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Answers Key:

III. Answer the following.

1. X = {a, b, c, d}
   R = {(a, a), (b, b), (a, c)}
   (i) To make R reflexive we need to include (c, c) and (d, d)
   (ii) To make R symmetric we need to include (c, a)
   (iii) R is transitive
   (iv) To make R reflexive we need to include (c, c)
   To make R symmetric we need to include (c, c) and (c, a) for transitive
   ∴ The relation now becomes
   R = {(a, a), (b, b), (a, c), (c, c), (c, a)}
   ∴ R is equivalence relation.
2. The relation is defined by aRb if a + b $\leq$ 6 for all a, b $\in$N.
   a+b $\leq$6 $\Rightarrow$ a $\leq$ 6 - b

a	5	4	3	2	1
b	1	2	3	4	5

   ∴ The list of ordered pairs are (5,1) (4, 2) (3, 3) (2, 4) and (1, 5).
   Hence, R is not an equivalence relation.
3. Given A = {a, b,c}
   (i) Let R = {(a, a) (b, b) (c,c)}
   R is reflexive
   R is symmetric and R is transitive $\Rightarrow$ R is an equivalence relation.
   This is the equivalence relation of smallest cardinality on A.
   ∴ n(R) = 3
   (ii) Let R = {(a, a) (a, b) (a,c) (b,a) (b, b) (b, c) (c, a) (c, b) (c,c)}
   R is reflexive since (a, a) (b, b) and (c, c) $\in$ R
   R is symmetric since (a, b) $\in$ R $\Rightarrow$(b, a) $\in$ R
   (b, c) $\in$ R $\Rightarrow$ (c, a) $\in$ R
   (c, a) $\in$ R $\Rightarrow$ (a, c) $\in$R
   R is also transitive since (a, b) (b, c) $\in$ R $\Rightarrow$ (a, c) $\in$R
   Hence R is are equivalence relation of largest cardinality on A.
   ∴ n(R) = 9

II. Answer the following.

4. Given n(p(A)) = 1024 = 2^{10}
   $\Rightarrow$ n(A) = 10 [∴ if n(A) = n, then n(p(A)) = 2^n]
   n(p(B)) = 32 = 2^5
   $\Rightarrow$ n(B) = 5.
   We know that,
   n(A$\cup$ B) = n(A) + n(B) - n(A$\cap$B)
   $\Rightarrow$ 15 = 10+5 - (A$\cap$B)
   $\Rightarrow$ n(A$\cap$B) = 0.
5. To find n(P(A)), we need n(A)
   Let $n(A \cap B) = k$. Then $n(A - B) = 2k$ and $n(B - A) = 4k$
   Now $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B) = 7k$.
   It is given that $n(A \cup B) = 14$. Thus 7k = 14 and hence k = 2
   So n(A - B) = 4 and n(B - A) = 8. As $n(A) = n(A - B) + n(A \cap B)$, we get n(A) = 6 and hence n(P(A)) = 2^6 = 64

I. Answer the following.

6. The set of all positive roots of the equation (x-1)(x+1)(x^2-1)=0.
   Let B = { the set of positive roots of the equation (x-1)(x+10(x^2-1)=0}
   $\Rightarrow$ x = 1, -1
   B = {1}.
7. n(A$\times$ A) = 16. $\Rightarrow$ n(A) = 4
   Given S = {a,b) $\in$ A$\times$A : a
   ∴ A = {0, 1, 2, -1}
   (A$\times$A) = {(0, 0)(0, 1)(0, 2)(0, -1),(1, 1) (1, 2)(1, -1)(2, 0)(2, 1)(2, 2)(2, -1)(-1, 0)(-1, 1)(-1, 2)(-1, -1)}
   Now, S = {(0, 1)(0, 2)(-1, 0)(-1, 1)(-1, 2)}
   ∴ The remaining elements of S are (0, 2)(-1, 0)(-1, 1)(1, 2)(0, 1) (1, 2)(0, 2)