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Answers Key:

Long Answer Type Questions

1. $$x = \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}}$$
$$= \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}-\sqrt{a-1}} \cdot \frac{\sqrt{a+1}+\sqrt{a-1}}{\sqrt{a+1}+\sqrt{a-1}}$$
$$= \frac{(\sqrt{a+1}+\sqrt{a-1})^2}{(\sqrt{a+1})^2-(\sqrt{a-1})^2}$$
$$= \frac{(a+1)+(a-1)+2\sqrt{a+1}\sqrt{a-1}}{(a+1)-(a-1)}$$
$$= \frac{2a+2\sqrt{a^2-1}}{2} = a+\sqrt{a^2-1}$$
$x - a = \sqrt{a^2-1}$
Squaring both sides, we get
$x^2 - 2ax + a^2 = a^2 - 1$
$x^2 - 2ax + 1 = 0$

2. Let $x = 0.4\overline{7} = 0.47777\ldots$(i)
Multiplying equation (i) by 10, we get
$10x = 10 \times (0.477777\ldots)$
$\Rightarrow 10x = 4.7777$ .....(ii)
Subtracting eq. (i) from eq. (ii), we get
10x -x= 4.7777.. -0.47777.....
⇒ 9x = 4.3000
$\Rightarrow x = \frac{43}{90}$
So $0.4\overline{7} = \frac{43}{90}$

3. (A) Prove that $\sqrt{10}$ is irrational.
Assume that $\sqrt{10} = \frac{p}{q}$
where p and q are co-prime integers and q ≠ 0.
Squaring both sides, $p^2 = 10q^2$
Since 10 = 2 x 5, both 2 and 5 divide p^2.
Therefore, 2 divides p and 5 also divides p. Hence, 10 divides p.
Let p = 10m
Substituting in the equation:
$(10m)^2 = 10q^2$
$100m^2 = 10q^2$
$q^2 = 10m^2$
Thus, 10 also divides q.
So both p and q are divisible by 10, which contradicts the assumption that p and q are co-prime.
Hence, $\sqrt{10}$ is irrational.
(B) For $\sqrt{4}$: $p^2 = 4q^2$
Taking p = 2k,
$(2k)^2 = 4q^2$
$4K^2 = 4q^2$
k = q
Therefore,
$\sqrt{4}$ = 2 which is rational,
Similarly, for $\sqrt{9}$:
$p^2 = 9q^2$
Taking p = 3k,
$9k^2 = 9q^2$
So k = q, giving
$\sqrt{9} = 3$
which is also rational.
The proof by contradiction works only when the number is not a perfect square. For perfect squares such as and 9, the steps simplify to a rational number instead of producing a contradiction.

4. (A) Natural Numbers → Integers → Rational Numbers → Real Numbers
Examples:
Integer but not a natural number: -3
Rational number but not an integer $\frac{1}{2}$
Real number but not a rational number: $\sqrt{2}$
(B) r_1 and r_2 be two rational numbers such that $r_1 < r_2$
Consider:
$m = r_1 + \frac{(r_2 - r_1)\sqrt{2}}{2}$
Since $\sqrt{2}$ is irrational, m is also irrational.

Also,

$0 < \frac{\sqrt{2}}{2}$

Therefore, $r_1 < m < r_2$

Hence, an irrational number lies between r_1 and r_2.

(C) Irrational numbers are not closed under addition. C counter-example$\sqrt{2} + (-\sqrt{2}) = 0 \in Q$.

Both addends are irrational, but their sum is rational.

5. By using Baudhāyana-Pythagoras Theorem at every step,we get

T1: $\sqrt{(1+1}} = \sqrt{2}$ (irrational)

T2: $\sqrt{(2+1}} = \sqrt{3}$ (irrational)

T3: $\sqrt{(3+1}} = \sqrt{4} = 2$ (rational)

T4: $\sqrt{(4+1}} = \sqrt{5}$ (irrational)

T5: $\sqrt{(5+1}} = \sqrt{6}$ (irrational)

T6: $\sqrt{(6+1}} = \sqrt{7}$ (irrational)

T7: $\sqrt{(7+1}} = \sqrt{8} = 2\sqrt{2}$ (irrational)

T8: $\sqrt{(8+1}} = \sqrt{9} = 3$ (rational)

T9: $\sqrt{(9+1}} = \sqrt{10}$ (irrational)

T10: $\sqrt{(10+1}} = \sqrt{11}$ (irrational)

Construction of $\sqrt{5}$ on the Number Line

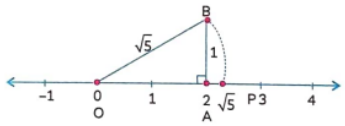
1. On the number line, mark O(0) and A(2) (since OA = 2).

2.At A,draw a perpendicular AB of length 1 unit.

3.Join O to B. Then OB $= \sqrt{2^2 + 1^2} = \sqrt{5}$ by Pythagoras theorem

4. With centre O and radius OB,cut the number line at P.

Then OP = $\sqrt{5}$



Hence, the point P on the number line represents $\sqrt{5}$

Short Answer Type Questions

6. Let the number of copper ingots the merchant has be x. Using the ratio, we have

2 bags: 15 ingots = 12 bags: x ingots

$x = \frac{(15\times12)}{2} = \frac{180}{2} = 90$ ingots

Hence, the merchant will leave with 90 copper ingots.

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7. Suppose you have ₹ 10 and also owe a friend ₹ 5.

This debt is represented by -5

Now imagine your friend forgives the debt

You no longer have to pay ₹ 5,so your financial position improves by ₹5.

Hence,

$10 - (-5) = 10 + 5$

$= 15$

So, subtracting a debt is equivalent to adding the same amount.

8. LHS $= \frac{1}{2} + \frac{3}{4} = \frac{5}{4}$;

$$\left(\frac{5}{4}\right)\left(\frac{8}{3}\right) = \frac{10}{3}$$

$$\text{RHS: } \left(\frac{1}{2}\right)\left(\frac{8}{3}\right) + \left(\frac{3}{4}\right)\left(\frac{8}{3}\right) = \frac{4}{3} + 2$$

$$= \frac{4}{3} + \frac{6}{3} = \frac{10}{3}$$

$$\text{LHS} = \text{RHS} = \frac{10}{3}$$

Property: Distributive Property: a x (b+ c)= a x b+a x c for rational numbers.

9. (A) $\sqrt{23}$ cannot be written in $\frac{p}{q}$ form and here, p =$\sqrt{23}$ which is not an integer and also not a perfect square. Hence, it is an irrational number.

(B) Here, e. $\sqrt{225} = \sqrt{15^2} = 15$

Thus $\sqrt{225}$ is a perfect square and can be written in $\frac{p}{q}$ form, where p is an integer and $q = 1(\neq 0)$.

Hence, $\frac{15}{1}$ is a rational number.

(C) 0.3796 is a rational number because it has a terminating decimal number.

10. Let us assume $5 + 2\sqrt{3}$ is rational, then it must be in the form of

where p and q are co-prime integers and $q \neq 0$

i.e., $5 + 2\sqrt{3} = \frac{p}{q}$

So

Since p,g, 5 and 2 are integers and $q \neq 0$,

RHS of equation (i) is rational. But LHS of (i) is $\sqrt{3}$ which is irrational.

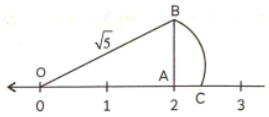
This is not possible. This contradiction has arisen due to our wrong assumption that $5 + 2\sqrt{3}$ is rational.

So, $5 + 2\sqrt{3}$ is irrational

11. Step 1: Draw a number line, mark 2 units from origin and represent the point as A

Step 2: At point A, draw a perpendicular and mark B such that AB = 1 unit.

Step 3: Now, keeping O as the centre and OB as radius, draw an arc to cut the number line at C.



Step 4: Point C represents

on the number line. $\sqrt{5}$ In △OAB, using Pythagoras theorem, we get

$$OB^2 = OA^2 + AB^2$$
$$= 2^2 + 1^2$$
$$= 4 + 1 = 5$$
$$OB = \sqrt{5} \text{ units}$$
$$OB = OC$$
$$= \sqrt{5} \text{ units}$$

Since

OB= 0C= $\sqrt{5}$ units, therefore, point C represents $\sqrt{5}$ on the number line.

12. To locate $\sqrt{3}$ on the number line, we first construct $\sqrt{2}$, and then use it as the base to construct $\sqrt{3}$. This is called the square-root spiral method.

Drawing $\sqrt{2}$

Step 1: Draw a number line. Mark the origin as O (at 0) and mark point A at 1.

Step 2: At point A, draw a perpendicular AB of length 1 unit, upward.

Step 3: Join OB. Now △OAB is a right-angled triangle with OA = 1 and AB = 1.

By Pythagoras theorem:

$OB^2 = OA^2 + AB^2 = 12 + 12 = 2$

$\Rightarrow OB = \sqrt{2}$

Step 4: With O as the centre and radius OB = $\sqrt{2}$, draw an arc that cuts the number line on the positive side at point P.

Then OP = $\sqrt{2}$, so point P represents $\sqrt{2}$ on the number line. (It lies between 1 and 2.)

For $\sqrt{3}$

Now using point P (which represents $\sqrt{2}$), we construct $\sqrt{3}$

Step 1: At point P (which is at distance $\sqrt{2}$ from O), draw a perpendicular PQ of length 1 unit, upward.

Step 2: Join OQ. Now △OPQ is a right-angled triangle with OP = $\sqrt{2}$ and PQ = 1.

By Pythagoras theorem:

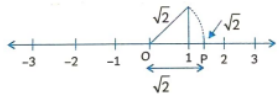
$OQ^2 = OP^2 + PQ^2$

$= (\sqrt{2})^2 + 1^2 = 2 + 1 = 3$

$\Rightarrow OQ = \sqrt{3}$

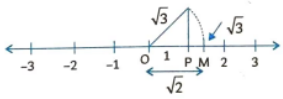
Step 3: With O as the centre and radius OQ = $\sqrt{3}$, draw an arc that cuts the number line on the positive side at point M.

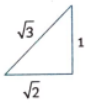
Then OM = $\sqrt{3}$, so point M represents $\sqrt{3}$ on the number line. (It lies between 1 and 2, since $\sqrt{3} \approx 1.732$.) Drawing $\sqrt{2}$





For $\sqrt{3}$





13. **Rational:** When dividing p by q. possible remainders are 0, 1, 2, .., (g - 1) a finite set. Either remainder becomes 0 (terminating) or a remainder repeats (by Pigeonhole Principle), causing digits to cycle.

Example: $\frac{3}{8} = 0.375$ (terminating); $\frac{1}{7} = 0.\overline{142857}$ (repeating).

Irrational: Cannot be written as $\frac{p}{q}$. so no long division process generates the decimal. The expansion continues forever without any repeating block.

Example: $\sqrt{2}$ = 141421356. (never terminates, never repeats).

Very Short Answer Type Questions

14. $2^{1/4} \times 8^{1/4} = (2 \times 8)^{1/4}$

$= (2 \times 2 \times 2 \times 2)^{1/4} = (2^4)^{1/4}$

$= 2^{4 \times \frac{1}{4}} = 2^1 = 2.$

15. $\left(\frac{27}{125}\right)^{-\frac{2}{3}}$

$= \left(\frac{3 \times 3 \times 3}{5 \times 5 \times 5}\right)^{-\frac{2}{3}} = \left(\frac{3^3}{5^3}\right)^{-\frac{2}{3}}$

$= \left\{\left(\frac{3}{5}\right)^3\right\}^{-\frac{2}{3}} = \left(\frac{3}{5}\right)^{3 \times (-\frac{2}{3})}$

$= \left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}.$

16. $\frac{4}{5\sqrt{5}} = \frac{4}{5\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{25}$

17. Let us assume that $\sqrt{5}$ be a rational number. Then, it can be written in the form of $\frac{p}{q}$, where p and q are co-prime and $q \neq 0$.
$\therefore \sqrt{5} = \frac{p}{q}$
$\Rightarrow \sqrt{5}q = p$
On squaring both sides, we get
$5q^2 = p^2$ ..(i)
Here, p^2 is divisible by 5. So, p is divisible by 5.
Let p = 5c
On squaring both sides, we get
$p^2 = 25c^2$
Substituting the value of p^2 in equation (i), we get
$5q^2 = 25c^2$
$\Rightarrow q^2 = 25c^2$
Here, q^2 is divisible by 5.
So, q is divisible by 5.
Thus, p and g have a common factor of 5.
So, there is a contradiction as per our assumption.
We have assumed p and q are co-prime but here, they have a common factor of 5.
The above statement contradicts our assumption.
Therefore, $\sqrt{5}$ is an irrational number.

18. (A) $\sqrt{169} = 13 \Rightarrow$ Rational ($169 = 13^2$, perfect square)
(B) $\sqrt{75} = 5\sqrt{3} \Rightarrow$ Irrational (75 not a perfect square)
(C) 0.12112111211112... $\Rightarrow$ Irrational (pattern grows no fixed block repeats)
(D) 3.205 $\Rightarrow$ Rational (non-terminating recurring repeating block '05')

Assertion and reason

19. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion
Explanation:
For any a, b e N, we always get a + be N. Example: 3 + 5 = 8 e N. The reason directly explains why the assertion holds.

20. (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
Explanation:
Brahmnagupta's Brahmasphutasiddhanta (628 AD) laid down sign rules: debt x debt = fortune, i.e., negative x negative = positive.
Hence (-3) x (-4) = +12.

21. (a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

22. (d) Assertion is incorrect but Reason is correct.
Explanation:
A recurring decimal is rational $\left(\text{e.g. } 0.333\ldots = \frac{1}{3}\right)$.
Only non-terminating non-recurring decimals are irrational.

23. (d) Assertion is incorrect but Reason is correct.
Explanation:
$\sqrt{36} = 6$
Since 6 is rational, the assertion is false. The reason is correct because 36 is a perfect square:
$36 = 6^2$

2 Marks

24. (i) False, because zero is a whole number but not a natural number.
(ii) True, because every integer m can be expressed in the form m/1, and so it is a rational number.
(iii) False because 3/5 is a rational number but not an integer.

25. 3.3333....= $3.\overline{3}$; 0.875; 0.142857 142857...
$= 0.\overline{142857}$

26. $8\sqrt{15} \div 2\sqrt{3} = \frac{8\sqrt{3}\times\sqrt{5}}{2\sqrt{3}} = 4\sqrt{5}$

27. Let x = 1.272727... Since two digits are repeating, we multiply x by 100 to get
100 x = 127.2727
So, 100 x = 126 + 1.272727 = 126 + x
Therefore, 100 x − x = 126, i.e., 99 x = 126
$x = \frac{126}{99} = \frac{14}{11}$
You can check the reverse that $\frac{14}{11} = 1.\overline{27}$

28. $\left(3^{\frac{1}{5}}\right)^4 = 3^{\frac{4}{5}}$

Multiple Choice Question

29. (d) 5

30. (c) 3

31. (d) $-\frac{1}{\sqrt{2}}$

32. One-to-one correspondence means pairing each object with exactly one counting unit (such as a pebble or tally mark), and vice versa. This idea forms the foundation of natural numbers. For example, an ancient herder placed one pebble for each cow leaving to graze; if pebbles remained in the evening, some cows had not returned. Counting began through matching,even before formal number words existed.

33. The Bakhshali Manuscript uses a bold dot called bindu ("dot" or "point") as a Symbol for zero. This dot acted as a placeholder in calculations and was an important early step toward zero becoming a complete number.

34. By Brahmagupta's rule, the product of two debts (negative numbers) is a fortune (positive). Removing 4 debts of 5 each gives a gain of R 20, so (-5) x (-4) = +20
    Thus, in modern notation:(-) x (-) = (+). This rule was stated by Brahmagupta over 1,300 years ago and is still correct today
35. Since, 40 = 2^3 x 5 the denomingtor contains only the prime factors 2 and 5, so the decimal expansion is terminating.
    Convert the denominator into a power of 10:
    $\frac{3}{40} = \frac{3\times25}{40\times25} = \frac{75}{1000} = 0.075$
    The decimal number 0.075 has 3 decimal places.
36. Irrational numbers are represented by non-terminating and non-repeating decimal expansions.
37. The sequence is:
    $T_1 = \sqrt{2},$
    $T_2 = \sqrt{3},$
    $T_3 = \sqrt{4} = 2,$
    $T_4 = \sqrt{5},$
    $T_5 = \sqrt{6}.$
38. $\sqrt{\frac{144}{49}} = \frac{12}{7}$
    Since $\frac{12}{7}$ is rational,$\sqrt{(\frac{144}{49})}$ is rational, whereas all the remaining given numbers are irrational.

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