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Answers Key:

Answer any FIVE questions

1. Given F $(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$

Expanding along R_1 we get,

$|F(\alpha)| = \cos\alpha \begin{vmatrix} 1 & 0 \\ 0 & cos\alpha \end{vmatrix} - 0 + sin\alpha \begin{vmatrix} 0 & 1 \\ -sin\alpha & 0 \end{vmatrix}$

$= \cos\alpha(\cos - 0) + \sin\alpha(0 + \sin\alpha)$

$= \cos^2 + \sin^2\alpha = 1 \neq 0$

Since F (α) is a non-singular matrix, $[F(\alpha)]^{-1}$ exists

Now, adj $(F(\alpha)) = \begin{bmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & cos\alpha \end{vmatrix} & - \begin{vmatrix} 0 & 0 \\ sin\alpha & cos\alpha \end{vmatrix} & + \begin{vmatrix} 0 & 1 \\ -sin\alpha & 0 \end{vmatrix} \\ - \begin{vmatrix} 0 & sin\alpha \\ 0 & cos\alpha \end{vmatrix} & + \begin{vmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{vmatrix} & - \begin{vmatrix} cos\alpha & 0 \\ sin\alpha & 0 \end{vmatrix} \\ + \begin{vmatrix} 0 & sin\alpha \\ 1 & 0 \end{vmatrix} & - \begin{vmatrix} cos\alpha & sin\alpha \\ 0 & 0 \end{vmatrix} & + \begin{vmatrix} cos\alpha & 0 \\ 0 & 1 \end{vmatrix} \end{bmatrix}^T$

$= \begin{bmatrix} +(cos\alpha - 0) & -(0) & +(0 + sin\alpha) \\ -(0) & +(cos^2\alpha + sin^2\alpha & -(0) \\ +(0 - sin\alpha) & -(0) & +(cos - 0) \end{bmatrix}^T$

$\begin{bmatrix} cos\alpha & 0 & +sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix} = \begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ +sin\alpha & 0 & cos\alpha \end{bmatrix}$

$\therefore F(\alpha)^{-1} = \frac{1}{|F(\alpha)|}$ adj $(F(\alpha))$

$[F(\alpha)]^{-1} = \frac{1}{1} \begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ +sin\alpha & 0 & cos\alpha \end{bmatrix}$

$= \begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ +sin\alpha & 0 & cos\alpha \end{bmatrix}$ .....(1)

Now, $F(-\alpha) = \begin{bmatrix} cos(-\alpha) & 0 & sin(-\alpha) \\ 0 & 1 & 0 \\ -s9n(-\alpha) & 0 & cos(-\alpha) \end{bmatrix}$

$= \begin{bmatrix} cos\alpha & 0 & -sin\alpha \\ 0 & 1 & 0 \\ sin\alpha & 0 & cos\alpha \end{bmatrix}$ .....(2)

[$\because \cos\alpha$ is an even function, $\cos(-\alpha) = \cos\alpha$ and $\sin\alpha$ is an odd function, $\sin(-\alpha) = -\sin\alpha$]

From (1) and (2)

$[F(\alpha)]^{-1} = F(-\alpha)$

2. The matrix form of the system is AX = B, where

$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$.

We find $|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2(4 + 1) - 3(-2 - 3) + 3(-1 + 6) = 10 + 15 + 15 = 40 \neq 0.$

So, A^{-1} exists and

$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{40} \begin{bmatrix} +(4+1) & -(-2-3) & +(-1+6) \\ -(-6+3) & +(-4-9) & -(-2-9) \\ +(3+6) & -(2-3) & +(-4-3) \end{bmatrix}^T = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$

Then, applying $X = A^{-1}B$, we get

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

So, the solution is $(x_1 = 1, x_2 = 2, x_3 = -1)$.

3. Given $A = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$AB = \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-3 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4.I_3$

$BA = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4.I_3.$

So, we get AB = BA = $4.I_3$

$\Rightarrow (\frac{1}{4}A)B = B(\frac{1}{4}A) = 1$

$\Rightarrow B^{-1} = \frac{1}{4} = 1$

Writing the given set of equations in matrix form we get,

$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = [\frac{1}{4}A] \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

$= \frac{1}{4} \begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$

$= \frac{1}{4} \begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

$\therefore$ x = 2, y = 1, z = -1

Hence, the solution set is {2, 1, - 1}.

4. Let the cost of one dosa be Rs. x

The cost of one idli be Rs. y

and the cost of one vadai be Rs. z

By the given data,

2x+ 3y + 2z = 150

2x + 2y + 4z = 200

5x + 4y + 2z = 250

$\therefore \Delta = \begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix}$

$= 2\begin{vmatrix} 2 & 4 \\ 4 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 4 \\ 5 & 2 \end{vmatrix} + 2\begin{vmatrix} 2 & 2 \\ 5 & 4 \end{vmatrix}$

= 2(4 - 16) - 3(4 - 20) + 2(8 - 10)

= 2(- 12) - 3(- 16) + 2(- 2)

= - 24 + 48 - 4 = 20

$\Delta_1 = \begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 250 & 4 & 2 \end{vmatrix}$

Taking 50 common from C3 we get,

$= 100\begin{vmatrix} 3 & 3 & 1 \\ 4 & 2 & 2 \\ 5 & 4 & 1 \end{vmatrix}$

$= 100\left[3\begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & 2 \\ 5 & 4 \end{vmatrix}\right]$

= 100[3(2 - 8) - 3(4 - 10) + 1(16 - 10)]

= 100[3(-6) - 3(- 6) + 6]

= 100[- 18 + 18 + 6] = 600

$\Delta_2 = \begin{vmatrix} 2 & 150 & 2 \\ 2 & 200 & 4 \\ 5 & 250 & 2 \end{vmatrix} = 100\begin{vmatrix} 2 & 3 & 1 \\ 2 & 4 & 2 \\ 5 & 5 & 1 \end{vmatrix}$

$= 100\left[3\begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & 2 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 5 & 5 \end{vmatrix}\right]$

= 100[2(4 - 10) - 3(2 - 10) + 1(10 - 20)]

= 100[2(- 6) - 3(- 8) + 1(- 10)]

= 100[- 12 + 24 - 10] = 100 [2] = 200

$\Delta_3 = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 5 & 4 & 250 \end{vmatrix} = 50\begin{vmatrix} 2 & 3 & 3 \\ 2 & 2 & 4 \\ 5 & 4 & 5 \end{vmatrix}$

$= 50\left[2\begin{vmatrix} 2 & 4 \\ 4 & 5 \end{vmatrix} - 3\begin{vmatrix} 2 & 4 \\ 5 & 5 \end{vmatrix} + 3\begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix}\right]$

= 50 [2(10 - 16) - 3(10 - 20) + 3(8 - 10)]

= 50[2(- 6) - 3(- 10) +3(- 2)]

= 50 [- 12 + 30 - 6] = 50 [12] = 600

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$\therefore x = \frac{\triangle_1}{\triangle} = \frac{600}{20} = 30$

$y = \frac{\triangle_2}{\triangle} = \frac{200}{20} = 10$

$z = \frac{\triangle_3}{\triangle} = \frac{600}{20} = 30$

Hence, the price of one dosa be Rs. 30, one idli be Rs. 10 and the price of 1 vadai be Rs. 30.

Also the cost on dosa, six idlies and six vadai is

= 3x + 6y + 6z = 3(30) + 6(10) + 6(30)

= 90 + 60 + 180 = Rs. 330

Since the family had Rs. 350 in hand, they will be able to manage to pay the bill.

5. Transforming the augmented matrix to echelon form, we get

$$\left[\begin{array}{ccc|c} 4 & 3 & 6 & 25 \\ 1 & 5 & 7 & 13 \\ 2 & 9 & 1 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 4 & 3 & 6 & 25 \\ 2 & 9 & 1 & 1 \end{array}\right] \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 4R_1} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -27 \\ 0 & -1 & -13 & -25 \end{array}\right] \xrightarrow[R_3 \to R_3 \div (-1)]{R_2 \to R_2 \div (-1)} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & 17 & 22 & 27 \\ 0 & 1 & 13 & 25 \end{array}\right] \xrightarrow{R_3 \to 17R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & 17 & 22 & 27 \\ 0 & 0 & 199 & 398 \end{array}\right].$$

The equivalent system is written by using the echelon form:

x + 5y + 7 = 13, ... (1)

17y + 22z = 27, ... (2)

199z = 398..... (3)

From (3), we get $z = \frac{398}{199} = 2$.

Substituting z = 2 in (2), we get $y = \frac{27 - 22 \times 2}{17} = \frac{-17}{17} = -1$

Substituting z = 2, y = -1, in (1), we get x = 13 - 5 x (-1) - 7 × 2 = 4.

So, the solution is (x = 4, y = -1, z = 2).

6. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$

Applying Gauss - Jordan method, we get

$$[A|I_3] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 2R_1 R_3 \to R_3 - R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + 8R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 \times (-1)} \left[\begin{array}{ccc|ccc} 11 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

So we get $A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$.

7. $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$

Put $\frac{1}{x} = u, \frac{1}{y} = v, \frac{1}{z} = w$

We get 3u - 4v - 2w = 1, u + 2v + w = 2, 2u - 5v - 4w = -1

$$\therefore \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix} = 3\begin{vmatrix} 2 & 1 \\ -5 & -4 \end{vmatrix} + 4\begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & -5 \end{vmatrix}$$

= 3(- 8 + 5) + 4'(- 4 - 2) - 2(- 5 - 4)

= 3(- 3) + 4(- 6) - 2(- 9)

= - 9 - 24 + 18 = -15

$$\Delta_1 = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix}$$

$$= 1\begin{vmatrix} 2 & 1 \\ -5 & -4 \end{vmatrix} + 4\begin{vmatrix} 2 & 1 \\ -1 & -4 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ -1 & -5 \end{vmatrix}$$

= 1(- 8 + 5) + 4(- 8 + 11 -2(-10 + 2)

= 1(- 3) + 4(-7) - 2(- 8)

= - 3 - 28 + 16= -15

$$\Delta_2 = \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix}$$

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$3\begin{vmatrix} 2 & 1 \\ -1 & -4 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 2 & -4 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

= 3(- 8 + 1) - 1(- 4 - 2) - 2(- 1 - 4)

= 3(-7) - 1(- 6) - 2(- 5)

= -21 + 6 + 10 = -5

$\Delta_3 = \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix}$

$3\begin{vmatrix} 2 & 2 \\ -5 & -1 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 2 & -5 \end{vmatrix}$

= 3(-2 + 10) + 4(-1 - 4)+ 1(-5 - 4)

= 3(8) + 4(- 5) + 1(- 9)

= 24 - 20 - 9 = - 5

$\therefore \frac{\triangle_1}{\triangle} = \frac{-15}{-15} = 1 \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1$

$v = \frac{\triangle_2}{\triangle} = \frac{-5}{-15} = \frac{1}{3} \Rightarrow \frac{1}{y} = \frac{1}{3} \Rightarrow y = 3$

$w = \frac{\triangle_3}{\triangle} = \frac{5}{-15} = \frac{1}{3} \Rightarrow \frac{1}{z} = \frac{1}{3} \Rightarrow z = 3$

$\therefore$ Solution set is {1, 3, 3}

Answer any THREE questions

8. For the given A, We get |A| = (2)(7) - (9)(1) = 14 - 9 = 5. So, $A^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -9 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{7}{5} & -\frac{9}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$.

   Then, $(A^{-1})^T = \begin{bmatrix} \frac{7}{5} & -\frac{1}{5} \\ -\frac{9}{5} & \frac{2}{5} \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$. ....(1)

   For the given A, We get $A^T = \begin{bmatrix} 2 & 1 \\ 9 & 7 \end{bmatrix}$. So $|A^T| = (2)(7) - (1)(9) = 5$

   Then, $(A^T)^{-1} = \frac{1}{5}\begin{bmatrix} 7 & -1 \\ -9 & 2 \end{bmatrix}$. ...(2)

   From (1) and (2), we get $(A^{-1})^T = (A^T)^{-1}$. Thus, we have verified the given property.

9. Given adj $A = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$

   We know that $A = \pm\frac{1}{\sqrt{|adj A|}}$ adj (adj A)..(1)

   $|\text{adj } A| = 2\begin{vmatrix} 12 & -7 \\ 0 & 2 \end{vmatrix} + 4\begin{vmatrix} -3 & -7 \\ -2 & 2 \end{vmatrix} + 2\begin{vmatrix} -3 & 12 \\ -2 & 0 \end{vmatrix}$

   [Expanded along R_1]

   = 2(24-0)+4(-6-14)+2(0+24)

   = 2(24)+4(-20)+2(24) = 48-80+48

   = 96-80 = 16

   Now, adj (adj A)

$$= \begin{bmatrix} +\begin{vmatrix} 12 & -7 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} -3 & -7 \\ -2 & 2 \end{vmatrix} & +\begin{vmatrix} -3 & 12 \\ -2 & 0 \end{vmatrix} \\ -\begin{vmatrix} -4 & 2 \\ 0 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 2 \\ -2 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & -4 \\ -2 & 0 \end{vmatrix} \\ +\begin{vmatrix} -4 & 2 \\ 12 & -7 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ -3 & -7 \end{vmatrix} & +\begin{vmatrix} 2 & -4 \\ -3 & 12 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} +(24-0)-(6-14)+(0+24) \\ -(-8-0)+(4+4)-(0-8) \\ +(28-24)-(-14+6)+(24-12) \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{bmatrix}^T = \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix}$$

$$= 4\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

   Substituting (2) and (3) in (1) we get,

   $A = \frac{1}{\sqrt{16}}.4\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$

   $A = \pm\frac{4}{4}\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix} = \pm\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$

10. The matrix form of the system is AX = B , where $A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

    We find $|A| = \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = 10 - 6 = 4 \neq 0$. So, A^{-1} exists and $A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

    Then, applying the formula $X = A^{-1}B$, we get

    $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix} = \begin{bmatrix} \frac{-4}{4} \\ \frac{16}{4} \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$.

    So the solution is (x = -1, y = 4).

11. 5x - 2y + 16 = 0, x + 3y - 7 = 0

    Given $\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix}$ = 15+2 = 17

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$\Delta_1 = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix}$ = -48+14 = -34

$\Delta_2 = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix}$ = 35+16 = 51

$\therefore x = \frac{\Delta_1}{\Delta} = \frac{-34}{7} = -2$

$y = \frac{\Delta_2}{\Delta} = \frac{51}{17}$

$\therefore$ Solution set is {-2, 3}

Answer any THREE questions

12. We first find adj A. By definition, we get adj A = $\begin{bmatrix} +M_{11} & -M_{12} \\ -M_{21} & +M_{22} \end{bmatrix}^T = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T = \begin{bmatrix} d & -c \\ -c & a \end{bmatrix}$.

Since A is non-singular, |A| = ad - bc $\neq$ 0.

As $A^{-1} = \frac{1}{|A|}$ adj A, we get $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

13. Let A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. Then, $A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

So, we get

$AA^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

Similarly, we get $A^TA = I_2$. Hence $AA^T = A^TA = I_2 \Rightarrow$ A is orthogonal.

14. $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

Let A = $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

A is a matrix of order (2×4)

$\therefore \rho(A) \leq \min(2, 4) = 2$

The highest order of minor of A is 2

It is $\begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix}$ = -6 + 6 = 0

Also, $\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix}$ = -1 + 0 = -1 $\neq$ 0

$\therefore \rho(A) = 2$

15. Given A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$...(1)

We know A (adj A) = (adj A) A = |A|. I_2 ...(2)

Comparing (1) and (2), we get |A| = 10

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Multiple Choice Question

16. (c) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$
17. (b) -80
18. (d) -1
19. (a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$
20. (b) $(A^T)^2$
21. (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$
22. (a) $\frac{-4}{5}$
23. (b) $\left(\cos^2\frac{\theta}{2}\right)A^T$
24. (c) 19
25. (b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$