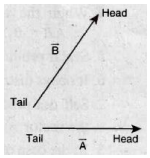
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Answers Key:

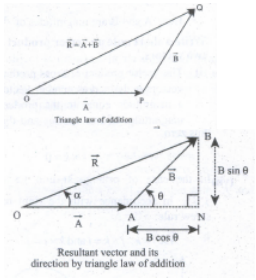
5 Marks

1. If two vectors which are inclined to each other are represented in magnitude and direction by the two adjacent sides of a triangle taken in order, then their resultant is the closing side of the triangle taken in the reverse order.

Let us consider two vectors, $\vec{A}$ and $\vec{B}$ inclined an angle θ with each other as shown in figure



To find the resultant, the head of the first vector $\vec{A}$ is connected to the tail of the second vector $\vec{B}$. Let θ be the angle between them. The resultant vector $\vec{R}$ is the line drawn connecting the tail of the first vector, $\vec{A}$ to the head of the second vector $\vec{B}$. Thus $\vec{R}$ = $\vec{A}$+ $\vec{B}$. The magnitude is $\vec{R}$ (resultant) is given geometrically by the length of $\vec{R}(\theta)$. The direction of the resultant is the angle between $\vec{R}$ and $\vec{A}$.
To find the magnitude of the resultant, the triangle ABN is obtained by extending OA to ON and drawing the perpendicular drop from B.



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ABN is a right angled triangle. From the figure, $\cos\ \theta = \frac{AN}{B}\ \therefore AN = B\ \cos\ \theta$
$\sin\ \theta = \frac{BN}{B}\ \therefore BN = B\ \sin\theta$
For $\triangle OBN$, we have $OB^2 + ON^2 + BN^2$
$R^2 = (A + B\ \cos\theta)^2 + (B\sin\theta)^2$
$= A^2 + B^2 cos^2\theta + 2AB\ \cos\theta + B^2\sin^2\theta$
$= A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\ \cos\theta$
$R\ = \sqrt{A^2 + B^2 + 2AB\cos\theta}$
Let $\vec{R}$ makes an angle α with $\vec{A}$, then in $\triangle OBN$,
$\tan\ \alpha = \frac{BN}{ON} + \frac{BN}{OA+AN} = \frac{B\ sin\theta}{A+B\ cos\theta}$
$\therefore \alpha = \tan^{-1}\left[\frac{B\sin\theta}{A+B\cos\theta}\right]$

2. **Scalar product:**
Scalar product or dot product of two vectors in defined as the product of the magnitudes of both the vectors and the cosine of the angle between them.
If $\vec{A}$ and $\vec{B}$ are two vector having an angle θ between them, then $\vec{A}\vec{B} = AB\cos\theta$ where A and B are magnitudes of $\vec{A}$ and $\vec{B}$.
Example: work, energy and electric flux.
Properties:
1. $\vec{A}.\vec{B}$ is always a scalar. It is positive if θ<90 and it is negative if $90^\circ < \theta < 180^\circ$
2. When the vectors are parallel, $\theta = 0^\circ\ and\ \cos 0^\circ = 1\ \therefore (\vec{A}\cdot\vec{B})_{mat}\ = AB$.
3. When the vectors are anti-parallel, $\theta = 180^\circ\ and\ \cos 180^\circ = -1\ \therefore (\vec{A}\cdot\vec{B})_{min} = -AB$
4. When the vectors are perpendicular to each other, $\theta = 90^\circ\ and\ \cos 90^\circ = 0\ \therefore \vec{A}\vec{B} = 0$.
5. Scalar product is commutative i.e, $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
6. It obeys distributive law i.e., $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
7. Self dot product is given by $\vec{A}\cdot\vec{A} = AA\cos\theta = A^2,\ here\ \theta = 0^\circ$. The magnitude of the vector $\vec{A}\ is\ (\vec{A}) = A = \sqrt{\vec{A}\cdot\vec{A}}$
8. In the case of orthogonal unit vectors $\vec{i}, \vec{j}\ and\ \vec{k}$
$\hat{i}\hat{j} = \hat{j}\hat{j} = \hat{k}\cdot\hat{k} = 1$ and
$\vec{i}\cdot\vec{j} = \hat{j}\hat{k} = \hat{k}\hat{i} = 0$
9. The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{AB}\right]$
10. In terms of components,
$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$
$= A_xB_x + A_yB_y + A_iB_z$, with all other terms zero.
The magnitude of A is given by $|\vec{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ and $|\vec{B}| = B = \sqrt{B_x^2 + B_y^2 + B_z^2}$ Vector product:
The vector product or cross product of two vectors is defined as another vector having a magnitude equal to the product of the magnitudes of two vectors and the sine of the angle between them.
If $\vec{A}$ and $\vec{B}$ are two vectors, then $\vec{A}\times\vec{B} = \vec{C} = (AB\sin\theta)\hat{n}$.

The direction $\hat{n}\ of\ \vec{A}\times\vec{B}$ is perpendicular to the plane containing the vectors $\vec{A}$ and $\vec{B}$ and is determined by the right hand screw rule or right hand thumb rule.

Example: Torque $\tau=\vec{r}\times\vec{F}$ and Angular momentum $\vec{L}=\vec{r}\times\vec{p}$

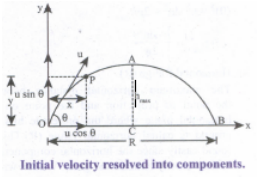
Properties:

1. The resultant of the vector product is always another vector whose direction is perpendicular to the plane containing these two vectors $\vec{A}$ and $\vec{B}$ even though the vectors $\vec{A}$ and $\vec{B}$ may or may not be mutually orthogonal.
2. It is not commutative. $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$. But $\vec{A}\times\vec{B}=-[\vec{B}\times\vec{A}]$
3. When the vectors $\vec{A}$ and $\vec{B}$ are orthogonal to each other the vector product will have maximum magnitude as $\theta=90^{\circ}\ and\ \sin\theta=1$.
$(\vec{A}\times\vec{B})_{\max}=AB\hat{n}$
4. The vector product of two non-zero vectors will be minimum when (sin θ)=0, i.e., $\theta=0^{\circ}\ or\ 180^{\circ}(\vec{A}\times\vec{B})_{\min}=0$.
It means that the vector product of two non-zero vectors vanishes if the vectors are parallel or anti parallel.
5. The self-cross product is a null vector. $\vec{A}\times\vec{A}=AA\sin 0^{\circ}\hat{n}=\overrightarrow{0}$
6. The self-vector products of unit vectors are then zero $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$.
7. In the case of orthogonal unit vectors, $\vec{i},\vec{j}\ and\ \hat{k}$
$\hat{i}\times\hat{j}=\hat{k},\hat{j}\times\hat{k}=\hat{i}\ and\ \hat{k}\times\hat{i}=\hat{j}$ and
$\hat{j}\times\hat{i}=-\hat{k},\hat{k}\times\hat{j}=-\hat{i}\ and\ \hat{i}\times\hat{k}=-\hat{j}$
8. In terms of components,

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\A_x&A_y&A_z\\B_x&B_y&B_z\end{vmatrix}=\begin{matrix}+\hat{i}\,(A_yB_z-A_zB_y)\\+\hat{j}\,(A_zB_x-A_xB_z)\\+\hat{k}\,(A_0B_y-A_zB_x)\end{matrix}$$

9. If two vectors $\vec{A}$ and $\vec{B}$ form adjacent sides of a parallelogram, then magnitude $(\vec{A}\times\vec{B})$ is equal to the area of the parallelogram.
10. If two vectors $\vec{A}$ and $\vec{B}$ are represented by the two sides of a triangle taken in order, then the area of the triangle is equal to $\frac{1}{2}|\vec{A}\times\vec{B}|$

3. Consider an object thrown with initial velocity $\overrightarrow{u}$ at an angle θ with the horizontal.
Then,
$\overrightarrow{u}=u,\overrightarrow{i}+u,\overrightarrow{j}$
where u_x =u cos θ is the horizontal component and u_y = u sin θ the vertical component of velocity.



Initial velocity resolved into components.

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Since the acceleration due to gravity acts in the direction opposite to the direction of vertical component u_y, this component will gradually reduce to zero at the maximum height of the projectile. At this maximum height, the same gravitational force will push the projectile to move downward and fall to the ground.
But, there is no acceleration along the x direction throughout the motion. So, the horizontal component of velocity (u_x = u cos θ) remains the same till the object reaches the ground.
After anytime t, the velocity along horizontal motion
$v_x = u_x + a_xt = u_x$ = u cosθ [$\therefore a_x = 0$]
The horizontal distance travelled by projectile in time t is
$s_x = u_xt+\frac{1}{2}a_nt^2$
[$\therefore s_x$ = x and a_n = 0]
$\therefore$ t = $\frac{x}{u\cos\theta}$ .....(1)
For the vertical motion the velocity after time t is $v_y = u_y + a_yt$
v_y = u sin θ - gt.....(2)
($\therefore u_g$ = u sin θ and a_y = - g)
The vertical distance travelled by the projectile in the same time r is
$s_y = u_yt\frac{1}{2}a_yt^2$
$y = \sin\theta\ t-\frac{1}{2}gt^2$
(Here, (Here, s_y = y, u_y = u sin θ and a_y = - g)
Substitute the value of t from equation (1) in equation (3), we have
$y=u\ \sin\theta\frac{x}{u\ \cos\theta}-\frac{1}{2}g\frac{x^2}{u^2cos^2\theta}$
$y=x\ \tan\theta-\frac{1}{2}g\frac{x^2}{u^2cos^2\theta}$
Thus the path followed by the projectile is an inverted parabola.

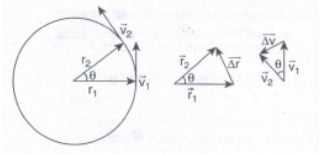
2. Maximum height: The maximum vertical distance travelled by the projectile during its journey is called maximum height.
For the vertical part of the motion. $v_y^2=u_y^2+2a_ys$
Here, u_y = u sin θ, a_y = -g, S = h_{max} and at the maximum height v_y = 0.
Hence,
$\therefore$ 0 = $u^2\sin^2$ = $2gh_{max}$
$h_{max}=\frac{u^2sin^2\theta}{2g}$

3. Horizontal Range (R): The maximum horizontal distance between the point of projection and the point on the horizontal plane where the projectile hits the ground is called horizontal range (R).
Range R = Horizontal component of velocity x time of flight
= u cos $\theta\times T_f$

$R = u\ \cos\theta \times \frac{2u\ \sin\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g}$ $\quad [\because T_f = \frac{2u\ sin\theta}{g}]$

$\therefore\ R = \frac{u^2 \sin 2\theta}{g}$

4. Consider a uniform circular motion of a point object moving on a circular path of radius r. In uniform circular motion, the velocity vector turns continuously without changing its magnitude (speed). Let v be its velocity.
The velocity is tangential at every point in the circle. But the acceleration 'a' is acting towards the centre of the circle. This is called centripetal acceleration. It always points towards the centre of the circle.
Let the direction of position and velocity vectors shift through the same angle θ in a small interval of time $\triangle t$ as shown in figure. For uniform circular motion $r = |\overrightarrow{r_1}| = r_2|$ and $vr = |\overrightarrow{v_1}| = \overrightarrow{v_2}|$



If the particle moves from position vector $\overrightarrow{r_1}$ to $\overrightarrow{r_2}$, the displacement is $\triangle\overrightarrow{r} = \overrightarrow{r_2} - \overrightarrow{r_1}$ and change in velocity from $\overrightarrow{v_1}$ to $\overrightarrow{v_2}$ is $\triangle = \overrightarrow{v_1} = \overrightarrow{v_2} - \overrightarrow{v_1}$ then the magnitude of the displacement $\triangle r\ and\ of \triangle v$ satisfy the following relation:
$\frac{\triangle r}{r} = -\frac{\triangle v}{v} = \theta$
Here the negative sign implies that $\triangle r$ points radially inward, towards the centre of the circle
$\triangle v = -v.\ \frac{\triangle r}{r}$
Then $a = \frac{\triangle r}{\triangle t} = \frac{r}{r}\left(\frac{\triangle r}{\triangle t}\right) = -\frac{v^2}{r}$
For uniform circular motion v = ωr. where ω - angular velocity of the object about the centre. Then the centripetal acceleration can be written as
$a = -\omega^2 r$

5. Consider an object moving in a straight line with uniform or constant acceleration 'a'. Let u be the velocity of the body at a time t = 0 and v be the velocity at a later time t.
1. Velocity - time relation: The acceleration of the body at any instant is given by the first derivative of the velocity with respect to time.
$a = \frac{dv}{dt}$ or $dv = adt$
Integrating both sides with the condition that an time changes from 0 to t, the velocity changes from u to v. For the constant acceleration,
$\int_u^v dv = a\int_0^t dt \Rightarrow [v]_u^r = a[t]_0^t$
$v - u =$ at or $v = u + at \ldots$ (1)
2. Displacement - time relation: The velocity of the body is given by the first derivative of the displacement with respect to time.
Since,
$v = \frac{ds}{dt}$ or $ds = vdt$
v = u + at
ds = (u + at) dt
Assume that initially at time t=0, the particle started from the origin and at a later time t, the particle displacement is s. Further assuming that acceleration is time independent, we have
$\int_0^t ds = \int_0^t udt + \int_0^t atdt$
$s = at + \frac{1}{2}at^2 \quad \ldots$ (2)
3. Velocity - displacement relation: The acceleration is given by the first derivative of velocity with respect to time.
$a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}$, v Since $\frac{ds}{dt} = v$ where s is displacement.
This is rewritten as, $a = \frac{1}{2}\frac{dv^2}{ds}$ or $ds = \frac{1}{2a}d\left(v^2\right)$
Integrating using the fact that when velocity changes from u² to v², displacement changes from 0 to s. We get
$\int_0^s ds = \int_u^v \frac{1}{2a}d\left(v^2\right)$
$s = \frac{1}{2a}\left(v^2 - u^2\right)$
$v^2 = u^2 + 2as$ .....(3)
4. $s = \frac{(u+v)t}{2}$
From the eqn. (1) a t = v-u
Substitute this is eqn. (2), we get
$s = ut + \frac{1}{2}(v-u)t$
$s = \frac{(u+v)t}{2}$ .....(4)

3 Marks

6. (i) Displacement is the difference between the final and initial positions of the object in a given interval of time. It can also be defined as the shortest distance between these two positions of the object and its direction is from the initial to final position of the object, during the given interval of time. It is a vector quantity.
(ii) Distance is the actual path length travelled by an object in the given interval of time during the motion. It is a positive scalar quantity.

7. **Velocity:**
Velocity is equal to the rate of change of position vector with respect to time.
It is a vector quantity $\overrightarrow{v} = \frac{d\overrightarrow{r}}{dt}$
Speed:
The magnitude of velocity is called speed and is given by $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$. It is a positive scalar.

8.

Velocity	Average Velocity
It refers to the rate of change of position vector with respect to time.	It refers to the rate of change of displacement vector with respect to time
$\overrightarrow{v} = \lim_{\Delta t \to 0} \frac{\Delta\overrightarrow{r}}{\Delta t}$	$\overrightarrow{v}_{avg} = \frac{\Delta\overrightarrow{r}}{\Delta t}$

9.

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Angular displacement: While a particle is revolving around a point in a circular path, the angle described by the particle about the axis of rotation (or at the centre of the circle) in a given time is called angular displacement. Its unit is radian.
Angular velocity: The rate of change of angular displacement is called angular velocity. Its unit is rad s^{-1}
$\omega = \frac{d\theta}{dt}$

10.

1. $\omega = \omega_0 + \alpha t$	ω = Final angular velocity
2. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	ω_0 = initial angular velocity
3. $\omega^2 = \omega_0^2 + 2\alpha\theta$	θ = Angular displacement
4. $\theta = \frac{(\omega_0+\omega)t}{2}$	α = angular acceleration t = time

11. $\vec{r} = 3\hat{i} + 2\hat{j}$
Unit vector $= \frac{\vec{r}}{|\vec{r}|}$
$|\vec{r}| = \sqrt{3^2 + 2^2} = \sqrt{13}$
$\therefore$ Unit vector $= \frac{3\hat{i}+2\hat{j}}{\sqrt{13}}$
12. 360º degrees in 24 hours.
$\therefore$ angular displacement $= \frac{360}{24} = 15^\circ$ *or* $\frac{2\pi}{24} = \frac{\pi}{12}$ radius.
13. 360º = 27 days
1 day = $\frac{360^\circ}{27} = 13^\circ.3'$
14. mass = m
$\alpha = 0.2 \text{rads}^{-2}$
$\omega_0 = 0$
t = 3s
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$= 0 + \frac{1}{2} \times 0.2 \times 9$
= 0.9 rad
or $= 0.9 \times \frac{180}{22} \times 7 = 51.5^\circ$

2 Marks

15. (i) Cartesian Coordinate system is a frame of reference in which the position of an object at any given instant is described in terms of its distances along x, y and z axes.
(ii) Conventionally right - handed Cartesian Coordinate system where the x, y and z axes are drawn in anti clockwise direction is followed in physics.
16. (i) A quantity which is described by both its magnitude and direction is called a vector quantity.
(ii) Geometrically, a vector is a directed line segment
Examples: Force, velocity displacement, acceleration, position vector, linear momentum and angular momentum.
17. Scalar is a property which can be described only by magnitude.
Examples:
Distance, mass, temperature, speed and energy.
18. The acceleration of a particle at any instant 'r' is equal to the rate of change of velocity.
Acceleration $\vec{a} = \frac{d\vec{v}}{dt}$. It is a vector quantity.
19. When a point object is moving on a circular path if the velocity of the object changes both in speed (magnitude) and direction the motion is said to be non uniform circular motion.
20. $\vec{A} = 4\hat{i} - 2\hat{j} + \hat{k}$
$\vec{B} = 5\hat{i} + 3\hat{j} - 4\hat{k}$
Resultant vector = $\vec{A} + \vec{B}$
$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 1 \\ 5 & 3 & -4 \end{vmatrix}$
$= \hat{i}(8-3) + \hat{j}[5-(-16)] + \hat{k}(12+10)$
Resultant vector = $5\hat{i}$ +$21\hat{j}$ + $22\hat{k}$
21. Max.height = $\frac{u^2 sin^2\theta}{2g}$
Range $R = \frac{u^2 sin2\theta}{2g}$
Here, R = 4 x Max.height
$\therefore \frac{u^2 sin2\theta}{g} = \frac{4 \times u^2 sin^2\theta}{2g}$
$sin2\theta = 2sin^2\theta$
$2sin\theta cos\theta = 2sin^2\theta$
$\therefore cos\theta = sin\theta$
$\therefore \theta = 45^\circ$
22. Sprinkled water takes circular motion.
The centripetal acceleration $= \frac{v^2}{r}$. The centripetal acceleration is provided by the acceleration due to gravity.
$\therefore g = \frac{v^2}{r}$ *or* $r = \frac{v^2}{g}$
$\therefore$ Area around the fountain that gets wet = πr^2
$= \pi\left(\frac{r^2}{g}\right)^2 = \frac{\pi r^4}{g^2}$
23. h = 100m
u = 10ms^{-1}

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Time to reach the ground $t = \sqrt{\frac{2h}{g}}$

∴ Horizontal range = $u_x \times t$

$= u_x\sqrt{\frac{2h}{g}}$

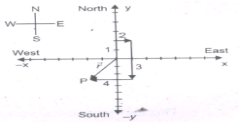
$= 10 \times \sqrt{\frac{2\times100}{9.8}}$

= 10 ×4.51 = 45.1 m

24. $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ 3 & -2 & 4 \end{vmatrix}$

$\vec{\tau} = (12-(10))\hat{i} + (15-8)\hat{j} + (-4-9)\hat{k}$

$\vec{\tau} = 22\hat{i} + 7\hat{j} - 13\hat{k}$

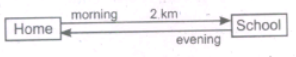


25.

As shown in the Figure, the positive x-axis is taken as east direction, positive y-direction is taken as north. After the trip, the person reaches the point P whose position vector given by

$\vec{r} = -2\hat{i} - 3\hat{j}$

The displacement direction is south west.



26.

The displacement covered is zero. It is because your initial and final positions are the same. But the distance travelled is 4 km.

27. The total distance the athlete covered = 3×circumference of track

Distance = 3× 2π× 50 = 300π m (or)

Distance = 300×3.14 = 942 m

The displacement is zero, since the athlete reaches the same point A after three rounds from where he started.



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28. We use p = mv

For the mass of 10 g, m = 0.01 kg

p = 0.01×10 = 0.1 kg m s^{-1}

For the mass of 1 kg

p = 1×10 = 0.1 kg m s^{-1}

Thus even though both the masses have the same speed, the momentum of the heavier mass is. 100 times greater than that of the lighter mass.

29. $u = 5ms^{-1}$

$\theta = 30^\circ$

$h_{\max} = \frac{u^2\sin^2\theta}{2g} = \frac{25\times(\frac{1}{2})^2}{2\times9.8} = 0.318$ m

Range $= \frac{u^2\sin2\theta}{g} = \frac{25\times\sqrt{3}}{9.8\times2} = 2.21$ m