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## Answers Key:

**ANSWER ALL QUESTIONS**

1. $a_{11} = 30, a_{12} = 40, x_1 = 120$
   $a_{21} = 20, a_{22} = 10, x_2 = 60$
   $b_{11} = \frac{a_{11}}{x_1} = \frac{30}{120} = \frac{1}{4}$
   $b_{12} = \frac{a_{12}}{x_2} = \frac{40}{60} = \frac{2}{3}$
   $b_{21} = \frac{a_{21}}{x_1} = \frac{20}{120} = \frac{1}{6}$
   $b_{22} = \frac{a_{22}}{x_1} = \frac{10}{60} = \frac{1}{6}$
   The technology matrix is $B = \begin{bmatrix} \frac{1}{4} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$
   $I - B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{4} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & -\frac{2}{3} \\ -\frac{1}{6} & \frac{5}{6} \end{bmatrix}$
   $|I - B| = \frac{3}{4} \times \frac{5}{6} - \frac{2}{3} \times \frac{1}{6} = \frac{5}{8} - \frac{1}{9} = \frac{37}{72} = 0$
   Since diagonals of I - B are positive and | I - B | is positive, the system is viable
   $(I - B)^{-1} = \frac{1}{|I-B|} adj\,(I - B) = \frac{72}{37} \begin{bmatrix} \frac{5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{3}{4} \end{bmatrix}$
   X = $(I - B)^{-1}$ D where D = $\begin{bmatrix} 80 \\ 40 \end{bmatrix}$
   $= \frac{72}{37} \begin{bmatrix} \frac{5}{6} & \frac{2}{3} \\ \frac{1}{6} & \frac{3}{4} \end{bmatrix} \begin{bmatrix} 80 \\ 40 \end{bmatrix}$
   $= \frac{72}{37} \begin{pmatrix} 66.67 & +26.67 \\ 13.33 & +30 \end{pmatrix} = \frac{72}{37} \begin{pmatrix} 93.34 \\ 43.33 \end{pmatrix}$
   $= \begin{pmatrix} 181.63 \\ 84.32 \end{pmatrix}$
   The output for production section X and Y are 181.63 and 84.32 respectively.
2. Given equations are 2x - z = 0, 5x + y = 4 and y + 3z = 5
   Given equations can be written in matrix form as
   $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$
   $\Rightarrow$ AX = B
   Where $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$
   $|A|$ = 2 ( 3 - 0 ) - 0 - 1 ( 5 - 0 ) = 6 - 5 = $1 \neq 0$
   $\therefore$ $A^{-1}$ exists.
   Co-factor matrix $= \begin{pmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{pmatrix}$
   $A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{1} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$
   $\therefore\ X = A^{-1}B = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \\ 5 \end{bmatrix}$
   $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 4 + 5 \\ 0 + 24 - 25 \\ 0 - 8 + 10 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$
   $x = 1, y = -1, z = 2$
3. Given $A = \begin{bmatrix} 3 & 7 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$
   $= \begin{bmatrix} 18 + 49 & 24 + 63 \\ 12 + 35 & 16 + 45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$
   $|AB| = \begin{bmatrix} 61 & 87 \\ 47 & 67 \end{bmatrix} = 4087 - 4089 = -2 \neq 0 (AB)^{-1}$ exists.
   $\Rightarrow\ (AB)^{-1} = \frac{1}{|AB|} adj(AB) = \frac{-1}{2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$
   $|B| = 54 - 56 = -2 \neq 0$
   $\therefore$ $B^{-1}$ exists
   $B^{-1} = \frac{1}{|B|} adj\ B = -\frac{1}{2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$
   $|A| = 15 - 14 = 1 \neq 0$
   $\therefore$ $A^{-1}$ exists
   $A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{1} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$
   $B^{-1}A^{-1} = \frac{-1}{2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= \frac{-1}{2}\begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$

$= -\frac{1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1}$.

4. $a_{11} = 10, a_{12} = 25, x_1 = 50$

$a_{21} = 20, a_{22} = 30\ x_2 = 60$

$b_{11} = \frac{a_{11}}{x_1} = \frac{10}{50} = \frac{1}{5}, b_{12} = \frac{a_{12}}{x_2} = \frac{25}{60} = \frac{5}{12}$

$b_{21} = \frac{a_{21}}{x_1} = \frac{20}{50} = \frac{2}{5}, b_{22} = \frac{a_{22}}{x_1} = \frac{30}{60} = \frac{1}{2}$

$\therefore$ The technology matrix is

$B = \begin{bmatrix} \frac{1}{5} & \frac{5}{12} \\ \frac{2}{5} & \frac{1}{2} \end{bmatrix}$

$I - B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{1}{5} & \frac{5}{12} \\ \frac{2}{5} & \frac{1}{2} \end{pmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{-5}{12} \\ \frac{-2}{5} & \frac{1}{2} \end{bmatrix}$, elements of main diagonal are positive

Now, $|I - B| = \begin{vmatrix} \frac{4}{5} & -\frac{5}{12} \\ -\frac{2}{5} & \frac{1}{2} \end{vmatrix}$

$= \frac{7}{30} > 0$

Main diagonal elements of $|I - B|$ are positive and $|I - B|$ is positive

$\therefore$ The problem has a solution

$\text{adj}(I - B) = \begin{bmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix}$

$(I - B)^{-1} = \frac{1}{|I-B|}\text{ adj }(I - B)$

(Since $|I - B| \neq 0, (I - B)^{-1}$ exist)

$= \frac{1}{\frac{7}{30}}\text{adj}(I - B)$

$= \frac{30}{7}\begin{bmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 15 & \frac{25}{2} \\ 12 & 24 \end{bmatrix}$

$X = (I - B)^{-1}\text{D}$, where $\text{D} = \begin{bmatrix} 35 \\ 42 \end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix} 15 & \frac{25}{2} \\ 12 & 24 \end{bmatrix}\begin{bmatrix} 35 \\ 42 \end{bmatrix} = \begin{bmatrix} 150 \\ 204 \end{bmatrix}$

The output of industry $P_1$ should be Rs. 150 crores and $P_2$ should be Rs. 204 crores.

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**ANSWER ALL QUESTIONS**

5. Expanding the given determinant along $R_1$ we

$\begin{vmatrix} 7 & 4 & 11 \\ -3 & 5 & x \\ -x & 3 & 1 \end{vmatrix} = 0$

$\Rightarrow 7(5 - 3x) - 4(-3 + x^2) + 11(-9 + 5x) = 0$

$\Rightarrow 35 - 21x + 12 - 4x^2 - 99 + 55x = 0$

$\Rightarrow -4x^2 + 34x - 52 = 0$

$\div$ by $-2$

$2x^2 - 17x + 26 = 0$

$2x^2 - 4x - 13x + 26 = 0$

$2x(x - 2) - 13(x - 2) = 0$

$(2x - 13)(x - 2) = 0$

$x = 2, \frac{13}{2}$

6. $2x + 3y = 5$

$x - 2y = -1$

The given system can be written as

$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

Where $A = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

$|A| = -4 - 3 = -7 \neq 0$

$\therefore A^{-1}$ exists.

$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{-1}{7}\begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}$

$X = A^{-1}B$

$= \frac{-1}{7}\begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$= \frac{-1}{7}\begin{pmatrix} -10 & +3 \\ -5 & -2 \end{pmatrix} = \frac{-1}{7}\begin{pmatrix} -7 \\ -7 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solution is $x = 1, y = 1$

7. $A = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$

$|A| = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$

=16 ≠ 0

Since A is a nonsingular matrix, $A^{-1}$ exists

Now adj $A = \begin{bmatrix} 2 & -4 \\ 3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} adjA$

$= \frac{1}{16}\begin{bmatrix} 2 & -4 \\ 3 & 2 \end{bmatrix}$

8. $a_{11}$ = 500 $a_{12}$ = 1600 $x_1$ = 2500

$a_{21}$ = 1750 $a_{22}$ = 1600 $x_2$ = 8000

$b_{11} = \frac{a_{11}}{x_1} = \frac{500}{2500} = 0.20$; $b_{12} = \frac{a_{12}}{x_2} = \frac{1600}{8000} = 0.20$

$b_{21} = \frac{a_{21}}{x_1} = \frac{1750}{2500} = 0.70$;$b_{22} = \frac{a_{22}}{x_2} = \frac{1600}{8000} = 0.20$

The technology matrix is $\begin{bmatrix} 0.2 & 0.2 \\ 0.7 & 0.2 \end{bmatrix}$

I - B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.2 & 0.2 \\ 0.7 & 0.2 \end{bmatrix}$

= $\begin{bmatrix} 0.8 & -0.2 \\ -0.7 & 0.8 \end{bmatrix}$ ,elements of main diagonal are positive

Now, |I - B| = $\begin{vmatrix} 0.8 & -0.2 \\ -0.7 & 0.8 \end{vmatrix}$

= (0.8)(0.8) – (–0.7)(–0.2)

= 0.64 – 0.14

= 0.50 >0

Since diagonal elements of I - B are positive and $|I - B|$ is positive,

Hawkins – Simon conditions are satisfied

Hence this system has a solution.

**ANSWER ALL QUESTIONS**

9. B = $\begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix}$

I - B = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.50 & 0.30 \\ 0.41 & 0.33 \end{bmatrix} = \begin{bmatrix} 0.50 & -0.30 \\ -0.41 & 0.67 \end{bmatrix}$

= (0.50) (0.67) - (0.30) (0.41)

$|I - B|$ = 0.335 - 0.123 = 0.212 > 0

Since the main diagonal elements of I - B are positive and |I–B| is positive. Hawkins Simon conditions are satisfied. Therefore given system is viable

10. Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5 \neq 0$

$\therefore A^{-1}$ exists

Now, $A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{5}\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}$

11. $\begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$ = 1 (Minor of 1) –2 (Minor of 2) + 4 (Minor of 4)

$= 1\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} - 2\begin{vmatrix} -1 & 0 \\ 4 & 0 \end{vmatrix} + 4\begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix}$

= 0 – 0 – 52 = –52.

12. $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$

⇒(x – 1)(x – 2)(x – 3) = 0

x = 1, x = 2, x = 3

**Multiple Choice Question**

13. $\begin{vmatrix} 0 & 1 & 0 \\ x & 2 & x \\ 1 & 3 & x \end{vmatrix} = 0 \Rightarrow -1\left[x^2 - x\right] = 0$

$\Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$

14. Co-factor of $-7 = \begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix} = 0 + 18 = 18$

15. (Since $|kA| = k^n|A|$, n is the order of matrix A

16. $|A| = \frac{1}{ad-bc}$; $A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

17. Since $|A| = 4 - 4 = 0$

18. $|A| = 4 - 2 = 2$

$A(\text{adj } A) = |A|I = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

19. (b) $\frac{1}{det(A)}$

20. $|A| = \cos^2\theta + \sin^2\theta = 1$

$|2A| = 2^2(1) = 4$

21. $\begin{vmatrix} 20 & 15 \\ 15 & 5 \end{vmatrix} = 5 \times 5\begin{vmatrix} 4 & 3 \\ 3 & 1 \end{vmatrix}$

$= 5 \times 5 \times -5$

$= -125$

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22. Inverse of identity matrix is identity matrix itself and $(k\ \mathrm{A})^{-1} = \frac{1}{k}I^{-1} = \frac{1}{k}I$ and also. If A is a scalar matrix with scalar k, then A =kI. Thus $\mathrm{A}^{-1} = (k\mathrm{I})^{-1} = \frac{1}{k}I$.

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