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Answers Key:

PART - D

Answer all the questions

1. Given C(x) = $\int C'(x)dx+k_1$
   = $\int \left(50+\frac{x}{50}\right)dx+k_1$
   C(x) = 50x + $\frac{x^2}{100}+k_1$
   When quantity produced is zero, then the fixed cost is 200.
   i.e. When x = 0, c = 200
   $\Rightarrow k_1 = 200$
   Cost function is C(x) = 50x + $\frac{x^2}{100}$ + 200 (1)
   The Revenue R'(x) = 60
   R(x) = $\int R'(x)dx + k_2$
   $\int 60dx + k_2$
   = 60x + k_2
   When no product is sold, revenue = 0
   i.e. When x = 0, R = 0
   Revenue R(x) = 60x (2)
   Profit P = Total Revenue – Total cost
   = 60x - 50x - $\frac{x^2}{100}$ - 200
   = 10x - $\frac{x^2}{100}$ - 200
   $\frac{dp}{dx} = 10 - \frac{x}{50}$
   To get profit maximum, $\frac{dp}{dx}$ = 0 $\Rightarrow$ x = 500
   $\frac{d^2P}{dx^2} = \frac{-1}{50} < 0$
   ∴ Profit is maximum when x = 500 and
   Maximum Profit is P = 10(500) - $\frac{(500)^2}{100}$ - 200
   = 5000 – 2500 – 200
   = 2300
   Profit = Rs. 2,300.
2. We know that,
   $\int_{-\infty}^{\infty} f(x)dx = 1$
   $\int_{-\infty}^{\infty} ke^{-|x|}dx = 1$
   $k\int_{-\infty}^{\infty} ke^{-|x|}dx = 1$
   $2k\int_{-\infty}^{\infty} e^{-|x|}dx = 1$ ($\because x^2e^{-|x|}$ is an function)
   $2k\int_0^{\infty}\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$
   $k = \frac{1}{2}$
   Mean of the random variable is
   $E(X) = \int_{-\infty}^{\infty} xf(x)dx$
   $E(X) = \int_{-\infty}^{\infty} xke^{-|x|}dx$ ($\because xe^{-|x|}$ is an odd function of x)
   $= \frac{1}{2}\int_{-\infty}^{\infty} xe^{-|x|}$
   = 0
   $E\left(x^2\right) = \int_{-\infty}^{\infty} x^2f(x)dx$
   $= \int_{-\infty}^{\infty} x^2ke^{-|x|}dx$
   $= \frac{1}{2}\int_{-\infty}^{\infty} x^2e^{-|x|}dx$
   $= \int_0^{\infty} x^2e^{-x}$ ($\because x^2e^{-|x|}$ is an even function)
   $= \Gamma 3\left(\because \Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1}e^{-x}dx, \alpha > 0; \Gamma n = (n-1)!\right)$
   = 2
   $V(X) = E\left(x^2\right) - [E(X)]^2$
   $= 2 - [0]^2$
   = 2
3. Let I = $\int_0^3 \frac{xdx}{\sqrt{x+1}+\sqrt{5x+1}}$
   $I = \int_0^3 \frac{x\left(\sqrt{x+1}-\sqrt{5x+1}\right)}{\left(\sqrt{x+1}+\sqrt{5x+1}\right)\left(\sqrt{x+1}-\sqrt{5x+1}\right)}$
   = $\int_0^3 \frac{x\left(\sqrt{x+1}-\sqrt{5x+1}\right)dx}{(x+1)-(5x+1)}$
   $\left[\because (a+b)(a-b) = a^2-b^2\right]$

$= \int_0^3 \frac{x\left(\sqrt{x+1}-\sqrt{5x+1}\right)dx}{x+1-5x-1}$

$= \int_0^3 \frac{x\left(\sqrt{x+1}-\sqrt{5x+1}\right)dx}{-4x}$

$= -\frac{1}{4}\int_0^3 \left(\sqrt{x+1}-\sqrt{5x+1}\right)dx$

$= -\frac{1}{4}\left[\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(5x+1)^{\frac{3}{2}}}{5\left(\frac{3}{2}\right)}\right]_0^3$

$= -\frac{1}{4}\left[\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{1}{15}(5x+1)^{\frac{3}{2}}\right]_0^3$

$= -\frac{1}{2}\left[\frac{1}{3}(x+1)^{\frac{3}{2}} - \frac{1}{15}(5x+1)^{\frac{3}{2}}\right]_0^3$

$= -\frac{1}{2}\left[\frac{1}{3}(4)^{\frac{3}{2}} - \frac{1}{15}(16)^{\frac{3}{2}}\right] - \left[\frac{1}{3}(1)^{\frac{3}{2}} - \frac{1}{15}(1)^{\frac{3}{2}}\right]$

$= -\frac{1}{2}\left[\left(\frac{1}{3}(4)\sqrt{4} - \frac{1}{15}16\sqrt{16}\right) - \left(\frac{1}{3} - \frac{1}{15}\right)\right]$

$= \frac{1}{2}\left[\left(\frac{8}{3} - \frac{64}{15}\right) - \frac{1}{3} + \frac{1}{5}\right]$

$= -\frac{1}{2}\left[\frac{8}{3} - \frac{64}{15} - \frac{1}{3} + \frac{1}{15}\right]$

$= -\frac{1}{2}\left[\frac{8}{3} - \frac{1}{3} - \frac{64}{15} + \frac{1}{15}\right]$

$= -\frac{1}{2}\left[\frac{7}{3} - \frac{63}{15}\right] = -\frac{1}{2}\left[\frac{105-189}{45}\right]$

$= -\frac{1}{2}\left[\frac{-84}{45}\right] = \frac{42}{45} = \frac{14}{15}$

$\therefore I = \frac{14}{15}$

4. The given differential equation is of this form

$\frac{dy}{dx}$ +Py=Q where

P = tan x; Q = $\cos^3 x$

$\therefore \int pdx = \int \tan x dx$ = log secx

$\therefore$ Integrating factor (I.F) = $e^{\int pdx}$ = $e^{\log x \sec x}$

= sec x

Hence, the solution is

$ye^{\int pdx} = \int Q.e^{\int pdx}dx + c$

$\Rightarrow$ y sec x = $\int cos^3 x$dx+c

[$\therefore$ cos 2x = $2\cos^2 x$-1]

$\Rightarrow$ $2\cos^2 x = 1 + \cos 2x$

$\Rightarrow$ $\cos^2 x = \frac{1+cos2x}{2}$]

$\Rightarrow$ y secx = $\int \cos^3 x. \frac{1}{\cos x}$dx+c

$\Rightarrow$y sec x = $\int cos^2 x dx + c$

$\Rightarrow$ y sec x = $\int \left(\frac{1+cos2x}{2}\right)dx + c$

$\Rightarrow$ y sec x = $\frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]$ +c

5. The forward interpolation formula is

$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!} = \triangle y_0 + \frac{n(n-1)}{2!}\triangle^2 y_0 + \frac{n(n-1)(n-2)}{3!}\triangle^3 y_0 + \ldots$

Here x_0 + nh = x $\Rightarrow$ x_0 = 0, h = 1

$\therefore$ 0 + n = x $\Rightarrow$ n = x.

The difference table is

x	y = f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		1		
1	2		-2	
		-1		12
2	1		10	
		9		
3	10			

$y_{(n=x)} = y_0 + \frac{n}{1!} = \triangle y_0 + \frac{n(n-1)}{2!}\triangle^2 y_0 + \frac{n(n-1)(n-2)}{3!}\triangle^3 y_0 + \ldots$

$y = 1 + \frac{n}{1!}(1) + \frac{n(n-1)}{2!}(-2) + \frac{n(n-1)(n-2)}{6}(12)$

$\Rightarrow$ y = 1 + x + (x^2 - x)(-1) + x(x^2 - 3x + 2) (2)

$\Rightarrow$ y = 1 + x - x^2 + x + $2x^3$ - $6x^2$ + 4x

$\Rightarrow$ y = 1+ 6x - $7x^2$ + $2x^3$

Hence, the cubic polynomial is $2x^3$ - $7x^2$+ 6x + 1.

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6. $\Delta = \begin{vmatrix} 1 & 4 & 3 \\ 2 & -6 & 6 \\ 5 & -2 & 3 \end{vmatrix}$

$= 1\begin{vmatrix} -6 & 6 \\ -2 & 3 \end{vmatrix} - 4\begin{vmatrix} 2 & 6 \\ 5 & 3 \end{vmatrix} + 3\begin{vmatrix} 2 & -6 \\ 5 & -2 \end{vmatrix}$

= 1(-18 + 12) - 4(6 - 30) +3 (- 4 +30)

= 1(- 6) - 4(- 24) + 3(26)

= - 6 + 96 + 78 = 168 $\neq 0$

Since $\Delta \neq 0$ the system is consistent with unique solution and Cramer's rule can be applied

$\Delta x = \begin{vmatrix} 2 & 4 & 3 \\ -3 & -6 & 6 \\ -5 & -2 & 3 \end{vmatrix}$

$= 2\begin{vmatrix} -6 & 6 \\ -2 & 3 \end{vmatrix} - 4\begin{vmatrix} -3 & 6 \\ -5 & 3 \end{vmatrix} + 3\begin{vmatrix} -3 & -6 \\ -5 & -2 \end{vmatrix}$

= 2 (- 18 + 12) - 4(- 9 +30) + 3(6 -30)

= 2(- 6) - 4(21) + 3(- 24)

= -12-84-72 =-168

$\Delta y = \begin{vmatrix} 1 & 2 & 3 \\ 2 & - & 6 \\ 5 & -5 & 3 \end{vmatrix} = 1\begin{vmatrix} -3 & 6 \\ -5 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & 6 \\ 5 & 3 \end{vmatrix} + 3\begin{vmatrix} 2 & -3 \\ 5 & -5 \end{vmatrix}$

= 1 (-9+30)-2(6-30)+3(- 10+ 15)

= 1(21) - 2(- 24) + 3(5)

= 21 + 48 + 15 = 84

$\Delta z = \begin{vmatrix} 1 & 4 & 2 \\ 2 & -6 & -3 \\ 5 & -2 & -5 \end{vmatrix}$

$= 1\begin{vmatrix} -6 & -3 \\ -2 & -5 \end{vmatrix} - 4\begin{vmatrix} 2 & -3 \\ 5 & -5 \end{vmatrix} + 2\begin{vmatrix} 2 & -6 \\ 5 & -2 \end{vmatrix}$

= 1(30-6)-4(-10+ 15)+2(-4+30)

= 24 - 4(5) + 2(26)

= 24 - 20 + 52 = 56

$x = \frac{\Delta x}{\Delta} = \frac{-168}{168} = -1$

$y = \frac{\Delta y}{\Delta} = \frac{84}{168} = \frac{1}{2}$

$z = \frac{\Delta z}{\Delta} = \frac{56}{168} = \frac{1}{3}$

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Solution set is $\left\{-1, \frac{1}{2}, \frac{1}{3}\right\}$

PART - C

Answer the following questions

7. Let the hourly charge for horse riding be Rs. x and the hourly charge for quad bike be Rs. y from the given data,

3x + 4y = 780

2x+ 3y = 560

$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = 3(3) - 2(4) = 9 - 8 = 1 \neq 0$

Since $\Delta \neq 0$ the system is consistent with unique solution and Cramer's rule can be applied

$\Delta x = \begin{vmatrix} 780 & 4 \\ 560 & 3 \end{vmatrix} = 780(30 - 4(560)$

= 2340 - 2240 = 100

$\Delta y = \begin{vmatrix} 3 & 780 \\ 2 & 560 \end{vmatrix} = 39560) - 2(780)$

= 1680 − 1560 = 120

$\therefore x = \frac{\Delta x}{\Delta} = \frac{100}{1} = 100$

$y = \frac{\Delta y}{\Delta} = \frac{120}{1} = 120$

$\therefore$ Hourly charges for the two rides are Rs. 100 and Rs. 120 respectively.

8. Given MC = 2 + $5e^x$

$C = \int MC dx + k$

$= \int (2 + 5e^x) dx + k$

= 2x + $5e^x$ + k

x = 0 ⇒ C = 100,

100 = 2(0)+ 5(e^0)+ k

k = 95

C = 2x + 5 e^x + 95.

9. The sample space (S) = { (HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}

X takes the values: 3, 2, 2, 1, 2, 1, 1, and 0

Range of X(Rx)	0	1	2	3
$P_x(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
$F_x(x)$	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	1

Thus, we have

$$F_x(x)=\begin{cases}0, & for\ x<0\\ \frac{1}{8}, & for\ 0\le x<1\\ \frac{4}{8}, & for\ 1\le x<2\\ \frac{7}{8}, & for\ 2\le x<3\\ 1, & for\ x\le 3\end{cases}$$

10. Let $I = \int (x+1)^2 \log x\, dx$

Let $u = \log x$ $dv = (x+1)^2 dx$

$du = \frac{1}{x}dx \quad v = \frac{(x+1)^3}{3}$

$\therefore$ Using integration by parts,

$I = \int udv = uv - \int vdu$

$I = \int (x+1)^2 \log x dx$

$= \frac{(x+1)^3}{3}\log x - \int \frac{(x+1)^3}{3}.\frac{1}{x}dx$

$= \frac{(x+1)^3}{3}\log x - \frac{1}{3}\int \frac{x^3+3x^2+3x+1}{x}$

$= \frac{(x+1)^3}{3}\log x - \frac{1}{3}\int \left(x^2+3x+3+\frac{1}{x}\right)dx$

$= \frac{(x+1)^3}{3}\log x - \frac{1}{3}\left[\frac{x^3}{3}+\frac{3x^2}{2}+3x+\log|x|\right]+c$

$= \left[(x+1)^3\log x - \frac{x^3}{3}-\frac{3x^2}{2}-3x-log|x|\right]+c$

11. Since only four values of f(x) are given, the polynomial which fits the data is of degree 3.

Hence fourth differences are zero.

$\therefore \Delta^4 (y_0) = 0$

$\Rightarrow (E-1)^4 (y_0) = 0$

$\Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1)\, y_0 = 0$

$\Rightarrow y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$\Rightarrow 81 - 4(y_3) + 6(9) - 4(3) + 1 = 0$

$\Rightarrow 81 - 4y_3 + 54 - 12 + 1 = 0$

$\Rightarrow 81 + 54 - 11 = 4y_3$

$\Rightarrow 124 = 4y_3 \Rightarrow \frac{124}{4} = 31$

$\Rightarrow y_3 = 31$

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12. Given $\frac{dy}{dx} = 1 + \frac{y}{x}$

$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = 1 + \frac{vx}{x}$

$x\frac{dv}{dx} = 1$

$\int dv = \int \frac{dx}{x}$

$v = \log|x| + c$

$y/x = \log|x| + c$

It passes through (1, 0)

0 = c

Required equation $\frac{y'}{x} = \log|x|$

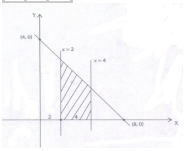
$y = x\log|x|$

PART- B

Answer the following questions

13. 2y + x = 8

x	0	8
y	4	0



Given 2y + x = 8

2y = 8-x

y = $\frac{1}{2}$ (8-x)

Given limits are x = 2 and x = 4

Area of the shaded region between the given limits

$A = \int_a^b y \quad dx = \int_2^4 \frac{1}{2}(8-x)dx$

$\frac{1}{2}\int_2^4 (8-x)dx = \frac{1}{2}\left[8x - \frac{x^2}{x}\right]_2^4$

$= \frac{1}{2}\left[\left(8(4) - \frac{4^2}{2}\right)\left(8(2) - \frac{2^2}{2}\right)\right]$

$= \frac{1}{2}[(32-8)-(16-21)]$

$= \frac{1}{2}[24-14]$

$= \frac{1}{2} \times 2(10)$

A = 5 sq. units.

14. $Let\ I = \int \frac{6x+7}{\sqrt{3x^2+7x-1}}dx$

$Let\ t = 3x^2+7x-1$

$\Rightarrow dt = (6x+7)dx$

$\therefore I = \int \frac{dt}{\sqrt{t}} = \int t^{\frac{1}{2}}dt$

$= \frac{t^{\frac{1}{2}+1}}{-\frac{1}{2}+1} + c = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2\sqrt{t} + c$

$= 2\sqrt{3x^2+7x-1} + c$

$\left[\because t = 3x^2+7x+c\right]$

15. LHS = V[aX] = E[aX - E(aX)]2

= E(aX-a.E(X)]2[∵E(aX) = a.E(X)]

= E[aX-a.E(X)]2[∵E(aX) = a.E(X)]

= E[a{X-E(X)}]^2

= a^2.E[X-E(X)]^2 = a^{2}E(X-$\bar{x}$)^2

= a^2.[E(X)^2-[E(X)]^2]

= a^2.V(X) [∵V(X) = E(X^2)-[E(X)]^2]

= RHS Hence proved.

16. Given (9D^2-12D + 4)y = 0

The auxiliary equation is (3m - 2)^2 = 0

(3m-2) (3m-2) = 0 ⇒ m = $\frac{2}{3}, \frac{2}{3}$

Roots are real and equal

The C.F. is (Ax + B)$e^{\frac{2}{3}x}$

The general solution is y = (Ax + B)$e^{\frac{2}{3}x}$

17. Given non-homogeneous equations are x + y + z = 6, x + 2y + 3z = 14, x + 4y + 7z = 30

Augmented matrix	Elementary Transformation
$\begin{pmatrix} 1 & 2 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{pmatrix}$	
$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{pmatrix}$	$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$
$-\begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_1$

Here $\rho(A) = \rho(A,B) = 2$

∴ The given system is consistent

18. $y = f(x) = x^2 + 2x + 2$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1	5			
		5		
2	10		2	
		7		
3	17		2	
		9		0
4	26		2	
		11		
5	37			

PART - A

Choose the correct answer

19. (d) $\left(\frac{\triangle_1}{\triangle_2}, \frac{\triangle_1}{\triangle_3}\right)$

20. (b) $\frac{1}{2}$ sq.unit

21. (a) zero

22. (a) probability distribution

23. (c) $\int_a^b f(x)dx$

24. (b) $\frac{1}{x}$

25. (b) 0

26. (a) e^2-1

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27. (b) 1
28. (b) 0

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