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Answers Key:

5 Marks

1. The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & a \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ b \end{pmatrix}$$

AX = B

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{pmatrix}$	$R_2 \to R_2 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{pmatrix}$	$R_3 \to R_3 - R_1$

Case (i) For no solution:
The system possesses no solution only when $\rho(A) \neq ([A, B])$ which is possible only when a–3 = 0 and b –10 $\neq$ 0.
Hence for a = 3, b $\neq$ 10, the system possesses no solution.
Case (ii) For a unique solution:
The system possesses a unique solution only when $\rho(A) = ([A, B])$=number of unknowns.
i.e when $\rho(A) = \rho([A, B])$ = 3
Which is possible only when a–3 $\neq$ 0 and b may be any real number as we can observe .
Hence for a $\neq$ and b $\in$ R, the system possesses a unique solution.
Case (iii) For an infinite number of solutions:
The system possesses an infinite number of solutions only when
$\rho(A) = \rho([A, B])$ i,e when $\rho(A) = \rho([A, B])$ = 2 < 3 (number of unknowns) which is possible only when a - 3 = 0, b–10 = 0
Hence for a = 3, b = 10, the system possesses infinite number of solutions.

2. Transition probability matrix

$$T = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} \cdot 6 & \cdot 4 \\ \cdot 2 & \cdot 8 \end{pmatrix} \end{array}$$

By the given data
A = 50% = ·5
B = 50% = ·5
Shares after one week

$$(\cdot 5 \quad \cdot 5) \begin{pmatrix} \cdot 6 & \cdot 4 \\ \cdot 2 & \cdot 8 \end{pmatrix}$$

$= (\cdot 5)(\cdot 5)(\cdot 2) \cdot 5(4) + \cdot 5(\cdot 8)$
$= (\cdot 30 + \cdot 10 \cdot 20 + 40)$
$= (\cdot 40 \quad \cdot 60)$
∴ Shares after one week for products A and Bare 40% and 60% respectively.
Shares after two weeks

$$(\cdot 4 \quad \cdot 6) \begin{pmatrix} \cdot 6 & \cdot 4 \\ \cdot 2 & \cdot 8 \end{pmatrix}$$

$((\cdot 4)(\cdot 6) + (\cdot 6)(\cdot 2) \cdot (\cdot 4)(\cdot 4) + \cdot 6(0.8))$
$= (\cdot 24 + \cdot 12 \quad \cdot 16 + \cdot 48)$
$= (\cdot 36 \quad \cdot 64)$
∴ Shares after two week for products A and Bare 36% and 64% respectively.
At equilibrium, we must have (A B) T = (A B)
where A+B = 1

$$(A \quad B) \begin{pmatrix} \cdot 6 & \cdot 4 \\ \cdot 2 & \cdot 8 \end{pmatrix} = (A \quad B)$$

$\Rightarrow$ = (–6A+·2B –4A+·8B) = (A B)
Equating the corresponding entries on both sides
we get,
·6A+ ·2B = A
$\Rightarrow$ ·6A + ·2(1 – A) = A
$\Rightarrow$ ·6A + ·2 – ·2A = A

$\Rightarrow$ ·2 = A - ·6A + ·2A
$\Rightarrow$ ·2 = A (1 - ·6 + ·2)
$\Rightarrow$ ·2 = A (·4 + ·2)
$\Rightarrow$ ·2 = A (·6)
$\Rightarrow A = \frac{\cdot 2}{\cdot 6} = \cdot 33 \Rightarrow A = 33$
and B = 1 - A = 1 - ·33 = ·67$\Rightarrow$ B = 67%
$\therefore$ Equilibrium is reached when A = 33% and B = 67%

3. Given C(x) = $\int C'(x)dx + k_1$
= $\int \left(50 + \frac{x}{50}\right)dx + k_1$
C(x) = $50x + \frac{x^2}{100} + k_1$
When quantity produced is zero, then the fixed cost is 200.
i.e. When x = 0, c = 200
$\Rightarrow k_1 = 200$
Cost function is C(x) = $50x + \frac{x^2}{100} + 200$ (1)
The Revenue R'(x) = 60
R(x) = $\int R'(x)dx + k_2$
$\int 60dx + k_2$
= $60x + k_2$
When no product is sold, revenue = 0
i.e. When x = 0, R = 0
Revenue R(x) = 60x (2)
Profit P = Total Revenue − Total cost
= 60x - 50x - $\frac{x^2}{100}$ - 200
= 10x - $\frac{x^2}{100}$ - 200
$\frac{dp}{dx} = 10 - \frac{x}{50}$
To get profit maximum, $\frac{dp}{dx}$ = 0 $\Rightarrow$ x = 500
$\frac{d^2P}{dx^2} = \frac{-1}{50} < 0$
$\therefore$ Profit is maximum when x = 500 and
Maximum Profit is P = 10(500) - $\frac{(500)^2}{100}$ - 200
= 5000 − 2500 − 200
= 2300
Profit = Rs. 2,300.

4. Given probability function is

Value of X = x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

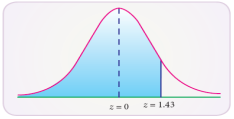
i) Since the given function is a probability function, each $\rho_i > 0$ and $\Sigma\rho_i = 1$
$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$
$\Rightarrow 10k^2+9k = 1$
$\Rightarrow 10k^2+9k-1 = 0$
on factoring we get
(k+1)(10k-1) = 0
$\Rightarrow$k = -1 or k = $\frac{1}{10}$
k = -1 is not possible [each $\rho_i > 0$]
we get k = $\frac{1}{10}$
ii) P(X<6) = P(X = 0)+P(X = 1)+P(X=2+P(X=3)+P(X=4)+P(X=5)
P(X<6) = 0 + k + 2k + 2k + 3k + k^2
= $8k+k^2$
= $8\left(\frac{1}{10}\right)\left(\frac{1}{10}\right)^2$
= $\frac{8}{10} + \frac{1}{100} = \frac{80+1}{100} = \frac{81}{100}$
$\therefore P(X < 6) = \frac{81}{100}$
Now P(X≥6) = P(X=6)+P(X=7)
= $2k^2+7k^2+k$
= $9k^2+k$
= $9\left(\frac{1}{10}\right)^2 + \frac{1}{10}$
= $\frac{9}{100} + \frac{1}{10} = \frac{9+10}{100} = \frac{19}{100}$
$\therefore P(X \geq 6) = \frac{19}{100}$
And P(0X<5) = P(X = 1)+P(X = 2)+P(X = 3)+P(X = 4)
= k+2k+2k+3k
$8k = 8\left(\frac{1}{10}\right) = \frac{8}{10}$
$\therefore P(0)$
iii) Given $P(X \leq x) \geq \frac{1}{2}$
$\Rightarrow$P(X = 0)+P(X = 1)+P(X = 2)+P(X = 3)+P(X = 4)
= 0+k+2k+2k+3k
$= 8k = 8\left(\frac{1}{10}\right) = \frac{8}{10} = \frac{4}{5} > \frac{1}{2}$
$\therefore P(X \leq 4)\frac{1}{2} \leq x = 4$
$\therefore$ The minimum value of x is 4.

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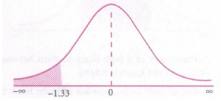
5. Given demand function P_d = 20 - 5x and
Supply function P_s = 4x + 8
Under market equilibrium $p_s = P_d$
⇒ 20-5x = 4x+8
⇒ 20-8 = 4x+5x
⇒ 12 = 9x
$\Rightarrow x = \frac{12}{9} = \frac{4}{3}$
When $x_0 = \frac{4}{3}, p_0 = 20 - 5\left(\frac{4}{3}\right) = 20 - \frac{20}{3}$
$= \frac{60-20}{3} = \frac{40}{3}$
$\therefore p_0 x_0 = \frac{40}{3} \times \frac{4}{3} = \frac{160}{9}$
Consumer Surplus (CS)
$= \int_0^x f(x)dx - p_0 x_0$
$= \int_0^{\frac{4}{3}} (20 - 5x)dx$
$= \left[20x - \frac{5x^2}{2}\right]_0^{\frac{4}{3}} - \frac{160}{9}$
$= 20\left(\frac{4}{3}\right) - \frac{5}{2}\left(\frac{16}{9}\right) - \frac{160}{9}$
$= \frac{80}{3} - \frac{40}{9} - \frac{160}{9}$
$= \frac{240-40-160}{9} = \frac{40}{9}$
$\therefore CS = \frac{40}{9}$units
Producer's Surplus
$(PS) = p_0 x_0 - \int_0^x g(x)dx$
$= \frac{160}{9} - \int_0^{\frac{4}{3}} (4x + 8)dx$
$= \frac{160}{9} - \left[\frac{4x^2}{2} + 8x\right]_0^{\frac{4}{3}}$
$= \frac{160}{9} - \left(2\left(\frac{16}{9}\right) + 8\left(\frac{4}{3}\right)\right)$
$= \frac{160}{9} - \left(\frac{32}{9} + \frac{32}{3}\right)$
$= \frac{160}{9} - \frac{32}{9} - \frac{32}{3}$
$PS = \frac{160-32-96}{9} = \frac{32}{9}$units

6. Given average number of customers, appear in a counter
λ = 2
∴ X follows a poisson distribution with
$p(x,\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$
(i) P (no customer appears)
= P(X = 0) = $\frac{e^{-\lambda}\lambda^0}{0!} = e^{-\lambda} = e^{-2}$ = 0.1353 [∵ e^{-2} = 0.1353]
(ii) P(3 or more customer appear)
= P(X≥3)
= 1-P(X<3)
= 1 - [P(X = 0) + P(X = 1) + P(X = 2)]
= $1 - \left[\frac{e^{-\lambda}\lambda^0}{0!} + \frac{e^{-\lambda}\lambda^1}{1!} + \frac{e^{-\lambda}\lambda^2}{2!}\right]$
= $1-e^{-\lambda}(1+\lambda+\frac{\lambda^2}{2})$
= $1-e^{-2}(1+2+\frac{4}{2})$
= $1-e^{-2}$ (5) = 1 - 0.1353(5)
= 1 - 0.6765 = 0.3235
Hence, probability of three or more customers appear in a counter of a certain bank is 0.3235.

7. Given μ = 68.0, σ = 3.0
(a) P(X>72)
When X = 72, Z = $\frac{X-\mu}{\sigma}$
= $\frac{72-68}{3} = \frac{4}{3}$ = 1.33
∴ P(X>72) = P(Z>1.33)
= P(0 = 0.5-0.4082 = 0.0918



∴ Prob of 1 student having height more than 72 inches is 0.0918.
Out of 500 students, number of students having height more than 72 inches is 0.0918 x 500 = 45.9
= 0.46 (approximately).
(b) P(X ≤ 64)
When X = 64, Z = $\frac{X-\mu}{\sigma}$
= $\frac{64-68}{3} = \frac{-4}{3}$ = -1.33
∴ P(X≤64) = P(Z≤-1.33)
= P(-∞ = 0.5-P(0 = 0.5-0.4082
= 0.0918



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∴ Probability of 1 student having height less than 64 inches is 0.0918

∴ Out of 500 students, number of students having height less than 64 inches is = 0.0918 x 500 = 45.9 = 46 (approximately).

(c) P(65 When X = 65, Z = $\frac{X-\mu}{\sigma}$

= $\frac{65-68}{3} = \frac{3}{3}$ = 1

When X = 71, Z = $\frac{X-\mu}{\sigma}$

= $\frac{71-68}{3} = \frac{3}{3}$ = 1

∴ P(65 = P(-1 = P(0 [By symmetry]

= 2. P(0 = 2(0.3413) = 0.6826)



∴ Probability of 1 student having height between 65 and 71 inches is 0.6826

∴ Out of 500 students, number of students having height between 65 and 71 inches is 0.6826 x 500 = 341.37

= 342 (approximately)

8. $\int_a^b f(x)dx = \lim_{n\to\infty h\to 0} \sum_{r=1}^{n} hf(a+rh)$

Here a =1, b = 2, h = $\frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$ and f (x) = x^2

Now, f (a + rh) = f$\left(1+\frac{r}{n}\right)$= $\left(1+\frac{r}{n}\right)^2 = 1+\frac{2r}{n}+\frac{r^2}{n^2}$

∴ $\int_1^2 x^2dx = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(1+\frac{2r}{n}+\frac{r^2}{n^2}\right)$

= $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{n}+\frac{2r}{n^2}+\frac{r^2}{n^3}\right)$

= $\lim_{n\to\infty} \left(\frac{1}{n}\sum_{r=1}^{n} 1+\frac{2}{n^2}\sum_{r=1}^{n} r+\frac{1}{n^3}\sum_{r=1}^{n} r^2\right)$

= $\lim_{n\leftarrow\infty} \left(\frac{1}{n}(n)+\frac{2}{n^2}\frac{n(n+1)}{2}+\frac{1}{n^3}\frac{n(n+1)(2n+1)}{6}\right)$

= $\lim_{n\to\infty} \left[1+\left(1+\frac{1}{n}\right)+\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}\right]$

= $\left[1+1+\frac{(1)(2)}{6}\right]$

∴ $\int_1^2 x^2 = \frac{7}{3}$

9. Computation of Seasonal Index by the method of simple averages.

Year	I Quarter	II Quarter	III Quarter	IV Quarter
2005	255	351	425	400
2006	269	310	396	410
2007	291	332	358	395
2008	198	289	310	357
2009	200	290	331	359
2010	250	300	350	400
Quarterly Total	1463	1872	2170	2321
Quarterly Averages	243.83	312	361.67	386.83

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S.I for I Quarter = $\frac{Average\ of\ I\ quarter}{Grand\ average} \times 100$

Grand Average = $\frac{1304.333}{4} = 326.0833$

S.I for I Q = $\frac{243.8333}{326.0833} \times 100 = 74.77$;

S.I for II Q = $\frac{312}{326.0833} \times 100 = 95.68$;

S.I for III Q = $\frac{361.6667}{326.0833} \times 100 = 110.91$;

S.I for IV Q = $\frac{386.833}{326.0833} \times 100 = 118.63$

10. Given $3e^x \tan y\,dx + (1+e^x)\sec^2 y\,dy = 0$

$3e^x \tan y\,dx = -(1+e^x)\sec^2 y\,dy$

$\frac{3e^x}{1+e^x}dx = -\frac{\sec^2 y}{\tan y}dy$

Integrating, we get $3\int \frac{3e^x}{1+e^x}dx = -\int \frac{\sec^2 y}{\tan y}dy + c$

$3\log(1+e^x) = -\log \tan y + \log c \quad \left[\because \int \frac{f'(x)}{f(x)}dx = \log f(x)\right]$

$\log(1+e^x)^3 + \log \tan y = \log c$

$\log[(1+e^x)^3 \tan y] = \log c$

$(1+e^x)^3 \tan y = c$ (1)

Given y(0) = $\frac{\pi}{4}$ (i.e) y = $\frac{\pi}{4}$ at x =0

(1) ⇒ $(1+e^0)^3 \tan\frac{\pi}{4} = c$

$2^3(1) = c$

⇒ c = 8

Hence the required solution is $(1+e^x)^3 \tan y = 8$

11. Given that $MR = \frac{dy}{dq} = \frac{q^2+3y^2}{2qy}$ (1)

Put y = vq and $\frac{dy}{dq} = v + q\frac{dv}{dq}$ in (1)

Now (1) becomes

$v + q\frac{dy}{dq} = \frac{q^2+3v^2q^2}{2yvq}$

$= \frac{1+3v^2}{2v}$

$q\frac{dv}{dq} = \frac{1+3v^2}{2v} - v$

$= \frac{1+3v^2-2v^2}{2v}$

$= \frac{1+v^2}{2v}$

$\frac{2v}{1+v^2}dv = \frac{dq}{q}$

On Integration

$\int \frac{2v}{1+v^2}dv = \int \frac{dq}{q}$

log (1+ v^2) = log q + log c

1+ v^2 = cq

Replace $v = \frac{y}{q}$

$1 + \frac{y^2}{q^2}$ = cq

$q^2 + y^2 = c\,q^3$ (2)

Given output is 1 unit and revenue is Rs. 5

∴ (2) ⇒ 1 + 25 = c ⇒ c = 26

∴ The total revenue function is $q^2 + y^2 = 26q^3$

12.

Commodity	2016		2017	
	p_0	q_0	p_1	q_1
Food	40	12	65	14
Fuel	72	14	78	20
Clothing	36	10	36	15
Wheat	20	6	42	4
Others	46	8	52	6

p_0q_0	p_1q_1	p_1q_0	p_0q_1
480	910	780	560
1008	1560	1092	1440
120	168	252	80
368	312	416	276
2336	**3490**	**2900**	**2896**

Fisher's price index number

$P_{01}^F = \sqrt{\frac{\sum p_1q_0 \times \sum p_1q_1}{\sum p_0q_0 \times \sum p_0q_1}} \times 100$

$= \sqrt{\frac{2900 \times 3490}{2336 \times 2896}} \times 100$

$= \sqrt{\frac{10121000}{6765056}} \times 100$

$= \sqrt{1.496} \times 100$

P_{01}^F = 122.31

Time Reversal Test

$P_{01} \times P_{10} = \sqrt{\frac{\sum p_1q_0 \times \sum p_1q_1 \times \sum p_0q_1 \times \sum p_0q_0}{\sum p_0q_0 \times \sum p_0q_1 \times \sum p_1q_1 \times \sum p_1q_0}}$

$= \sqrt{\frac{2900 \times 3490 \times 2896 \times 2336}{2336 \times 2896 \times 3490 \times 2900}}$

$= \sqrt{1} = 1$

Hence, it satisfies time reversal test.

13. Here $\sum a_i = \sum b_j = 80$

(i.e) Total Availability = Total Requirement

∴ The given problem is balanced transportation problem.

Hence there exists a feasible solution to the given problem.

First Allocation:

	I	II	III	IV	a_i	Penalty
A	5	1	3	3	34	2
B	3	3	5	4	15	0
C	6	4	4	3	12	1
D	4	(19) 1	4	5	19/0	3
b_j	21	25/6	17	17		
Penalty	1	0	1	0		

Second Allocation:

	I	II	III	IV	a_i	Penalty
A	5	(6) 1	3	3	34/28	2
B	3	3	5	4	15	0
C	6	4	4	3	12	1
b_j	21	6/0	17	17		
Penalty	2	2	1	0		

Third Allocation:

	I	III	IV	a_i	Penalty
A	5	3	3	28	0
B	(15) 3	5	4	15/0	1
C	6	4	3	12	1
b_j	21/6	17	17		
Penalty	2	1	0		

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Fourth Allocation:

	I	III	IV	a_i	Penalty
A	5	3	3	28	0
C	6	4	(12) 3	12/0	1
b_j	6	17	17/5		
Penalty	1	1	0		

Fifth Allocation:

	I	III	IV		Penalty
A	5	3	(5) 3	28/23	0
	6	17	5/0		
Penalty	-	-	-		

Sixth Allocation:

	I	III		Penalty
A	(6) 5	(17) 3	23/6/0	2
	6/0	17/0		
Penalty	-	-		

Thus we have the following allocations:

	I	II	III	IV	a_i
A	(6) 5	(6) 1	(17) 3	(5) 3	34
B	(15) 3	3	5	4	15
C	6	4	4	(12) 3	12
D	4	(19) 1	4	5	19
b_j	21	25	17	17	

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Transportation schedule :
A⟶I, A⟶II, A⟶III, A⟶IV, B⟶I, C⟶IV, D⟶II
Total transportation cost:
= (6×5)+(6×1)+(17×3)+(5×3)+(15×3)+(12×3)+(19×1)
= 30+6+51+15+45+36+19
= Rs.202

14. $\Delta\left[\frac{5x+12}{x^2+5x+6}\right]$

By Partial fraction method

$\frac{5x+12}{x^2+5x+6} = \frac{A}{x+3} + \frac{B}{x+2}$

$A = \frac{5x+12}{x+12}[x = -3] = \frac{-15+12}{-1} = \frac{-3}{-1} = -3$

$B = \frac{5x+12}{x+3}$[x = -2] $= \frac{2}{1} = 2$

$\frac{5x+12}{x^2+5x+6} = \left[\frac{3}{x+3} + \frac{2}{x+2}\right]$

$\Delta\frac{5x+12}{x^2+5x+6} = \Delta\left[\frac{3}{x+3} + \frac{2}{x+2}\right]$

$= \left[\frac{3}{x+1+3} - \frac{3}{x+3}\right] + \left\{\frac{2}{x+1+2} - \frac{2}{x+2}\right\}$

$= 3\left[\frac{1}{x+4} - \frac{1}{x+3}\right] + 2\left[\frac{1}{x+3} - \frac{1}{x+2}\right]$

$= \left[\frac{-3}{(x+4)(x+3)} - \frac{2}{(x+3)(2+3)}\right]$

$= \frac{-5x-14}{(x+2)(x+3)(x+4)}$

3 Marks

15. The matrix equation corresponding to the given system is

$\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$

AX = B

Matrix A	Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix}$ $\sim \begin{pmatrix} 3 & -2 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -2 & 6 \\ 6 & -4 & 10 \end{pmatrix}$ $\sim \begin{pmatrix} 3 & -2 & 6 \\ 0 & 0 & -2 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\rho(A) = 1$	$\rho([A, B]) = 2$	

$\therefore \rho([A, B]) = 2, \rho(A) = 1$

$\rho(A) \neq \rho([A, B])$

$\therefore$ The given system is inconsistent and has no solution.

16.

Let 'x' be the cost of a pencil

Let 'y' be the cost of an eraser

∴ By given data, we get the following equations

11x + 3y = 64

8x + 3y = 49

$\triangle = \begin{vmatrix} 11 & 3 \\ 8 & 3 \end{vmatrix} = 9 \neq 0$, It has unique solution.

$\triangle_x \begin{vmatrix} 64 & 3 \\ 49 & 3 \end{vmatrix} = 45$

$\triangle_y \begin{vmatrix} 11 & 64 \\ 8 & 49 \end{vmatrix} = 27$

∴ By Cramer's rule

$x = \frac{\triangle x}{\triangle} = \frac{45}{9} = 5$

$y = \frac{\triangle y}{\triangle} = \frac{27}{9} = 3$

∴ The cost of a pencil is Rs. 5 and the cost of an eraser is Rs. 3.

17. $\int \frac{1}{\sqrt{x+2}-\sqrt{x-2}} dx$

$= \int \frac{\sqrt{x+2}+\sqrt{x-2}}{4} dx$

$= \frac{1}{6}\left\{(x+2)^{\frac{3}{2}} + (x-2)^{\frac{3}{2}}\right\} + c$

By rationalisation,

$\frac{1}{\sqrt{x+2}-\sqrt{x-2}} = \frac{1}{\sqrt{x+2}-\sqrt{x-2}} \times \frac{\sqrt{x+2}-\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}} = \frac{\sqrt{x+2}-\sqrt{x-2}}{4}$

18. We know that,

$E(X) = \int_{-\infty}^{\infty} x f(x) dx$

$= \int_0^1 x 4x^3 dx$

$= 4\left[\frac{x^5}{5}\right]_0^1$

$E(X) = \frac{4}{5}$

$E\left(X^2\right) = \int_{-\infty}^{\infty} x^2 f(x) dx$

$= \int_0^1 x^2 4x^3 dx$

$= 4\left[\frac{x^6}{6}\right]_0^1$

$= \frac{4}{6} = \frac{2}{3}$

$V(X) = \left(x^2\right) - [E(X)]^2$

$= \frac{4}{6} - \left[\frac{4}{5}\right]^2$

$= \frac{2}{75}$

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19. Separating the variables, we get

$\frac{\sec^2 x}{\tan x} dx + \frac{\sec^2}{\tan y} dy = 0$

Integrating, we get

$\int \frac{\sec^2 x}{\tan x} dx + \int \frac{\sec^2}{\tan y} dy = c$

log tan x + log tan y = log c

log(tan x tan y) = log c

tan x tan y = c

20. Let k be the sixth term of the series in the difference table.

First we find the forward differences

x	y	Δ	Δ^2
1	8		
		4	
2	12		3
		7	
3	19		3
		10	
4	29		3
		13	
5	42		k-55
		k-42	
6	k		

Given that the second differences are constant

∴ k − 55 = 3

k = 58

∴ the sixth term of the series is 58

2 Marks

21. $\int \sqrt{1+\sin 2x}\, dx = \int \sqrt{(\sin x + \cos x)^2} dx$

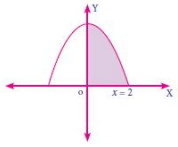
=∫(sinx + cosx)dx

=−cos x + sin x + c

[Change into simple integrands

1+ sin2x = $\sin^2 x + \cos^2 x + 2\sin x \cos x$
= $(\sin x + \cos x)^2$]

22. $y = 4 - x^2$
Required area = $\int_0^2 y dx = \int_0^2 (4 - x^2) dx$
= $\left[4x - \frac{x^2}{3}\right]_0^2 = 8 - \frac{8}{3}$
= $\frac{16}{3}$ sq.units



23. $F(x) = \int_{-\infty}^{x} f(t)dt$
$= \int_{100}^{x} \frac{100}{t^2} dt, \quad x \geq 100$
$= \left[\frac{100}{-t}\right]_{100}^{x}, \quad x \geq 100$
$F(x) = \left[1 - \frac{100}{x}\right], \geq 100$

24. Let X be a random variable follows binomial distribution with p = q = 1/2
P (3 heads) = $5C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{5-x}$
$= 5C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$
$= 5C_3 \left(\frac{1}{2}\right)^5$
$= \frac{5}{16}$

25. Given equation is y = cx + c - c^3 ....(1)
Differentiating w.r.t 'x' we get,
$\frac{dy}{dx}$ = c(1) + 0-0
$\Rightarrow \frac{dy}{dx}$ = c ....(2)
Substituting (2) in (1) we get,
$y = x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right) - \left(\frac{dy}{dx}\right)^3$ which is the required differential equation.

26. Time series has an important objective to identify the variations and try to eliminate the variations and also helps us to estimate or predict the future values.

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27. $\Delta^2 \left(\frac{1}{x}\right) = \Delta\left(\Delta\left(\frac{1}{x}\right)\right)$
Now $\Delta\left[\frac{1}{x}\right] = \frac{1}{1+x} - \frac{1}{x}$
$\Delta^2 \left(\frac{1}{x}\right) = \Delta\left(\frac{1}{1+x} - \frac{1}{x}\right)$
$= \Delta\left(\frac{1}{1+x}\right) - \Delta\left(\frac{1}{x}\right)$
Similarly $\Delta^2 \left(\frac{1}{x}\right) = \frac{2}{x(x+1)(x+2)}$

28. Let A = $\begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$
Order of A is 2×2 [Since minimum of (2,2) is 2]
Consider the second order minor $\begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix}$
= 8-8
= 0
Since the second order minor vanishes $\rho(A) \neq 2$
Consider a first order minor $[1] \neq 0$
There is a minor of order 1, which is not zero
$\therefore \rho(A) = 1$

Multiple Choice Question

29. (a) 1
30. (b) $R_i \rightarrow 2R_i + 2C_j$
31. (a) P = ∫(MR - MC) dx + k
32. (a) (e -1) sq.units
33. (d) $p(x) = -0.5$
34. (c) 1
35. (c) $\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^{e^{\left(\frac{1}{2}\right)}}$
36. (d) 1/81
37. (c) 0
38. (a) (n -1)!
39. (a) Harper
40. (c) Reject H_0 when it is true
41. (c) y = T + S + C + I
42. (d) 109
43. (b) of order 1 and degree 3
44. (c) A cos 2x + B sin 2x
45. (a) Equal to m+n–1
46. (d) all of the above

47. (d) 0

48. (c) both equal and unequal intervals

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