Visit QB365-SLMS - https://www.qb365.in/slms/ Complete Questions and Answers

Answers Key:

5 Marks

1. The technology matrix is given under.

	A	B	Final demand
A	0.4	0.1	6.8
B	0.7	0.7	10.2

$B = \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.7 \end{bmatrix}$

$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.7 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.3 \end{bmatrix}$

$|I - B| = \begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.3 \end{bmatrix}$

$|I - B| = 0.18 - 0.07 = 0.11 > 0$

Since the diagonal elements of (I - B) are positive and |I - B| is positive, the system is viable.

$\therefore (I - B)^{-1} = \frac{1}{|I-B|} adj(I - B) = \frac{1}{0.11}\begin{bmatrix} 0.3 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

Now $X = (I - B)^{-1} D$ where $D = \begin{bmatrix} 6.8 \\ 10.2 \end{bmatrix}$

$X = \frac{1}{0.11}\begin{bmatrix} 0.3 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}\begin{bmatrix} 6.8 \\ 10.2 \end{bmatrix} = \frac{1}{0.11}\begin{bmatrix} 0.3\times6.8+0.1\times10.2 \\ 0.7\times6.8+0.6\times10.2 \end{bmatrix}$

$= \frac{1}{0.11}\begin{bmatrix} 2.04+1.02 \\ 4.76+6.12 \end{bmatrix} = \frac{1}{0.11}\begin{bmatrix} 3.06 \\ 10.88 \end{bmatrix} = \begin{bmatrix} 27.81 \\ 98.90 \end{bmatrix}$

Gross production of commodity A and B are 27.81

Gross production of B is 98.91 tonnes

2. $a_{11} = 30, a_{12} = 40, x_1 = 120$

$a_{21} = 20, a_{22} = 10, x_2 = 60$

$b_{11} = \frac{a_{11}}{x_1} = \frac{30}{120} = \frac{1}{4}$

$b_{12} = \frac{a_{12}}{x_2} = \frac{40}{60} = \frac{2}{3}$

$b_{21} = \frac{a_{21}}{x_1} = \frac{20}{120} = \frac{1}{6}$

$b_{22} = \frac{a_{22}}{x_1} = \frac{10}{60} = \frac{1}{6}$

The technology matrix is $B = \begin{bmatrix} \frac{1}{4} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$

$I - B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{4} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{-2}{3} \\ \frac{-1}{6} & \frac{5}{6} \end{bmatrix}$

$|I - B| = \frac{3}{4} \times \frac{5}{6} - \frac{2}{3} \times \frac{1}{6} = \frac{5}{8} - \frac{1}{9} = \frac{37}{72} = 0$

Since diagonals of I - B are positive and | I - B | is positive, the system is viable

$(I - B)^{-1} = \frac{1}{|I-B|} adj\,(I - B) = \frac{72}{37}\begin{bmatrix} \frac{5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{3}{4} \end{bmatrix}$

$X = (I - B)^{-1} D$ where $D = \begin{bmatrix} 80 \\ 40 \end{bmatrix}$

$= \frac{72}{37}\begin{bmatrix} \frac{5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{3}{4} \end{bmatrix}\begin{bmatrix} 80 \\ 40 \end{bmatrix}$

$= \frac{72}{37}\begin{pmatrix} 66.67 & +26.67 \\ 13.33 & +30 \end{pmatrix} = \frac{72}{37}\begin{pmatrix} 93.34 \\ 43.33 \end{pmatrix}$

$= \begin{pmatrix} 181.63 \\ 84.32 \end{pmatrix}$

The output for production section X and Y are 181.63 and 84.32 respectively.

3. Let x, y, z denote the cost of 1 kg of Wheat, Sugar and Rice respectively.

$2x + y = 70$

$x + z = 70$

$3x + 2y + z = 17$

The given system can be written as

$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 70 \\ 70 \\ 170 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

Where $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 70 \\ 70 \\ 170 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix}$

$= 2(0 - 2) - 1(1 - 3) + 0 = -4 + 2 + 0 = -2 \neq 0$

$\therefore A^{-1}$ exists

Co-factor matrix $= \begin{pmatrix} -2 & 2 & 2 \\ -1 & 2 & -1 \\ 1 & -2 & -1 \end{pmatrix}$

adj $A = \begin{pmatrix} -2 & -1 & 1 \\ 2 & 2 & -2 \\ 2 & -1 & -1 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} adj\ A = \frac{-1}{2} \begin{bmatrix} -2 & -1 & 1 \\ 2 & 2 & -2 \\ 2 & -1 & -1 \end{bmatrix}$

$X = A^{-1}B = \frac{-1}{2} \begin{bmatrix} -2 & -1 & 1 \\ 2 & 2 & -2 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} 70 \\ 70 \\ 170 \end{bmatrix} = \frac{-1}{2} \begin{bmatrix} -40 \\ -60 \\ -100 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 50 \end{bmatrix}$

$\therefore$ Cost of 1kg of Wheat, Sugar and Rice are Rs. 20, Rs. 30 and Rs. 50 respectively.

4. $a_{11} = 50, a_{12} = 75, x_1 = 200$

$a_{21} = 100, a_{22} = 50, x_2 = 200$

$b_{11} = \frac{a_{11}}{x_1} = \frac{50}{200} = \frac{1}{4}; b_{12} = \frac{a_{12}}{x_{12}} = \frac{75}{200} = \frac{3}{8}$

$b_{21} = \frac{a_{11}}{x_1} \frac{100}{200} = \frac{1}{2}; b_{22} = \frac{a_{22}}{x_2} = \frac{50}{200} = \frac{1}{4}$

$\therefore$ The technology matrix is B $= \begin{bmatrix} \frac{1}{4} & \frac{3}{8} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$

$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{4} & \frac{3}{8} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & -\frac{3}{8} \\ -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$

$|I - B| = \frac{9}{16} - \frac{3}{16} = \frac{6}{16} = \frac{3}{8} > 0$

Now, X = $(I - B)^{-1}$ D Where D $= \begin{bmatrix} 300 \\ 600 \end{bmatrix}$

$X = \frac{8}{3} \begin{pmatrix} \frac{1}{4} & \frac{3}{8} \\ \frac{1}{2} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 300 \\ 600 \end{pmatrix}$

$= \frac{8}{3} \begin{pmatrix} 75 + 225 \\ 150 + 150 \end{pmatrix} = \begin{pmatrix} 800 \\ 800 \end{pmatrix}$

The output of P and Q must be 800 each.

5. $3x - y + 2z = 13; 2x + y - z = 3$

$x + 3y - 5z = -8$

The given system can be written as

$\begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 \\ 3 \\ -8 \end{pmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

Where $A = \begin{pmatrix} 3 & -1 & 2 \\ 2 & 1 & -1 \\ 1 & 3 & -5 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$B = \begin{pmatrix} 13 \\ 3 \\ -8 \end{pmatrix}$

$|A| = 3(-5 + 3) + 1(-10 + 1) + 2(6 - 1)$

$= -6 - 9 + 10 = -5 \neq 0$

$\therefore A^{-1} exists$

A_{11} = Co-factor of $3 = (-5 + 3) = -2$

A_{12} = Co-factor of $-1 = -(-10 + 1) = 9$

A_{13} = Co-factor of $2 = (6 - 1) = 5$

A_{21} = Co-factor of $2 = -(5 - 6) = 1$

A_{22} = Co-factor of $1 = -15 - 2 = -17$

A_{23} = Co-factor of $-1 = -(9 + 1) = -10$

A_{31} = Co-factor of $1 = 1 - 2 = -1$

A_{32} = Co-factor of $3 = -(-3 - 4) = 7$

A_{33} = Co-factor of $-5 = 3 + 2 = 5$

Co-factor matrix $= \begin{pmatrix} -2 & 9 & 5 \\ 1 & -17 & -10 \\ -1 & 7 & 5 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-5} \begin{pmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{pmatrix}$

$X = A^{-1}B = \frac{-1}{5} \begin{pmatrix} -2 & 1 & -1 \\ 9 & -17 & 7 \\ 5 & -10 & 5 \end{pmatrix} \begin{pmatrix} 13 \\ 3 \\ -8 \end{pmatrix}$

$= \frac{-1}{5} \begin{pmatrix} -26 & 3 & 8 \\ 117 & -51 & -56 \\ 65 & -30 & -40 \end{pmatrix}$

$= \frac{-1}{5} \begin{pmatrix} -15 \\ 10 \\ -5 \end{pmatrix}$

QB 365 TM Study Material
QUESTION BANK 365

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

$x = 3, y = -2, z = 1$

6. Given equations are

x - y + 2z = 3, 2x + z = 1 and 3x + 2y +z = 4.

The given equations can be written in matrix form as

$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

Where $A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$B = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$

$|A| = 1(0 - 2) + 1(2 - 3) + 2(4 - 0)$

$= -2 - 1 + 8 = 5 \neq 0$

$\therefore A^{-1}$ exists

Co-factor matrix $= \begin{pmatrix} -2 & 1 & 4 \\ 5 & -5 & -5 \\ -1 & 3 & 2 \end{pmatrix}$

$\mathrm{A}^{-1} = \frac{1}{|A|}\mathrm{adj\ A} = \frac{1}{5}\begin{pmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{pmatrix}$

$\mathrm{X} = \mathrm{A}^{-1}\,\mathrm{B}$

$= \frac{1}{5}\begin{bmatrix} -2 & 5 & -1 \\ 1 & -5 & 3 \\ 4 & -5 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -6+5-4 \\ 3-5+12 \\ 12-5+8 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -5 \\ 10 \\ 15 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$

$x = -1, y = 2, z = 3$

QB 365 TM Study Material
QUESTION BANK 365

7. Given equations are 2x - z = 0, 5x + y = 4 and y + 3z = 5

Given equations can be written in matrix form as

$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$

$\Rightarrow$ AX = B

Where A $= \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B $= \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$

$|A|$ = 2 (3 - 0) - 0 - 1 (5 - 0) = 6 - 5 = 1 $\neq$ 0

$\therefore$ A^{-1} exists.

Co-factor matrix $= \begin{pmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|}adj\ A = \frac{1}{1}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

$\therefore X = A^{-1}B = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0-4+5 \\ 0+24-25 \\ 0-8+10 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$x = 1, y = -1, z = 2$

8. We consider selling the units to be a positive earning and buying as a negative earning

$3x + 5y - 4z = 6000$

$2x - 3y + z = 5000$

$-x + 4y + 6z = 13000$

The given system can be written as

$\begin{pmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6000 \\ 5000 \\ 13000 \end{pmatrix}$

$AX = \mathrm{B} \Rightarrow X = A^{-1}B$

Where A $= \begin{pmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$B = \begin{pmatrix} 6000 \\ 5000 \\ 13000 \end{pmatrix}$

$|A| = 3(-18-4) - 5(12+1) - 4(8-3)$

$= -66 - 65 - 20 = -151 \neq 0$

$\text{Co-factor matrix} = \begin{pmatrix} -22 & -13 & 5 \\ -46 & 14 & -17 \\ -7 & -11 & -19 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{-1}{151}\begin{pmatrix} -22 & -13 & 5 \\ -46 & 14 & -17 \\ -7 & -11 & -19 \end{pmatrix}^T$

$X = \text{A}^{-1}\,\text{B}$

$= \frac{-1}{151}\begin{pmatrix} -22 & -46 & -7 \\ -13 & 14 & -11 \\ 5 & -17 & -19 \end{pmatrix}\begin{pmatrix} 6000 \\ 5000 \\ 13000 \end{pmatrix}$

$= \frac{-1}{151}\begin{pmatrix} -132000 & -230000 & -91000 \\ -78000 & +70000 & -143000 \\ 30000 & -85000 & -247000 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{151}\begin{pmatrix} -453000 \\ -151000 \\ -302000 \end{pmatrix} = \begin{pmatrix} 3000 \\ 1000 \\ 2000 \end{pmatrix}$

$x = Rs.3000, \text{y} = Rs.1000, \text{z} = Rs.2000$

9. Let the 3 numbers be x, y and z.

Given x + y + z = 20

2x + y - z = 23

3x+ y + z = 46

It can be rewritten as

$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix}$

$\Rightarrow \ AX = B \Rightarrow X = A^{-1}B$

Where $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix}$

= 1 (1 + 1) - 1 (2 + 3) + 1 (2 - 3)

= 2 - 5 - 1 = 2 - 6 = - 4 $\neq 0$

$\therefore \text{A}^{-1}$ exists.

$\text{Co-factor matrix} = \begin{pmatrix} 2 & -5 & -1 \\ 0 & -2 & 2 \\ -2 & 3 & -1 \end{pmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}adj\ A = \frac{-1}{4}\begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1 \end{bmatrix}$

$X = A^{-1}B = \frac{-1}{4}\begin{bmatrix} 2 & 0 & -2 \\ 5 & -2 & 3 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix}$

$= \begin{bmatrix} x \\ y \\ z \end{bmatrix}, = \frac{-1}{4}\begin{bmatrix} 40 + 0 - 92 \\ -100 - 46 + 138 \\ -20 + 46 - 46 \end{bmatrix} = \frac{-1}{4}\begin{bmatrix} -5 \\ -8 \\ -20 \end{bmatrix}$

$X = \begin{bmatrix} 13 \\ 2 \\ 5 \end{bmatrix}$

$x = 13, y = 2, z = 5$

$\therefore$ The 3 numbers are 13, 2 and 5.

10. Let x, y, z be the salary of each type of staff in A, B and C categories

4x + 2y + 3z = 4900

3x + 3y + 2z = 4500

4x + 3y + 4z = 5800

In matrix form $\begin{bmatrix} 4 & 2 & 3 \\ 3 & 3 & 2 \\ 4 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4900 \\ 4500 \\ 5800 \end{bmatrix}$

$\Rightarrow AX = B \Rightarrow X = A^{-1}B$

Where A $= \begin{bmatrix} 4 & 2 & 3 \\ 3 & 3 & 2 \\ 4 & 3^4 & \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4900 \\ 4500 \\ 5800 \end{bmatrix}$

= 4 (12 - 6) - 2(12 - 8) + 3 (9 - 12)

= 4 (6) - 2 (4) + 3 (-3) = 24 - 8 - 9 = 7 = 1

$\therefore \text{A}^{-1}$ exists.

$Co - factormatrix = \begin{bmatrix} 6 & -4 & -3 \\ 1 & 4 & -4 \\ -5 & 1 & 6 \end{bmatrix}$

$\therefore\ A^{-1} = \frac{1}{|A|}adj\ A = \frac{1}{7}\begin{bmatrix} 6 & 1 & -5 \\ -4 & 4 & 1 \\ -3 & -4 & 6 \end{bmatrix}$

QB 365 Study Material
QUESTION BANK 365

$\therefore\ X = A^{-1}B = \frac{1}{7}\begin{bmatrix} 6 & 1 & -5 \\ -4 & 4 & 1 \\ -3 & -4 & 6 \end{bmatrix}\begin{bmatrix} 4900 \\ 45000 \\ 5800 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 29400 & +45000 & -29,000 \\ -19600 & +18000 & +5800 \\ -14700 & -18,000 & +34800 \end{bmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7}\begin{bmatrix} 49000 \\ 4200 \\ 2100 \end{bmatrix} = \begin{bmatrix} 700 \\ 600 \\ 300 \end{bmatrix}$

Salary for each type of staff under category A, B and C are respectively Rs 700,Rs 2600 and Rs 7300

11. Let x, y and z be the cost of onion, wheat and rice per kg respectively.

Then, 4x + 3y + 2z = 320

2x + 4y + 6z = 560

6x + 2y + 3z = 380.

It can be written as

$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 320 \\ 560 \\ 380 \end{bmatrix}$

AX = B

where $A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 320 \\ 560 \\ 380 \end{bmatrix}$

$|A| = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} = 50 \neq 0$

A^{-1} exist

$A_{11} = 0\ A_{12} = 30\ A_{13} = -20$

$A_{21} = -5\ A_{22} = 0\ A_{23} = 10$

$A_{31} = 10\ A_{32} = -20\ A_{33} = 10$

$[A_{ij}] = \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}$

adj A = $[A_{ij}]^T$

$= \begin{bmatrix} 0 & 5 & -10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}adjA$

$= \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$

$X = A^{-1}B$

$= \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 320 \\ 560 \\ 380 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 32 \\ 56 \\ 38 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 0 & -280 & +380 \\ 960 & 0 & -760 \\ -640 & +560 & +380 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 40 \\ 60 \end{bmatrix}$

x = 20, y = 40, z = 60

Cost of 1 kg onion is Rs.20, cost of 1 kg wheat is Rs.40 and cost of 1 kg rice is Rs.60.

12. Here the technology matrix is given under

	Steel	Coal	Final demand
Steel	0.4	0.1	50
Coal	0.7	0.6	100
Labour days	5	2	-

The technology matrix is B = $\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

I - B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{bmatrix}$

$|I - B| = \begin{vmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{vmatrix}$

= (0.6)(0.4) – (–0.7)(–0.1)

= 0.24 – 0.07 = 0.17

Since the diagonal elements of I – B are positive and value of |I-B| is positive, the system is viable.

adj(I - B) = $\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

$(I - B)^{-1} = \frac{1}{|I-B|}$adj(I - B)

QB 365 Study Material
QUESTION BANK 365

$= \frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

X = $(I - B)^{-1}D$, where D = $\begin{bmatrix} 50 \\ 100 \end{bmatrix}$

$= \frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}\begin{bmatrix} 50 \\ 100 \end{bmatrix}$

$= \frac{1}{0.17}\begin{bmatrix} 30 \\ 95 \end{bmatrix}$

$= \begin{bmatrix} 176.5 \\ 558.8 \end{bmatrix}$

Steel output = 176.5 tonnes

Coal output = 558.8 tonnes

Total labour days required

= 5(steel output) + 2(coal output)

= 5(176.5) + 2(558.8)

= 882.5 + 1117.6 = 2000.1

$\simeq$ 2000 labour days.

3 Marks

13. $AB = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 9-1 & 0+2 \\ 6+1 & 0-2 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 7 & -2 \end{bmatrix}$

= -16 - 14 = -30

$\therefore$ |AB| = -30

14. Expanding the given determinant along R_1 we

$\begin{vmatrix} 7 & 4 & 11 \\ -3 & 5 & x \\ -x & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow$ 7 (5 - 3x) - 4 (-3 + x^2) + 11 (-9 + 5x) = 0

$\Rightarrow$ 35 - 21x + 12 - $4x^2$ - 99 + 55x = 0

$\Rightarrow$ - $4x^2$ + 34x - 52 = 0

$\div$ by - 2

$2x^2 - 17x + 26 = 0$

$2x^2 - 4x - 13x + 26 = 0$

$2x(x-2) - 13(x-2) = 0$

$(2x - 13)(x - 2) = 0$

$x = 2, \frac{13}{2}$

15. $\begin{vmatrix} x & 2 & -1 \\ 2 & 5 & x \\ -1 & 2 & x \end{vmatrix} = 0$

x(5x - 2x)-2(2x + x)-1 (4 + 5) = 0

$3x^2$ - 6x - 9 = 0

x^2 - 2x - 3 = 0 (Divided by 3)

(x - 3)(x + 1) = 0

$x = +3, -1$

16. $= \begin{vmatrix} 10041 & 10042 & 10043 \\ 10045 & 10046 & 10047 \\ 10049 & 10050 & 10051 \end{vmatrix}$

Applying the elementary transformation $R_2 \to R_2 \to R_1$, $R_3 \to R_3 \to R_2$ we get

$\begin{vmatrix} 10041 & 10042 & 10043 \\ 4 & 4 & 4 \\ 8 & 8 & 8 \end{vmatrix} = 0$ Since $R_2 \sim R_3$

17. $= \begin{vmatrix} 5 & 5^2 & 5^3 \\ 5^2 & 5^3 & 5^4 \\ 5^4 & 5^5 & 5^6 \end{vmatrix}$

Taking 5 and 5^2 common from R_1 and R_2

$5 \times 5^2 \begin{vmatrix} 1 & 5 & 5^2 \\ 1 & 5 & 5^2 \\ 5^4 & 5^5 & 5^6 \end{vmatrix} = 0$ (Since $R_1 = R_2$)

18. 2x + 3y = 5

x - 2y = -1

The given system can be written as

$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

AX = B $\Rightarrow X = A^{-1}B$

Where A = $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$,B= $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$

$|A| = -4 - 3 = -7 \neq 0$

$\therefore$ A^{-1} exists.

$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{-1}{7}\begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}$

$X = A^{-1}B$

QB 365 TM Study Material
QUESTION BANK 365

$= \frac{-1}{7}\begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$= \frac{-1}{7}\begin{pmatrix} -10 & +3 \\ -5 & -2 \end{pmatrix} = \frac{-1}{7}\begin{pmatrix} -7 \\ -7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solution is $x = 1, y = 1$

19. a_{11} = 500 a_{12} = 1600 x_1 = 2500

a_{21} = 1750 a_{22} = 1600 x_2 = 8000

$b_{11} = \frac{a_{11}}{x_1} = \frac{500}{2500} = 0.20$; $b_{12} = \frac{a_{12}}{x_2} = \frac{1600}{8000} = 0.20$

$b_{21} = \frac{a_{21}}{x_1} = \frac{1750}{2500} = 0.70$;$b_{22} = \frac{a_{22}}{x_2} = \frac{1600}{8000} = 0.20$

The technology matrix is $\begin{bmatrix} 0.2 & 0.2 \\ 0.7 & 0.2 \end{bmatrix}$

I - B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.2 & 0.2 \\ 0.7 & 0.2 \end{bmatrix}$

= $\begin{bmatrix} 0.8 & -0.2 \\ -0.7 & 0.8 \end{bmatrix}$,elements of main diagonal are positive

Now, |I - B| = $\begin{vmatrix} 0.8 & -0.2 \\ -0.7 & 0.8 \end{vmatrix}$

= (0.8)(0.8) – (–0.7)(–0.2)

= 0.64 – 0.14

= 0.50 >0

Since diagonal elements of I - B are positive and $|I - B|$ is positive,

Hawkins – Simon conditions are satisfied

Hence this system has a solution.

2 Marks

20. $a_{11} = 15, \quad a_{12} = 10, \quad x_1 = 35$

$a_{21} = 20, \quad a_{22} = 30, \quad x_2 = 65$

$b_{11} = \frac{a_{11}}{x_1} = \frac{15}{35} = \frac{3}{7}; \quad b_{12} = \frac{a_{12}}{x_2} = \frac{10}{65} = \frac{2}{13}$

$b_{21} = \frac{a_{21}}{x_1} = \frac{20}{35} = \frac{4}{7}; \quad b_{22} = \frac{a_{12}}{x_2} = \frac{30}{65} = \frac{6}{13}$

B = $\begin{bmatrix} \frac{3}{7} & \frac{2}{13} \\ \frac{4}{7} & \frac{6}{13} \end{bmatrix}$

$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{3}{7} & \frac{2}{13} \\ \frac{4}{7} & \frac{6}{13} \end{bmatrix} = \begin{bmatrix} \frac{4}{7} & \frac{-2}{13} \\ \frac{4}{7} & \frac{7}{13} \end{bmatrix}$

$|I - B| = \frac{4}{7} \times \frac{7}{13} - \left(\frac{2}{13} \times \frac{4}{7}\right) = \frac{28-8}{91} = \frac{20}{91}$

Since the diagonal elements of I - B are positive and |I - B| is positive the system is available

$(I - B)^{-1} = \frac{1}{|I-B|}\text{adj(I} - \text{B)} = \frac{91}{20}\begin{pmatrix} \frac{7}{13} & \frac{2}{13} \\ \frac{4}{7} & \frac{4}{7} \end{pmatrix}$

Now, X = $(I - B)^{-1}$ D where D = $\begin{bmatrix} 12 \\ 18 \end{bmatrix} = \frac{91}{20}\begin{bmatrix} \frac{7}{13} & \frac{2}{13} \\ \frac{4}{7} & \frac{4}{7} \end{bmatrix}\begin{bmatrix} 12 \\ 18 \end{bmatrix} = \frac{91}{20}\begin{pmatrix} \frac{84+36}{13} \\ \frac{48+72}{7} \end{pmatrix}$

$= \begin{pmatrix} 42 \\ 78 \end{pmatrix}$

The gross output for two sectors X and Y are 42 and 78 respectively

21. B = $\begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix}$

I - B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.30 \\ -0.41 & 0.67 \end{bmatrix}$

= (0.50) (0.67) - (0.30) (0.41)

$|I - B|$ = 0.335 - 0.123 = 0.212 > 0

Since the main diagonal elements of I - B are positive and |I-B| is positive. Hawkins Simon conditions are satisfied. Therefore given system is viable

22. B = $\begin{bmatrix} 0.6 & 0.9 \\ 0.20 & 0.80 \end{bmatrix}$

I - B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.6 & 0.9 \\ 0.20 & 0.80 \end{bmatrix} = \begin{bmatrix} 0.4 & -0.9 \\ -0.20 & 0.20 \end{bmatrix}$

|I - B| = $\begin{bmatrix} 0.4 & -0.9 \\ -0.20 & 0.20 \end{bmatrix}$

= 0.08 - 0.18 = - 0.1 < 0

Since |I - B| is negative, Hawkins - Simon conditions are not satisfied.

Therefore the given system is not viable.

23. The technology matrix is

$B = \begin{bmatrix} 0.50 & 0.25 \\ 0.40 & 0.67 \end{bmatrix}$

I - B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.50 & 0.25 \\ 0.40 & 0.67 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.25 \\ -0.40 & 0.33 \end{bmatrix}$

|I – B| = 0.165 - 0.1 = 0.065 > 0

Since the diagonal elements of (I - B) are positive and |I - B| is positive, Hawkins - Simon conditions are satisfied.

Therefore the given system is viable.

24. Let A = $\begin{vmatrix} 5 & 20 \\ 0 & -1 \end{vmatrix}$

Minor of 5 = M_{11} = -1

Minor of 20 = M_{12} = 0

QB 365 Study Material

Minor of 0 = M_{21} = 20
Minor of -1 = M_{22} = 5
Co-factor of 5 = A_{11} = -1
Co-factor of 20 = A_{12} = 0
Co-factor of 0 = A_{21} = -20
Co-factor of -1 = A_{22} = 5

25. $B = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{2}{5} & \frac{2}{3} \end{pmatrix}$

$I - B = \begin{pmatrix} \frac{1}{2} & \frac{-1}{4} \\ \frac{-2}{5} & \frac{1}{3} \end{pmatrix}$

Main diagonal element in I - B are positive

$|I - B| = \frac{1}{6} - \frac{2}{20} = \frac{8}{120} > 0$

$\therefore$ 2 Hawkin-Simon conditions are satisfied.

Hence the system is viable.

QB365 TM Study Material
QUESTION BANK 365