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Answers Key:

5 Marks

1. Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 7 \\ 1 & -1 & 1 \end{bmatrix}$

$= (4 + 7) - 1(3 - 7) + 1(-3 - 4)$

$= 11 + 4 - 7 = 8$

Co-factor matrix $= \begin{pmatrix} 11 & 4 & -7 \\ -2 & 0 & +2 \\ 3 & -4 & 1 \end{pmatrix}$

adj $A = \begin{pmatrix} 11 & -2 & 3 \\ 4 & 0 & -4 \\ -7 & 2 & 1 \end{pmatrix}$

$A(\text{adj } A) = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & 7 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 11 & -2 & 3 \\ 4 & 0 & -4 \\ -7 & 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} 11+4-7 & -2+0+2 & 3-4+1 \\ 33+16-49 & -6+0+14 & 9-16+7 \\ 11-4-7 & -2+0+2 & 3+4+1 \end{pmatrix}$

$= \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} = 8 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = |A| I_3$

$(adj\ A)\ A = \begin{bmatrix} 11 & -2 & 3 \\ 4 & 0 & -4 \\ -7 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 7 \\ 1 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 11-6+3 & 11-8-3 & 11-14+3 \\ 4+0-4 & 4+0+4 & 4+0-4 \\ -7+6+1 & -7+8-1 & -7+14+1 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ ...(2)

$|A|.\ I_3 = 8 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ ...(3)

From (1), (2) and (3)

A (adj A) = (adj A) A = $|A|I_3$.

2. Let x, y, z denote the cost of 1 kg of Wheat, Sugar and Rice respectively.

2x + y = 70

x + z = 70

3x + 2y + z = 17

The given system can be written as

$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 70 \\ 70 \\ 170 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

Where $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 70 \\ 70 \\ 170 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 2 \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 0 \begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix}$

$= 2(0 - 2) - 1(1 - 3) + 0 = -4 + 2 + 0 = -2 \neq 0$

$\therefore A^{-1}$ exists

Co-factor matrix $= \begin{pmatrix} -2 & 2 & 2 \\ -1 & 2 & -1 \\ 1 & -2 & -1 \end{pmatrix}$

adj $A = \begin{pmatrix} -2 & -1 & 1 \\ 2 & 2 & -2 \\ 2 & -1 & -1 \end{pmatrix}$

$A^{-1} = \frac{1}{|A|} adj\ A = \frac{-1}{2} \begin{bmatrix} -2 & -1 & 1 \\ 2 & 2 & -2 \\ 2 & -1 & -1 \end{bmatrix}$

$X = A^{-1}B = \frac{-1}{2} \begin{bmatrix} -2 & -1 & 1 \\ 2 & 2 & -2 \\ 2 & -1 & -1 \end{bmatrix} \begin{bmatrix} 70 \\ 70 \\ 170 \end{bmatrix} = \frac{-1}{2} \begin{bmatrix} -40 \\ -60 \\ -100 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 50 \end{bmatrix}$

$\therefore$ Cost of 1kg of Wheat, Sugar and Rice are Rs. 20, Rs. 30 and Rs. 50 respectively.

3. $a_{11} = 50, a_{12} = 75, x_1 = 200$

$a_{21} = 100, a_{22} = 50, x_2 = 200$

$b_{11} = \frac{a_{11}}{x_1} = \frac{50}{200} = \frac{1}{4}; b_{12} = \frac{a_{12}}{x_{12}} = \frac{75}{200} = \frac{3}{8}$

$b_{21} = \frac{a_{11}}{x_1} \frac{100}{200} = \frac{1}{2}; b_{22} = \frac{a_{22}}{x_2} = \frac{50}{200} = \frac{1}{4}$

$\therefore$ The technology matrix is B $= \begin{bmatrix} \frac{1}{4} & \frac{3}{8} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$

$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{4} & \frac{3}{8} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{-3}{8} \\ -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$

$|I - B| = \frac{9}{16} - \frac{3}{16} = \frac{6}{16} = \frac{3}{8} > 0$

Now, X = (I - B)$^{-1}$ D Where D $= \begin{bmatrix} 300 \\ 600 \end{bmatrix}$

$X = \frac{8}{3}\begin{pmatrix} \frac{1}{4} & \frac{3}{8} \\ \frac{1}{2} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} 300 \\ 600 \end{pmatrix}$

$= \frac{8}{3}\begin{pmatrix} 75 + 225 \\ 150 + 150 \end{pmatrix} = \begin{pmatrix} 800 \\ 800 \end{pmatrix}$

The output of P and Q must be 800 each.

4. Given A $= \begin{bmatrix} 3 & 7 \\ 2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$= \begin{bmatrix} 18 + 49 & 24 + 63 \\ 12 + 35 & 16 + 45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

$|AB| = \begin{bmatrix} 61 & 87 \\ 47 & 67 \end{bmatrix} = 4087 - 4089 = -2 \neq 0 (AB)^{-1}$ exists.

$\Rightarrow \ (AB)^{-1} = \frac{1}{|AB|} adj(AB) = \frac{-1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

$|B| = 54 - 56 = -2 \neq 0$

$\therefore$ B^{-1} exists

$B^{-1} = \frac{1}{|B|} adj\ B = -\frac{1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$|A| = 15 - 14 = 1 \neq 0$

$\therefore$ A^{-1} exists

$A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{-1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= \frac{-1}{2}\begin{bmatrix} 45 + 16 & -63 - 24 \\ -35 - 12 & 49 + 18 \end{bmatrix}$

$= -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

(AB)$^{-1}$ = B^{-1} A^{-1}.

5. Let x, y, z be the salary of each type of staff in A, B and C categories

4x + 2y + 3z = 4900

3x + 3y + 2z = 4500

4x + 3y + 4z = 5800

In matrix form $\begin{bmatrix} 4 & 2 & 3 \\ 3 & 3 & 2 \\ 4 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4900 \\ 4500 \\ 5800 \end{bmatrix}$

$\Rightarrow AX = B \Rightarrow X = A^{-1}B$

Where A $= \begin{bmatrix} 4 & 2 & 3 \\ 3 & 3 & 2 \\ 4 & 3^4 & \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4900 \\ 4500 \\ 5800 \end{bmatrix}$

= 4 (12 - 6) - 2(12 - 8) + 3 (9 - 12)

= 4 (6) - 2 (4) + 3 (-3) = 24 - 8 - 9 = 7 = 1

$\therefore$ A^{-1} exists.

$Co - factormatrix = \begin{bmatrix} 6 & -4 & -3 \\ 1 & 4 & -4 \\ -5 & 1 & 6 \end{bmatrix}$

$\therefore \ A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{7}\begin{bmatrix} 6 & 1 & -5 \\ -4 & 4 & 1 \\ -3 & -4 & 6 \end{bmatrix}$

$\therefore \ X = A^{-1}B = \frac{1}{7}\begin{bmatrix} 6 & 1 & -5 \\ -4 & 4 & 1 \\ -3 & -4 & 6 \end{bmatrix}\begin{bmatrix} 4900 \\ 45000 \\ 5800 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 29400 & +45000 & -29,000 \\ -19600 & +18000 & +5800 \\ -14700 & -18,000 & +34800 \end{bmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7}\begin{bmatrix} 49000 \\ 4200 \\ 2100 \end{bmatrix} = \begin{bmatrix} 700 \\ 600 \\ 300 \end{bmatrix}$

Salary for each type of staff under category A, B and C are respectively Rs 700,Rs 2600 and Rs 7300

3 Marks

6. Expanding along R_1 we get,

$\begin{vmatrix} 2 & x & 3 \\ 4 & 1 & 6 \\ 1 & 2 & 7 \end{vmatrix} = 0$

$\Rightarrow$ 2 (7 -12) -x (28- 6) + 3 (8 - 1) = 0

$\Rightarrow$ -10 - 22x + 21 = 0

$\Rightarrow$ 11 = 22x

$\Rightarrow x = \frac{11}{22} = \frac{1}{2}$

7.

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2x + 3y = 5

x - 2y = -1

The given system can be written as

$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix}=\begin{bmatrix} 5 \\ -1 \end{bmatrix}$

AX = B $\Rightarrow X = A^{-1}B$

Where A = $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$,B= $\begin{bmatrix} 5 \\ -1 \end{bmatrix}$

$|A| = -4 - 3 = -7 \neq 0$

$\therefore$ A^{-1} exists.

$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{-1}{7}\begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}$

$X = A^{-1}B$

$= \frac{-1}{7}\begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$= \frac{-1}{7}\begin{pmatrix} -10 & +3 \\ -5 & -2 \end{pmatrix} = \frac{-1}{7}\begin{pmatrix} -7 \\ -7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solution is $x = 1, y = 1$

8. $AB = \begin{pmatrix} 2 & 3 \\ 1 & -6 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ 1 & -2 \end{pmatrix}$

$= \begin{pmatrix} -2+3 & 8-6 \\ -1-6 & 4+12 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ -7 & 16 \end{pmatrix}$

$\text{LHS} = \text{adj(AB)} = \begin{pmatrix} 16 & -2 \\ 7 & 1 \end{pmatrix}$

$\text{RHS} = (\text{adj } B)(\text{adj A})$

$= \begin{pmatrix} -2 & -1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} -6 & -3 \\ -1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 12+4 & 6-5 \\ 6+1 & 3-2 \end{pmatrix} = \begin{pmatrix} 16 & -2 \\ 7 & 1 \end{pmatrix}$

$adj(AB) = (adj\ B)(adj\ A)$

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2 Marks

9. $B = \begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix}$

$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.30 \\ -0.41 & 0.67 \end{bmatrix}$

= (0.50) (0.67) - (0.30) (0.41)

$|I - B|$ = 0.335 - 0.123 = 0.212 > 0

Since the main diagonal elements of I - B are positive and |I-B| is positive. Hawkins Simon conditions are satisfied. Therefore given system is viable

10. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5 \neq 0$

$\therefore$ A^{-1} exists

Now, $A^{-1} = \frac{1}{|A|}adj\ A = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$

11. Let A = $\begin{bmatrix} 8 & 2 \\ 4 & 3 \end{bmatrix}$

|A| = $\begin{vmatrix} 8 & 2 \\ 4 & 3 \end{vmatrix}$

= 24 − 8 = 16 $\neq 0$

$\therefore$ A is a non-singular matrix