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Answers Key:

5 Marks

1. Let the amount of investment in each bond be
Rs. x, Rs. y, Rs. z respectively.
Given x + y + z = 5000 ..(1)
Also $\frac{6x}{100}+\frac{7y}{100}+\frac{8z}{100}=358$
$\therefore$ Interes $=\frac{PNR}{100}=\frac{x\times1\times6}{100}=\frac{6x}{100}$
$\Rightarrow\frac{6x+7y+8z}{100}=358$
$\Rightarrow 6x+7y++8z=35800$
Given that $\frac{6x}{100}+\frac{7y}{100}=70+\frac{8z}{100}$
$\Rightarrow 6x+7y=7008z$
$\Rightarrow 6x+7y-8z=7000$
The matrix equation corresponding to the given system is
$\begin{pmatrix}1&1&1\\6&7&8\\6&7&-8\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5000\\35800\\7000\end{pmatrix}$

Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix}1&1&1&5000\\6&7&8&35800\\6&7&-8&7000\end{pmatrix}$	
$\begin{pmatrix}1&1&1&5000\\0&1&2&5800\\0&1&-14&-2300\end{pmatrix}$	$R_2\to R_2-6R_1$ $R_3\to R_3-6R_1$
$\begin{pmatrix}1&1&1&5000\\0&1&2&5800\\0&0&-16&-28800\end{pmatrix}$	$R_3\to R_3-R_2$

The last equivalent matrix is in echelon form and ρ(A) = ρ([A,B]) = 3 = Number of unknowns
Thus, the given system is consistent with unique solution. To find the solution, let us rewrite the above echelon form into the matrix form
$\begin{pmatrix}1&1&1\\0&1&2\\0&0&-16\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5000\\5800\\-28800\end{pmatrix}$
$\Rightarrow x+y+z=5000$ ...(1)
$\Rightarrow y+2z=5800$ ..(2)
$\Rightarrow -16z=-28800$ ...(3)
$(3)\Rightarrow -16z=-28800$
$\Rightarrow z=\frac{28800}{-16}=1800$
Substituting z = 1800 in (2) we get,
y + 2(1800) = 5800
⇒ y + 3600 = 5800
⇒ y=5800-3600
⇒ y = 2200
Substituting y = 2200 and z = 1800 in (1) we get
⇒ x + 2200 + 1800 = 5000
⇒x + 4000 = 5000
⇒x = 5000 - 4000
⇒x = 1000
Hence, the amount of investment in each bond is Rs. 1000, Rs. 2200 and Rs. 1800 respectively

2. $\Delta=\begin{vmatrix}1&2&1\\2&-1&2\\1&1&-2\end{vmatrix}$
$=1\begin{vmatrix}-1&2\\1&-2\end{vmatrix}-2\begin{vmatrix}2&2\\1&-2\end{vmatrix}+1\begin{vmatrix}2&-1\\1&1\end{vmatrix}$
= 1(2 -2) - 2(-4 -2) + 1(2 + 1)
= 1(0)-2(-6)+1(3)
= 12 + 3 = 15≠0.
Since $\Delta\neq0$ Cramer's rule can be applied and thesystem is consistent with unique solution.
$\Delta x=\begin{vmatrix}7&2&1\\4&-1&2\\-1&1&-2\end{vmatrix}$

$= 7\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 4 & 2 \\ -1 & -2 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ -1 & 1 \end{vmatrix}$

= 7 (2 -2) -2 (-8 + 2) + 1 (4 - 1)

= 7 (0) - 2(-6) + 1(3)

= 12 + 3 = 15

$\Delta y = \begin{vmatrix} 1 & 7 & 1 \\ 2 & 4 & 2 \\ 1 & -1 & -2 \end{vmatrix}$

$= 1\begin{vmatrix} 4 & 2 \\ -1 & -2 \end{vmatrix} - 7\begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix}$

= 1 (- 8 + 2) -7(-4 -2) + 1(-2 -4)

= 1 (-6) -7 (-6) + 1 (-6)

= - 6 + 42 - 6 = 30

$\Delta z = \begin{vmatrix} 1 & 2 & 7 \\ 2 & -1 & 4 \\ 1 & 1 & -1 \end{vmatrix}$

$= 1\begin{vmatrix} -1 & 4 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix} + 7\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$

= 1 (1 - 4) - 2(- 2 - 4) + 7(2 + 1)

= 1(-3)-2(-6)+7(3)

= - 3 + 12 + 21 = 30

$x = \frac{\Delta x}{\Delta} = \frac{\cancel{15}}{\cancel{15}} = 1$

$y = \frac{\Delta y}{\Delta} = \frac{\cancel{30}^{2}}{\cancel{15}} = 2$

$z = \frac{\Delta z}{\Delta} = \frac{\cancel{30}^{2}}{\cancel{15}} = 2$

$\therefore$ Solution set is {1, 2, 2}

3. $\int \frac{3x^2+6x+1}{(x+3)(x^2+1)}dx = \int \left[\frac{1}{(x+3)} \frac{2x}{(x^2+1)}\right]dx$

$\int \frac{dx}{(x+3)} + \int \frac{2x}{(x^2+1)}dx$

$= \log|x+3| + \log|x^2+1| + c$

$= \log|(x+3)(x^2+1)| + c$

$= \log|x^3+3x^2+x+3| + c$

[By partial fractions,

$\frac{3x^2+6x+1}{(x+3)(x^2+1)} = \frac{A}{(x+3)} + \frac{Bx+C}{(x^2+1)} \Rightarrow \frac{3x^2+6x+1}{(x+3)(x^2+1)} = \frac{1}{(x+3)} + \frac{2x}{(x^2+1)}$

4. MR = a $(x + b)^{-2}$ - c

$R = \int (x+b)^2 dx - c\int dx$

R = $\frac{a(x+b)^{-2}}{-1}$ - cx + k

R = $\frac{a}{(x+b)}$ - cx + k

When x = 0, R = 0

$0 = \frac{a}{b} - c(0) + k$

$k = \frac{a}{b}$

$R = -\frac{a}{x+b}$ - cx+ $\frac{a}{b}$

$= \frac{-ab+a(x+b)}{b(x+b)} - cx$

$R = \frac{ax}{b(x+b)} - cx$

Demand function $P = \frac{R}{x}$

$P = \frac{a}{b(x+b)} - c$

$P + c = \frac{a}{b(x+b)}$

$b(x+b) = \frac{a}{P+c}$

$x = \frac{a}{b(P+c)} - b$

5. (a) We know that

$\int_{-\infty}^{\infty} f(x)dx = 1$

$\int_0^{10} Axdx + \int_{10}^{20} A(20-x)dx = 1$

$A\left\{\left[\frac{x^2}{2}\right]_0^{10} + \left[20x - \frac{x^2}{2}\right]_{10}^{20}\right\} = 1$

A[(50-0)+(400-200)-(200-50)] = 1

$A = \frac{1}{100}$

(b) (i) The probability that the number of pounds of bread that will be sold tomorrow is more than 10 pounds is given by

$P(10 \le X \le 20) = \int_{10}^{20} \frac{1}{100}(20-x)dx$

$= \frac{1}{100}\left[20x - \frac{x^2}{2}\right]_{10}^{20}$

$= \frac{1}{100}[(400-200)-(200-50)]$

= 0.5

(ii) The probability that the number of pounds of bread that will be sold tomorrow is less than 10 pounds, is given by

$P(0 \le X \le 20) = \int_0^{10} \frac{1}{100}xdx$

$= \frac{1}{100}\left[\frac{x^2}{2}\right]_0^{10}$

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$= \frac{1}{100}(50-0)$

= 0.5

(ii) The probability that the number of pounds of bread that will be sold tomorrow is between 5 and 15 pounds is

$P(5 \le X \le 15) = \int_5^{10} \frac{1}{100} x dx + \int_{10}^{15} \frac{1}{100}(20-x)dx$

$= \frac{1}{100}\left[\frac{x^2}{2}\right]_5^{10} + \frac{1}{100}\left[20x - \frac{x^2}{2}\right]_{10}^{15}$

= 0.75

6. $\frac{EY}{Ex} = \frac{-7x}{(1-2x)(2+3x)}$

$\frac{7}{(2x-1)(3x+2)} = \frac{A}{2x-1} + \frac{B}{3x+2}$

7 = A(3x+2)+B(2x-1)

$Put\ x = \frac{-2}{3} 7 = B\left(\frac{-4}{3} - 1\right) 7 = B\left(\frac{-7}{3}\right)$

$Put\ x = \frac{1}{2} 7 = A\left(\frac{3}{2} + 2\right) \Rightarrow 7 = A\left(\frac{7}{2}\right)$

$\therefore \frac{7}{(2x-1)(3x+2)} = \frac{2}{2x-1} - \frac{3}{3x+2}$

Also, it is given that x = 2, when y = $\frac{3}{8}$

$\Rightarrow \frac{x}{y}\frac{dy}{dx} = \frac{-7x}{(1-2x)(2+3x)}$

$\Rightarrow \frac{dy}{y} = \frac{-7x\,dx}{x(1-2x)(2+3x)}$

$= \frac{-7dx}{(1-2x)(2+3x)}$

$\Rightarrow \frac{dy}{y} = \frac{7dx}{(2x+1)(3x+2)}$

$\int \frac{dy}{y} = \int \frac{7dx}{(2x-)(3x+2)}$

$\int \frac{dy}{y} = \int \left(\frac{2}{2x-1} - \frac{3}{3x+2}\right)dx$

$log \quad y = 2\int \frac{1}{2x-1}dx - 3\int \frac{dx}{3x+2}$

$= 2\frac{log|2x-1|}{2} - 3\frac{log|3x+2|}{3} + logc$

$= log \quad y - log \quad c = log\left|\frac{2x-1}{3x+2}\right|$

$\Rightarrow log\left|\left(\frac{y}{c}\right)\right| = log\left|\frac{2x-1}{3x+2}\right|$

$\Rightarrow \frac{y}{c} = \frac{2x-1}{3x+2} y = c\left(\frac{2x-1}{3x+2}\right)$ ....(1)

When $x = 2, y = \frac{3}{8}$

$\Rightarrow \frac{3}{8} = c\left(\frac{4-1}{8}\right)$

$\frac{3}{8} = c\left(\frac{3}{8}\right) = c = 1$

$y = \left(\frac{2x-1}{3x+2}\right)$

$\Rightarrow y = \frac{2x-1}{3x+2}$

7. We know that,

$\int_{-\infty}^{\infty} f(x)dx = 1$

$\int_{-\infty}^{\infty} ke^{-|x|}dx = 1$

$k\int_{-\infty}^{\infty} ke^{-|x|}dx = 1$

$2k\int_{-\infty}^{\infty} e^{-|x|}dx = 1$ ($\because x^2e^{-|x|}$ is an function)

$2k\int_0^{\infty}\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$

$k = \frac{1}{2}$

Mean of the random variable is

$E(X) = \int_{-\infty}^{\infty} xf(x)dx$

$E(X) = \int_{-\infty}^{\infty} xke^{-|x|}dx$ ($\because xe^{-|x|}$ is an odd function of x)

$= \frac{1}{2}\int_{-\infty}^{\infty} xe^{-|x|}$

= 0

$E\left(x^2\right) = \int_{-\infty}^{\infty} x^2 f(x)dx$

$= \int_{-\infty}^{\infty} x^2 ke^{-|x|}dx$

$= \frac{1}{2}\int_{-\infty}^{\infty} x^2e^{-|x|}dx$

$= \int_0^{\infty} x^2e^{-x}$ ($\because x^2e^{-|x|}$ is an even function)

$= \Gamma 3\ (\because \Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1}e^{-x}dx, \alpha > 0; \Gamma n = (n-1)!)$

= 2

$V(X) = E\left(x^2\right) - [E(X)]^2$

$= 2 - [0]^2$

= 2

8. Let the probability for boy or girl is p = $\frac{1}{2}$ and q = $\frac{1}{2}$, n = 4

(i) P (atleast one boy) = P(X≥1)

= 1-P(X<1)

= 1-[P(X = 0)]

= 1-$\left[4_{C0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4\right]$ p(x) = $^nC_x\, p^x q^{n-x}$, n = 4, x = 0

= $\left(\frac{1}{2}\right)^4 = 1 - \frac{1}{16}$ = 1-.0625

= 0.9375

∴ Probability of one family having atleast one boy = 0.9375.

Hence, probability of 750 families having atleast one boy

= 0.9375 × 750 = 703

(ii) P (atmost 2 girls) = P(X ≤ 2)

= P(X = 0) + P(X = 1) + P(X = 2)

= $^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 + {}^4C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 + {}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$

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$= \left(\frac{1}{2}\right)^4 [{}^4C_0 + {}^4C_1 + {}^4C_1]$

$= \frac{1}{16}\left[1 + 4 + \frac{4\times3}{2\times1}\right] = 0.0625\ (5+6)$

$= 0.0625 \times 11 = 0.6815$

∴ Probability of one family having atmost 2 girls = 0.6875.

Hence, probability of 750 families having atmost 2 girls

$= 0.6875 \times 750 = 515.6 = 516.$

(iii) P(Chiidren of both sexes) = P (atleast one girl and atleast one boy)

$= 1[P(X = 0) + P(X = 4)]$

$= 1 - {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 - C_4\left(-\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0$

$= 1 - \frac{1}{16} - \frac{1}{16} = 1 - \frac{2}{16} = 1 - \frac{1}{8} = \frac{7}{8}$

∴ Probability of 1 family of having children of both sexes = 0.875.

∴ Probability of 750 families having children of both sexes = 0.875 × 75 = 656.

9. $\int_a^b f(x)dx = \lim_{n\to\infty, h\to0} \sum_{r=1}^n h.\, f\,(a + rh)$

Here a = 0, b = 1

$h = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$

and f(x) = x^2

Now, $f(a + rh) = f\left(0 + r.\frac{1}{n}\right) = f\left(\frac{r}{n}\right)$

$= \left(\frac{r}{n}\right)^2 = \frac{r^2}{n^2}$

$\therefore \int_0^1 x^2 dx = \lim_{n\to\infty} \sum_{r=1}^n \frac{1}{n}\left(\frac{r^2}{n^2}\right)$

$= \lim_{n\to\infty} \frac{1}{n^3}.\frac{n(n+1)(2n+1)}{6}$

$\left[\because \sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}\right]$

$\lim_{n\to\infty} \cdot \frac{1}{n^3} \cdot \frac{n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}$

[Taking n common from each bracket]

$= \frac{1}{6}.\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)$

$= \frac{1}{6}(1 + 0)\,(2 + 0)$

$[\because \text{when } n \to \infty, \frac{1}{n} \to 0]$

$= \frac{1}{6}(1)(2) = \frac{2}{6}$

$= \frac{1}{3}$

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10. Sample size n = 50,

Sample mean $\bar{X}$ = 1850

Population mean μ = 1800

Population standard deviation σ = 100

Null Hypotheses H_0:

μ = 1800 (i.e., To claim that the breaking strength of the cables have increased)

Alternative Hypotheses H_1:

μ≠1800(To claim that the breaking strength of the cables have not increased)

The level of significance ∝ = 1% =.001

Applying the test statistic,

$Z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}}$

$\Rightarrow Z = \frac{1850-1800}{\frac{100}{\sqrt{50}}} = \frac{50}{\frac{100}{7.07}} = \frac{50}{14.144} = 3.535$

$\Rightarrow \therefore Z = 3.535$

The Significant value $Z_{\frac{\alpha}{2}} = 2.58$

Here $Z < Z_{\frac{\alpha}{2}} i.e., 3.535 < 2.58$

Inference: Since $Z < Z_{\frac{\alpha}{2}}$ at 1% level of significance, the null hypothesis Ho is rejected.

Hence, we conclude that μ≠ 1800 and we cannot support the claim that the breaking strength of the cables have increased.

11. Computation of trend values by the method of least squares (ODD Years).

Year(x)	Production of Sugarcane(Y)	X=(x–2003)	X^2	XY	Trend values(Yt)
2000	40	-3	9	-120	42.04
2001	45	-2	4	-90	43.07
2002	46	-1	1	-46	44.11
2003	42	0	0	0	45.14
2004	47	1	1	47	46.18
2005	50	2	4	100	47.22
2006	46	3	9	138	48.25
N=7	$\sum Y$=316	$\sum X$=0	$\sum X^2$=8	$\sum XY$ =29	$\sum Yt$=316

$a = \frac{\sum Y}{n} = \frac{316}{7} = 45.143; \quad b = \frac{\sum XY}{\sum X^2} = \frac{29}{28} = 1.036$

Therefore, the required equation of the straight line trend is given by

Y = a + bX

Y = 45.143 + 1.036 (x - 2003)

The trend values can be obtained by

When X = 2000 , Yt = 45.143 + 1.036(2000–2003) = 42.035

When X = 2001, Yt = 45.143 + 1.036(2001–2003) = 43.071,

similarly other values can be obtained.

12. Given $Q_d = 13 - 6p + 2\frac{dp}{dt} + \frac{d^2p}{dt^2}$ $Q_s = -3 + 2p$

At equilibrium $Q_d = Q_s$

$\Rightarrow 13 - 6p + 2\frac{dp}{dt} + \frac{d^2p}{dt^2} = -3 + 2p$

$\Rightarrow \frac{d^2p}{dt^2} + 2\frac{dp}{dt} - 6p + 13 + 3 - 2p = 0$

$\Rightarrow \frac{d^2p}{dt^2} + 2\frac{dp}{dt} - 8p + 16 = 0$

$\Rightarrow \frac{d^2p}{dt^2} + 2\frac{dp}{dt} - 8p = -16$

The auxiliary equation is $m^2 + 2m - 8 = 0$

$\Rightarrow (m + 4)(m - 2) = 0$

$\Rightarrow m = -4, 2$

The roots are real and different.

$\therefore$ Complementary function CF is $Ae^{-4t} + Be^{2t}$

Particular Integral PI = $\frac{1}{\phi(D)}.f(t)$

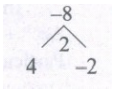
$= \frac{1}{D^2+2D-8}.(-16) = \frac{-16.e^{0x}}{(D+4)(D-2)}$

PI = $\frac{-16}{(0+4)(0-2)} = \frac{-16}{-8} = 2$

$\therefore$ P = CF + PI

$\therefore$ The general solution is

$P = Ae^{-4t} + Be^{2t} + 2$



13. Here the number of rows and columns are equal

$\therefore$ The given assignment problem is balanced.

Step 1 : Select a minimum element in each row and subtract this from all the elements in its row.

$\therefore$ The cost matrix of the given assignment problem is

	1	2	3	4	Row Min.
P	0	18	9	3	8
Q	9	24	0	22	4
R	23	4	3	0	15
S	0	17	15	1	9
Column Min.	0	4	0	0	

Column 2 has no zero. Go to Step 2.

Step 2 : Select the minimum element in each column and subtract this from all the elements in its column.

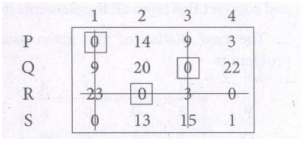
	1	2	3	4
P	0	14	9	3
Q	9	20	0	22
R	23	0	3	0
S	0	13	15	1

Since each row and column contains atleast one zero, assignments can be made.

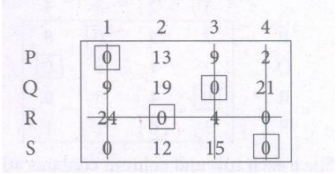
Step 3 : Examine the rows with exactly one zero, mark it by □ and draw a vertical line.

After examining the rows, examine the columns with exactly one zero.

Mark it by □ and draw a horizontal line.



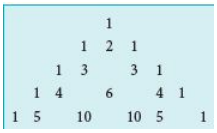
Only 3 assignments have been made.
The numbers not lying on the line are and min. is 1. Subtract 1 from all these numbers and add 1 to 23 which lies on the intersecting lines. Other numbers remain the same.
A new cost matrix is formed and repeat step 3.
∴ The new cost matrix is



Thus, all the 4 assignments have been made.
∴The optimal assignment schedule and total cost is

Subordinates	tasks	cost
P	1	8
Q	3	4
R	2	19
S	4	10
Total Cost		**Rs. 41**

14. Since five values are given, the polynomial which fits the data is of degree four.
Hence $\Delta^5 y_k = 0$ (i.e) $(E-1)^5 y_k = 0$
i.e., $(E^5 - 5E^4 + 10E^3 - 10E^2 + 5E - 1)y_k = 0$
$E^5 y_k - 5E^4 y_k + 10E^3 y_k - 10E^2 y_k + 5Ey_k - y_k = 0$ (1)
Put k = 0 in (1)
$E^5 y_0 - 5E^4 y_0 + 10E^3 y_0 - 10E^2 y_0 + 5Ey_0 - y_0 = 0$
$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 - y_0 = 0$
$y_5 - 5(350) + 10y_3 - 10(260) + 5(220) - 200 = 0$
$y_5 + 10y_3 = 3450$ (2)



Put k = 1 in (1)
$E^5 y_1 - 5E^4 y_1 + 10E^3 y_1 - 10E^2 y_1 + 5Ey_1 - y_0 = 0$
$y_6 - 5y_5 + 10y_4 - 10y_3 - y_1 = 0$
$430 - 5y_5 + 10(350) - 10y_3 + 5(260) - 220 = 0$
$5y_5 + 10y_3 = 5010$ (3)
(3) − (2) ⇒ $4y_5 = 1560$
$y_5 = 390$
From (1) $390 + 10y_3 = 3450$
$10y_3 = 3450 - 390$
$y_3 \cong 306$

3 Marks

15. Let $A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$
Order of A is 3×4
$\therefore \rho(A) \leq 3$
Consider the third order minors

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & 1 \\ 3 & 6 & 3 \end{vmatrix} = 0, \quad \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \\ 3 & 3 & -7 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & -2 \\ 3 & 6 & -7 \end{vmatrix} = 0, \quad \begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -2 \\ 6 & 3 & -7 \end{vmatrix} = 0$$

Since all third order minors vanishes, $\rho(A) \neq 3$
Now, let us consider the second order minors,

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Consider one of the second order minors $\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 6 \neq 0$

There is a minor of order 2 which is not zero.

$\therefore \rho(A) = 2$

16. The transition porbability matrix is T = $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

At equilibrium, (S H) $\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$ = (S H)

where S + H = 1

$\frac{4}{5}S + \frac{1}{3}H = S$

$\frac{1}{3}(1 - H) + \frac{4}{5}H = 1 - H$

On solving this, we get S = $\frac{6}{11}$ and H = $\frac{5}{11}$

In the long run, on a randomly selected day, his chances of being happy is $\frac{5}{11}$

17. Given f'(x) = $8x^3$-2x, f(2) = 8

f'(x) = $8x^3$-2x

$\Rightarrow \int f'(x)dx = \int (8x^3 - 2x)dx$

$\Rightarrow f(x) = \frac{2}{\cancel{8}}\frac{\cancel{8}x^4}{\cancel{4}} - \frac{\cancel{2}x^2}{\cancel{2}} + c$

$\Rightarrow$ f(x) = $2x^4$-x^2+c...(1)

Given f(2) = 8

$\Rightarrow$ 8 = 2(2^4)-2^2+c

$\Rightarrow$ 8 = 32 - 4+c

$\Rightarrow$ 8 = 32 - 4+c

$\Rightarrow$ 8 - 28 = c

$\Rightarrow$ c = -20

Substituting c = -20 in (1) we get,

f(x) = $2x^4$-x^2-20

18. Given probability distribution function is

$= \begin{cases} kx & x = 2, 4, 6 \\ k(x-2), & x = 8 \\ 0, & \text{otherwise} \end{cases}$

where k is a constant

X = x	2	4	6	8
P(X=x)	2k	4k	6k	k(8-2) = 6k

Since the given function is a probability distribution function, each $\rho_i > 0 \Sigma \rho_i = 1$

$\Rightarrow$ 2k+4k+6k+6k = 1

$\Rightarrow$ 18 k = 1

$\Rightarrow$ k = $\frac{1}{18}$

19. Given demand function p = e^{-x}

and p = 0.5

when p0 = 0.5, 0.5 = e^{-x}

$\Rightarrow \frac{1}{2} = e^{-}x$

$\Rightarrow \frac{1}{2} = \frac{1}{e^x} \Rightarrow ex = 2$

$\Rightarrow x = \log 2$

$\therefore p_0x_0 = 0.5 \log 2 = \frac{1}{2}\log 2$

Consumer's Surplus

$= \int_0^x f(x)dx - p_0x_0$

$= \int_0^{log2} e^{-x}dx - \frac{1}{2}log2$

$= \left[\frac{e^{-x}}{-1}\right]_0^{log2} - \frac{1}{2}log2$

$= -[e^{-x}]_0^{log2} - \frac{1}{2}log2$

$= -\left(e^{-log2} - e^0\right) - \frac{1}{2}log2$

$= -\left(e^{log\ \frac{1}{2}} - 1\right) - \frac{1}{2}log2$

$= -\left(\frac{1}{2} - 1\right) - \frac{1}{2}log2$

$= -\left(-\frac{1}{2}\right) - \frac{1}{2}log2$

$= \frac{1}{2} - \frac{1}{2}log2 = \frac{1}{2}(1 - log_e 2)$

$\therefore$ C.S = $\frac{1}{2}$[1-$\log_e$ 2] units

20. Probability p of guessing an answer correctly is p = $\frac{1}{2}$

$\Rightarrow$ q = $\frac{1}{2}$

Probability of guessing correctly x answers in 10 questions

$P(X = x) = p(x)^nC_xq^{n-x} = 10Cx\ (\frac{1}{2})\ (\frac{1}{2})$

The required probability P(X $\geq$ 6) = P(6) + P(7) + P(8) + P(9) + P(10)

$= (\frac{1}{2})^{10}\ [10C_6 + 10C_7 + 10C_8 + 10C_9 + 10C_{10}]$

$= [\frac{1}{1024}][210 + 120 + 45 + 10 + 1]$

$= \frac{193}{512}$

21.

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$\int_0^{\frac{\pi}{2}} x \sin x dx = \int_0^{\frac{\pi}{2}} u dv$

$= (uv)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} v du$

$= [-x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x dx$

$= 0 + [\sin x]_0^{\frac{\pi}{2}} = 1$

Take u = x	and dv = sin x dx
Differentiate	Integrate
du = dx	v = - cos x

22. $\Delta^2 U_0 = (E-1)^4 U_0$

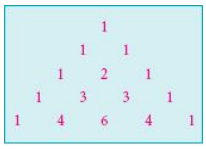
$= (E^4 - 4E^3 + 6E^2 - 4E + 1)U_0$

$= E^4U_0 - 4E^3U_0 + 6E^2U_0 - 4EU_0 + U_0$

$= U_4 - 4U_3 + 6U_2 - 4U_1 + U_0$

$= 29 - 4(28) + 6(21) - 4(11) + 1.$

$= 156 - 156 = 0$



23. Given equation is $y = c(x-c)^2$

$\Rightarrow y = c(x^2 - 2xc + c^2)$....(1)

Differentiating w.r.t 'x' we get

$\frac{dy}{dx} - 2c(x-c)$ ...(2)

(1) ÷ (2) gives

$\frac{y}{\frac{dy}{dx}} = \frac{c(x-c)^2}{2c(x-c)} = \frac{y}{\frac{dy}{dx}} = \frac{x-c}{2}$

$x-c = \frac{2y}{\frac{dy}{dx}} \Rightarrow c = x - \frac{2y}{\frac{dy}{dx}}$

Substituting the value of c in (1) we get

$y = \left(x - \frac{2y}{\frac{dy}{dx}}\right)\left(x - x + \frac{2y}{\frac{dy}{dx}}\right)^2$

$= \left(x - \frac{2y}{\frac{dy}{dx}}\right)\left(\frac{4y^2}{\left(\frac{dy}{dx}\right)^2}\right)$

$y = \left(\frac{x\frac{dy}{dx} - 2y}{\frac{dy}{dx}}\right)\left(\frac{4y^2}{\left(\frac{dy}{dx}\right)^2}\right)$

$\Rightarrow y\left(\frac{dy}{dx}\right)^3 = 4y^2\left(x\frac{dy}{dx} - 2y\right)$

$\Rightarrow \left(\frac{dy}{dx}\right)^3 = 4y\left(x\frac{dy}{dx} - 2y\right)$

$\Rightarrow \left(\frac{dy}{dx}\right)^3 = 4xy\left(\frac{dy}{dx}\right) - 8y^2$

$\Rightarrow \left(\frac{dy}{dx}\right)^3 - 4xy\left(\frac{dy}{dx}\right) + 8y^2 = 0$

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24. $\int \frac{dx}{\sqrt{3x+4} - \sqrt{3x+1}}$

$= \int \frac{1}{\sqrt{3x+4} - \sqrt{3x+1}} \times \frac{\sqrt{3x+4} + \sqrt{3x+1}}{\sqrt{3x+4} + \sqrt{3x+1}} dx$

$= \int \frac{\sqrt{3x+4} + \sqrt{3x+1}}{3x+4-3x-1} dx$

$= \frac{1}{3}\int \left((3x+4)^{\frac{1}{2}} + (3x+1)^{\frac{1}{2}}\right) dx$

$= \frac{1}{3}\left[\frac{(3x+4)^{\frac{3}{2}}}{\frac{3}{2}(3)} - \frac{(3x+1)^{\frac{3}{2}}}{\frac{3}{2}(3)}\right] + c$

$= \frac{2}{27}\left[(3x-4)^{\frac{3}{2}} + (3x-1)^{\frac{3}{2}}\right] - c$

2 Marks

25. $\int \left(e^x + \frac{1}{e^x}\right)^2 dx = \int \left(e^{2x} + \frac{1}{e^{2x}} + 2\right) dx$

$= \int \left(e^{2x} + e^{-2x} + 2\right) dx$

$= \frac{e^{2x}}{2} - \frac{e^{-2x}}{2} + 2x + c$

26. The given probability function is

X	1	2	3	4
P(X = x)	k	2k	3k	4k

Since the given function is a probability function, each $\rho_i > 0$ and $\Sigma\rho_i = 1$

$\Rightarrow k + 2k + 3k + 4k = 1$

$\Rightarrow 10k = 1 \Rightarrow k = \frac{1}{10}$

$\Rightarrow k = 0.1$

27. Given $MR = 20 - 5x + 3x^2$

$\Rightarrow \frac{dR}{dx} = 20 - 5x + 3x^2$

$\Rightarrow dR = (20 - 5x + 3x^2)dx$

$\Rightarrow \int dR = \int (20 - 5x + 3x^2)dx$

$\Rightarrow R = 20x - \frac{5x^2}{2} + \frac{3x^3}{3} + k$

When x = 0, R = 0 $\Rightarrow$ k = 0

$\therefore R = 20x - \frac{5x^2}{x} + x^3$

28. (i) E(a) = a where a is a constant

(ii) E(aX) = a. E(X)

(iii) E(aX + b) = a E(X) + b
(iv) If $X \geq 0$, then $E(X) \geq 0$
(v) V(a) = 0
(vi) $V(aX + b) = a^2 V(X)$.

29. (i) Number of lightnings per second.
(ii) Number of printing mistakes per page in a textbook.

30. Let $I = \int_0^\infty e^{-4x} x^4 dx$
Here n = 4 and a = 4
$\therefore I = \frac{4!}{4^{4+1}} = \frac{4!}{4^5}$
$\left[\because \int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}\right]$
$= \frac{\cancel{4} \times 3 \times \cancel{2} \times \cancel{1}}{\cancel{4} \times 4 \times 4 \times 4 \times \cancel{4}^2}$
$= \frac{3}{128}$

31. $y' + (y'')^2 = (x + y'')^2$
$y' + (y'') = x^2 + 2xy'' + (y'')^2$
$y' = x^2 + 2xy'' \Rightarrow \frac{dy}{dx} = x^2 + 2x\frac{d^2y}{dx^2}$
∴ order = 2, ∴ Degree = 1

32. log(1+h/ax)

Multiple Choice Question

33. (b) n
34. (c) $\triangle \neq 0$
35. (a) log 2 sq.units
36. (c) 2 sq.units
37. (d) c
38. (c) $\frac{1}{15}$
39. (c) $Z = \frac{X-9}{9}$
40. (d) all of the above statements are true.
41. (a) -cos 2x + c
42. (c) 4!
43. (b) finite subset
44. (c) $\frac{\sigma}{\sqrt{n}}$
45. (c) Either positive or negative
46. (c) CL, LCL, UCL
47. (a) 2 tan x
48. (a) y = v x
49. (a) Total supply ≠ Total demand
50. (b) 3
51. (a) $\Delta^{m+n} f(x)$
52. (a) $y(x) = \frac{x-x_1}{x_0-x_1} y_0 + \frac{x-x_0}{x_1-x_0} y_1$

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