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Answers Key:

ANSWER ANY 4 QUESTIONS

1. $\frac{1}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$
   $\Rightarrow \frac{1}{(x-1)(x+2)^2} = \frac{A(x+2)^2+B(x-1)(x+2)+C(x-1)}{(x-1)(x+2)^2}$
   $\Rightarrow 1 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$ ....(1)
   Putting x = 1 in (1) we get,
   $1 = A(3)^2 \Rightarrow 1 = 9A \Rightarrow A = \frac{1}{9}$
   Putting x = -2 in (1) we get
   $1 = C(-2-1) \Rightarrow 1 = -3 \Rightarrow C = \frac{-1}{3}$
   Equate co-efficient of x^2 on both sides of (1)
   $0 = A + B$
   $B = -A = \frac{-1}{9}$
   $\frac{1}{(x-1)(x+2)^2} - \frac{1}{9(x-1)} - \frac{1}{9(x+2)} - \frac{1}{3(x+2)^2}$
2. $\frac{1}{(x^2+4)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$
   (Since $x^2 + 4$ cannot be factorised into linear factors)
   $\frac{1}{(x^2+4)(x+1)} = \frac{A(x^2+4)+(Bx+C)(x+1)}{x^2+4}$
   $1 = A(x^2+4) + (Bx+C)(x+1)$ ...(1)
   putting x = -1 in(1) we get
   $1 = c(5) \Rightarrow c = \frac{1}{5}$
   Equate co-efficient of x^2 on both sides of (1)
   $0 = A + B \Rightarrow B = -A = \frac{-1}{5}$
   Putting x = 0 in (1) we get,
   $1 = 4A + C$
   $1 = \frac{4}{5} + C$
   $C = 1 - \frac{4}{5} = \frac{1}{5}$
   $\frac{1}{(x^2+4)(x+1)} = \frac{1}{5(x+1)} + \frac{\frac{-1}{5}x+\frac{1}{5}}{x^2+4}$
   $= \frac{1}{5(x+1)} + \frac{1-x}{5(x^2+4)}$.
3. $(1 + x)^{2n}$
   2n is even
   Middle term is $t_{\frac{n}{2}+1} = t_{\frac{2n}{2}+1} = t_{n+1}$
   $r = n$
   $t_{r+1} = nC_r x^{n-r} a^r$
   $t_{n+1} = 2nC_n (1)^{2n-n} x^n$
   $= 2nC_n \cdot x^n$
   $nC_r = \frac{n!}{n!(n-r)!}$
   $= \frac{(2n)!}{n!(2n-n)!} x^n$
   $= \frac{(2n)(2n-1)(2n-2)(2n-3)\ldots5\cdot4\cdot3\cdot2.1}{n!n!} x^n$
   $= \frac{(2n)(2n-2)\ldots4.2(2n-1)(2n-3)\ldots5.3.1}{n!n!} x^n$
   $= \frac{2^n(n(n-1)\ldots2.1)(2n-1)(2n-3)\ldots5.3.1}{n!n!} x^n$
   $= \frac{2^n.n!(2n-1)(2n-3)\ldots5\cdot3\cdot1}{n!n!} x^n$
   $= \frac{1.3.5\ldots(2n-3)(2n-1)2^n x^n}{n!}$
4. Let P (n) denote the statement $1.2 + 2.3 + 3.4 + \ldots + (n + 1) = \frac{n(n+1)(n+2)}{3}$
   Put n = 1
   LHS = 1 (2) = 2
   $= \frac{1(2)(3)}{3} \Rightarrow 2 = 2$
   LHS = RHs
   $\therefore$ P(1) is true.
   Let us assume that P(k) IS true i.e, to P.T. $1.2 + 2.3 + 3.4 + \ldots + k(k + 1)$
   $= \frac{k(k+1)(k+2)}{3}$ .....(1)
   To prove that P(k+1) is true i.e to P.T. $1.2 + 2.3 + 3.4 + \ldots + k(k + 1) + (k + 1)(k + 2)$
   $= \frac{k(k+2)(k+2)}{3} + (k + 1)(k + 2)$
   $= \frac{k(k+1)(k+2)+3(k+1)(k+2)}{3}$
   $= \frac{(k+1)(k+2)(k+3)}{3}$ = RHS
   $\therefore$ P(k + 1) is true whenever P(k) is true.
   $\therefore$ p(n) is true for all $n \in N$.
5. **Step-1 :** Let P (n) denote the statement a - b is divisible by a - b for all $n \in N$
   $\therefore a^1 - b^1 = a - b$ is divisible by a - b
   Put n = 1
   P(1) : a - b is divisible by a - b
   $\therefore$ P(1) is true.

Step-2:

Let us assume that P(k) is true.

$\therefore a^k - b^k$ is divisible by a - b

$\Rightarrow a^k - b^k = C(a - b)$ where C is a constant ...(1)

Step-3:

To prove that P(k+1) is true

$P(k + 1) = a^{k+1} - b^{k+1}$

$= a^{k+1} - a^kb + a^kb - b^{k+1}$ [Adding and subtracting ak b]

$= a^k (a - b) + b (a^k - b^k)$

$= a^k (a - b) + b.c. (a - b)$ [using (1)]

$= (a - b) (a^k + bc)$ which is divisible by a - b.

$\therefore$ P (k + 1) is true whenever P(k) is true

$\therefore$ P(n) is true for all $n \in N$.

ANSWER ANY 4 QUESTIONS

6. $(101)^4 = (100 + 1)^4$

$= 4C_0 (100)^4 + 4C_1 (100)^3 (1)^1 + 4C_2 (100)^2 (1)^2 + 4 C_3 (100)^1 (1)^3 + 4 C_4(1)^4$

$= 100,000,000 + 4 (1,000,000) + 6 (10000) + 4 (100) + 1$

$= 10,40,60,401$

7. $\left(x^2 - \frac{2}{3x}\right)^9$

n = 9

$t_{r+1} = nC_r x^{n-r} a^r$

$t_{r+1} = 9C_r((x^2)^{9-r} \left(-\frac{2}{3x}\right)^r$

$= 9C_r x^{18-2r}(-1)^r \frac{2^r}{3^r x^r}$

$= (-1)^r 9C_r \frac{2^r}{3^r} x^{18-2r-r}$

$= (-1)^r 9C_r \frac{2^r}{3^r} x^{18-3r}$

To find the term independent of x, equate the power of x as 0

18 - 3r = 0

$3r = 18 \Rightarrow r = \frac{18}{3} = 6$

$t_{r+1} = (-1)^6 9C_6 \frac{2^6}{3^6} = 9C_6 \frac{2^6}{3^6}$

8. $nP_r = 1680; nC_r = 70$

$\frac{nP_r}{r!} = 70$

$\frac{1680}{r!} = 70$

$r! = \frac{1680}{70} = 24$

$r! = 4! \Rightarrow r = 4$

$nP_4 = 1680$

$n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$

$n = 8$

9. General term of $(x + a)^n$ is

$t_{r+1} = nC_r x^{n-r} a^r$ .....(1)

Compare $\left(2x^2 - \frac{3}{x}\right)^{11}$ with $(x + a)^n$

$t_{r+1} = 11C_r (2x^2)^{11-r} \left(\frac{-3}{x}\right)^r$

$= 11C_r 2^{11-r} x^{2(11-r)} (-3)^r \left(\frac{1}{x}\right)^r = 11C_r 2^{11-r} (-1)^r . 3^r x^{22-3r}$

To find the co-efficient of x^{10}, take $22 - 3r = 10$

r = 4. Put r = 4 in (1)

$t_5 = 11C_4 2^{11-4} 3^4 x^{10} = 11C_4 . 2^7 3^4 x^{10}$

$\therefore$ Co-efficient of x^{10} is $11C_4 2^7 3^4$

10. We know that $nC_r = \frac{nP_r}{r!}$

$120 = \frac{720}{r!}$

$r! = \frac{720}{120} = 6 = 3!$

$\Rightarrow r = 3$

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11. $\frac{4x+1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$

$\Rightarrow \frac{4x+1}{(x-2)(x+1)} = \frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$

$\Rightarrow 4x + 1 = A(x + 1) + B(x - 2)$

If x = 2

9 = A(2 + 1)

$\Rightarrow A = \frac{9}{3} = 3$

If x = -1

-4 + 1 = B(-1 - 2)

$\Rightarrow$ -3 = -3B $\Rightarrow B = 1$

$\frac{4x+1}{(x-2)(x+1)} = \frac{3}{x-2} + \frac{1}{x+1}$

12. The letter of the word CHAT in alphabetical order are A, C, H, T.

(i) Number of words starting with A = 3! = 6 C begins

(ii) Number of words starting with CA are 2! = 2

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Now the words CH begins

(iii) After that we get the word CHAT = 1

∴ Rank of CHAT is = 6 + 2 + 1 = 9

13. $\frac{7!}{6!} = \frac{7 \times 6!}{6!} = 7$

14. Let n denote the number of sides of a regular polygon

Number of diagonals = 44

$\frac{n(n-3)}{2} = 44$

$n^2 - 3n - 88 = 0$

$(n - 11)(n + 8) = 0$

$n = 11$

15. $\frac{1}{9!} + \frac{1}{10!} = \frac{n}{11!}$

$\frac{1}{9!} + \frac{1}{10 \times 9!} = \frac{n}{11!}$

$\frac{1}{9!}\left[1 + \frac{1}{10}\right] = \frac{n}{11!}$

$\frac{1}{9!} \times \frac{11}{10} = \frac{n}{11!}$

$n = \frac{11! \times 11}{9! \times 10}$

$= \frac{11! \times 11}{10!}$

$= \frac{11 \times 10! \times 11}{10!}$

$n = 121$

Answer all questions

16. $nP_2 = 20$

$n(n - 1) = 5 \times 4$

$n = 5$

17. $nP_r = 720(nCr)$

$\frac{n!}{(n-r)!} = 720\left(\frac{n!}{r!(n-r)!}\right)$

$r! = 720$

$r! = 6!$

r = 6

18. (c) 5^{th}

19. $t_{r+1} = 6C_r n^{6-r}\left(\frac{2^r}{x^r}\right)$

$= 6C_r x^{6-2r} 2^r$

$6 - 2r = 0 = r = 3$

$t_{r+1} = 6C_3 2^3 = 160$

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20. Equality coefficient of x in the numerator

kx = 4 (2n) + 1 (x)

kx = 9x

21. 210 = 1024 (2.x 2... 10 terms)

22. The number of ways = 6!/3! = 120

23. (c) 8!

24. (a) 2^n

25. $nC_{12} = nC_8$

x + y = n

12 + 8 = n

n = 20