is even

⇒ (a - b) is even and (b - c) is even

⇒ (a - c) = (a - b) + (b - c) is even [Sum of two even integer is even]

⇒ |a - c | is even

∴ R is transitive.

Hence, R is an equivalence relation.

Now, all elements of the set {1, 3, 5} are related to each other as all the elements of this subset are odd. Thus, the modulus of the difference between any two elements will be even.

Similarly, all elements of the set {2, 4} are related to each other as all the elements of this subset are even.

Also, no element of the subset {1, 3, 5} can be related to any element of {2, 4} as all elements of {1, 3, 5} are odd and all elements of {2, 4} are even. Thus, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

6. We have: (a, b)R(c, d) if ad(b+c)(a+d).

(I) $(a,b)R(a,b) \Rightarrow ab(b+a) = ba(a+b)$

$\Rightarrow ab(a+b) = ab(a+b)$, which is true.

Thus R is reflexive.

(II) $(a,b)R(c,d) \Rightarrow ad(b+c) = bc(a+d)$

$\Rightarrow bc(a+d) = ad(b+c)$

$\Rightarrow cb(d+a) = da(c+b)$

$[\because \quad bc = cb \quad and \quad a+d = d+a; \quad etc. \quad \forall \quad a,b,c,d \in N]$

⇒ (c,d) R (a,b).

Thus R is symmetric.

(III) Let (a,b) R (c,d) and (c,d) R (e,f).

∴ ad(b+c) = bc(a+d) and cf(d+e) = de(c+f)

$\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad}$ and $\frac{d+e}{de} = \frac{c+f}{cf}$

$\Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ and $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

$\Rightarrow \frac{1}{c} - \frac{1}{d} = \frac{1}{a} - \frac{1}{b}$ and $\frac{1}{c} + \frac{1}{d} = \frac{1}{e} + \frac{1}{f}$

$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{e} - \frac{1}{f} \Rightarrow \frac{1}{a} + \frac{1}{f} = \frac{1}{b} + \frac{1}{e}$

⇒ be (a+f) = af(b+e) ⇒af(b+e)=be(a+f)

⇒ (a,b) R (e,f).

Thus R is transitive.

Hence, R is an equivalence relation.

7. We have:

$f(x) = x^2 + 2$ and $g(x) = \frac{x}{x-1}$

$\therefore fog(x) = f(g(x)) = (g(x))^2 + 2$

$= \left(\frac{x}{x-1}\right)^2 + 2 = \frac{x^2+2(x-1)^2}{(x-1)^2}$

$= \frac{x^2+2(x^2-2x+1)}{(x-1)^2} = \frac{3x^2-4x+2}{(x-1)^2}$

Hence, fog (2)= $\frac{3(4)-4(2)+2}{(2-1)^2} = \frac{12-8+2}{(1)^2} = \frac{6}{1} = 6$

And $gof(x) = g(f(x)) = \frac{f(x)}{f(x)-1}$

$= \frac{x^2+2}{(x^2+2)-1} = \frac{x^2+2}{x^2+1}$

Hence, $gof(-3) = \frac{(-3)^2+2}{(-3)^2+1} = \frac{9+2}{9+1} = \frac{11}{10}$.

8. We have: $f : N \to S$, $f(x) = 9x^2 + 6x - 5$.

Now $f(x_1) = f(x_2) \Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9(x_1 - x_2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)[9x_1 + 9x_2 + 6]$

$\Rightarrow x_1 = x_2 \quad [\because x_1, x_2 \in N]$

$\Rightarrow f \ \ is \quad one - one.$

Let $y \in S$ be an arbitrary number.

Now $y = f(x) \Rightarrow y = 9x^2 + 6x - 5 \Rightarrow y = (3x+1)^2 - 6$

$\Rightarrow \sqrt{y+6} = 3x + 1 \Rightarrow x = \sqrt{\frac{y+6-1}{3}} \in N$

$\Rightarrow \ x = f^{-1}(y)$

Since f is one-one and onto ⇒ f is invertible.

Also $f^{-1}(y) = \sqrt{\frac{y+6-1}{3}}$

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$\Rightarrow \quad f^{-1}(x) = \sqrt{\frac{x+6-1}{3}}$

$\therefore \quad f^{-1}(43) = \frac{\sqrt{49}-1}{3} = \frac{7-1}{3} = 2$

and $\quad f^{-1} = \frac{\sqrt{169}-1}{3} = \frac{13-1}{3} = 4.$

9. Given a relation 5 in $N \times N$, defined as
(a, b) S(c, d), if a+d=b+c.
Reflexive Let (a, b) be any arbitrary element of $N \times N$
i.e. $(a, b) \in N \times N$, where $a, b \in N$
Now, as a+b=b+a
[$\because$ addition is commutative]
$\therefore$ (a, b) S(a, b)
So, S is reflexive.
Symmetric Let$(a, b), (c, d) \in N \times N$, such that (a, b)
S(c, d). Then, a+d=b+c
$\Rightarrow b + c = a + d \Rightarrow c + b = d + a$
$\Rightarrow (c, d)S(a, b)$
So, S is symmetric.
Transitive Let $(a, b), (c, d), (e, f) \in N \times N$ such that (a, b) S(c, d) and (c, d) S(e, f).
Then, a+d=b+c and c+f=d+e
On adding the above equations, we get
a+d+c+f=b+c+d+e
$\Rightarrow a + f = b + e \Rightarrow (a, b)S(e, f)$
So, S is transitive.
Thus, S is reflexive, symmetric and transitive. Hence, S is an equivalence relation.

2 Marks

10. $fog(x) = f[g(x)]$
$= f(x-7)$ .....[Since, $g(x) = x - 7$]
$= x - 7 + 7$ .....[Since, $f(x) = x + 7$]
x
$\Rightarrow fog(x) = \Rightarrow fog(7) = 7$
11. $a * b = 2a + b - ab3 * 4 = 2 \times 3 + 4 - 3 \times 43 * 4 = -2$
12. Given R=[(a,a^2) : a is a prime number less than 5}
$\Rightarrow$ R =[(2,8),(3,27)]
Range=[8,27]
13. $\frac{1}{2} > \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}, \frac{1}{2}\right) \in R$
Hence R is reflexive.

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