Visit QB365-SLMS - https://www.qb365.in/slms/ Complete Questions and Answers

Answers Key:

Answer all

1. We find that $|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{vmatrix}$ = 8(21 - 16) + 6(-18 + 8) + 2(24 - 14) = 40 - 60 + 20 = 0

   By the definition of adjoint, we get

   $\text{adj A} = \begin{bmatrix} (21-16) & -(-18+8) & (24-14) \\ -(-18+8) & (24-4) & -(32+12) \\ (24-14) & -(-32+12) & (56-36) \end{bmatrix}^T = \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$

   So, we get

   $\text{A(adj A)} = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$

   $= \begin{bmatrix} 40-60+20 & 80-120+40 & 80-120+40 \\ -30+70-40 & -60+140-80 & -60+140-80 \\ 10-40+30 & 20-80+60 & 20-80+60 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0I_3 = |A|I_3,$

   Similarly, we get

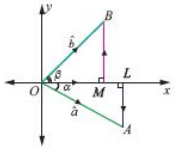
   $\text{(adj A)A} = \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

   $= \begin{bmatrix} 40-60+20 & -30+70-40 & 10-40+30 \\ 80-120+40 & -60+140-80 & 20-80+60 \\ 80-120+40 & -60+140-80 & 20-80+60 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0I_3 = |A|I_3.$

   Hence, A(adj A) = (adj A)A = $|A|I_3$.

2. Let the general equation of the circle be
   $x^2 + y^2 + 2gx + 2fy + c = 0$ ..... (1)
   It passes through points (1, 1), (2, -1) and(3, 2) .
   Therefore,
   2g+2f+c = -2 ..... (2)
   4g-2f+c = -5 .....(3)
   6g+4f+c = -13 ..... (4)
   (2) – (3) gives -2g + 4f = 3 ... (5)
   (4) – (3) gives 2g + 6f = -8 ....(6)
   (5) + (6) gives $f = -\frac{1}{2}$
   Substituting $f = -\frac{1}{2}$ in (6), $g = -\frac{5}{2}$
   Substituting $f = -\frac{1}{2}$ and $g = -\frac{5}{2}$ in (2), c = 4 .
   Therefore the required equation of the circle is
   $x^2 + y^2 + 2\left(-\frac{5}{2}\right)x + 2\left(-\frac{1}{2}\right)y + 4 = 0$ and $x^2 + y^2 - 5x - y + 4 = 0$.

3. Let $\hat{a} = \vec{OA}$ and $\hat{b} = \vec{OB}$ be the unit vectors and which make angles α and β, respectively, with positive x-axis, where A and B are as in the diagram.
   Draw AL and BM perpendicular to the x-axis. Then $\left|\vec{OL}\right| = \left|\vec{OA}\right| \cos\alpha = \cos\alpha$, $\left|\vec{LA}\right| = \left|\vec{OA}\right| \sin\alpha = \sin\alpha$
   So, $\vec{OL} = \left|\vec{OL}\right|\hat{i} = \cos\alpha\,\hat{i}$, $\overrightarrow{LA} = \sin\alpha\,(-\hat{j})$
   Therefore, $\hat{a} = \overrightarrow{OA} = \overrightarrow{OL} + \overrightarrow{LA} = \cos\alpha\,\hat{i} - \sin\alpha\,\hat{j}$ ..(1)
   Similarly $\hat{b} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$ ....(2)
   The angle between $\hat{a}$ and $\hat{b}$ is α + β and so,
   $\hat{a}.\hat{b} = |\hat{a}|\,|\hat{b}| \cos(\alpha+\beta) = \cos(\alpha+\beta)$ ... (3)

   On the other hand, from (1) and (2)
   $\hat{a}.\hat{b} = (\cos\alpha\hat{i} - \sin\alpha\hat{j})(\cos\beta\hat{i} - \sin\beta\hat{j}) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$....(4)
   From (3) and (4), we get $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

4. Given $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$, $|z_1 + z_2 + z_3| = 1$
   $|z_1|^2 = 1^2 \Rightarrow z_1\overline{z_1} = 1 \Rightarrow z_1 = \frac{1}{\overline{z_1}}$
   $|z_2|^2 = 4 \Rightarrow z_2\overline{z_2} = 1 \Rightarrow z_2 = \frac{4}{\overline{z_2}}$
   $|z_3|^2 = 9 \Rightarrow z_3\overline{z_3} = 1 \Rightarrow z_3 = \frac{9}{\overline{z_3}}$
   $\therefore \left|9, \frac{1}{\overline{z_1}}.\frac{4}{\overline{z_2}} + 4.\frac{1}{\overline{z_1}}.\frac{9}{\overline{z_3}} + \frac{4}{\overline{z_2}}.\frac{9}{\overline{z_3}}\right|$

$\left|\frac{36}{z_1 z_2}+\frac{36}{z_1 z_3}+\frac{36}{z_2 z_3}\right|=\left|36\left(\frac{\overline{z_3}+\overline{z_2}+\overline{z_1}}{z_1 z_2 z_3}\right)\right|$

$\left[\because|\overline{z_1}+\overline{z_2}+\overline{z_3}|=|z_1+z_2+z_3|\right]$

$=\frac{36|z_1+z_2+z_3|}{|z_1||z_2||z_3|}=36\frac{|\overline{z_1}+\overline{z_2}+\overline{z_3}|}{|z_1||z_2||z_3|}$

$\left[\because|\overline{z_1}|=|z_1|,|\overline{z_2}=|z_{21}|,|\overline{z_3}|=|\overline{z_3}|\right]$

$=\frac{36(1)}{1(2)(3)}=\frac{36}{6}=6$

$\therefore |9z_1 + z_2 + 4z_1z_3 + z_2z_3| = 6$

5. We find AB = $\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}=\begin{bmatrix}-4+4+8&4-8+4&-4-8+12\\-7+1+6&7-2+3&-7-2+9\\5-3-2&-5+6-1&5+6-3\end{bmatrix}$

$=\begin{bmatrix}8&0&0\\0&8&0\\0&0&8\end{bmatrix}=8I_3$

and BA = $\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}=\begin{bmatrix}-4+7+5&4-1-3&4-3-1\\-4+14-10&4-2+6&4-6+2\\-8-7+15&8+1-9&8+3-3\end{bmatrix}$

$=\begin{bmatrix}8&0&0\\0&8&0\\0&0&8\end{bmatrix}=8I_3$

So we get AB = BA = $8I_3$. That is, $(\frac{1}{8}A)B = B(\frac{1}{8}A) = I_3$. Hence, $B^{-1} = \frac{1}{8}A$.

Writing the given system of equations in matrix form, we get

$\begin{bmatrix}1&-1&1\\1&-2&-2\\2&1&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\9\\1\end{bmatrix}$.

That is $B\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}4\\9\\1\end{bmatrix}$.

So, $\begin{bmatrix}x\\y\\z\end{bmatrix}=B^{-1}\begin{bmatrix}4\\9\\1\end{bmatrix}$

$=(\frac{1}{8}A)\begin{bmatrix}4\\9\\1\end{bmatrix}=\frac{1}{8}\begin{bmatrix}-4&4&4\\-7&1&3\\5&-3&-1\end{bmatrix}\begin{bmatrix}4\\9\\1\end{bmatrix}=\frac{1}{8}\begin{bmatrix}-16+36+4\\-28+9+3\\20-27-1\end{bmatrix}=\frac{1}{8}\begin{bmatrix}24\\-16\\-8\end{bmatrix}=\begin{bmatrix}3\\-2\\-1\end{bmatrix}$

Hence, the solution is (x = 3, y = -2, z = -1).

6. Given z = x + iy and $\arg(\frac{z-i}{z+2})=\frac{\pi}{4}$

$\Rightarrow \arg(z-i) - \arg(z+2) = \frac{\pi}{4}$

$\Rightarrow \arg(x + iy-i) - \arg(x+iy+2) = \frac{\pi}{4}$

$\Rightarrow \arg(x+i(y-1)-\arg((x+2)+iy) = \frac{\pi}{4}$

$\Rightarrow tan^{-1}\left(\frac{y-1}{x}\right)-tan^{-1}\left(\frac{y}{x+2}\right)=\frac{\pi}{4}$

$\Rightarrow tan^{-1}\left(\frac{\frac{y-1}{x}-\frac{y}{x+2}}{1+\frac{y-1}{x}\cdot\frac{y}{x+2}}\right)$

$=\frac{\pi}{4}\left[\because tan^{-1}x-tan^{-1}y=tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$\Rightarrow \frac{\left(\frac{(x+2)(y-1)-xy}{x(x+2)}\right)}{\left(\frac{x(x+2)+y(y-1)}{x(x+2)}\right)}=\tan\frac{\pi}{4}=1$

$\Rightarrow \frac{(x+2)(y-1)-xy}{x(x+2)+y(y-1)} = 1$

$\Rightarrow$ -x + 2y-2 = x^2+ 2x + y^2-y

$\Rightarrow x^2 + 2x + y^2-y + x-2y + 2 = 0$

$\Rightarrow x^2 + y^2+3x-3y + 2 = 0$

Hence proved.

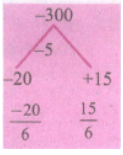
7. This equation is Type- 2 even degree reciprocal equation. Hence, it can be rewritten as

$6(x^2+\frac{1}{x^2})-5(x+\frac{1}{x})-38=0\ldots(1)$

Put $x+\frac{1}{x}=y\Rightarrow\left(x+\frac{1}{x}\right)^2=y^2$

$\Rightarrow x^2+\frac{1}{x^2}+2=y^2\Rightarrow x^2+\frac{1}{x}=y^2-2$

$\therefore$ (1)becomes as,



$6(y^2-2)-5y-38=0$

$\Rightarrow 6y^2-12-5y-38=0$

$\Rightarrow 6y^2-5y-50=0$

$\Rightarrow (3y-10)(2y+5)=0$

$\Rightarrow y=\frac{10}{3},\frac{-5}{2}$

Case (i) when $y=\frac{+10}{3}$

$x+\frac{1}{x}=\frac{+10}{3}$

$\frac{x^2+1}{x}=\frac{+10}{3}\Rightarrow 3x^2+3=10x$

$3x^2-10x+3=0$

$(x-3)(x-\frac{1}{3})=0\Rightarrow x=3,\frac{1}{3}$

QB365 TM Study Material
QUESTION BANK 365

Case (ii) when $y = -\frac{5}{2} \Rightarrow x + \frac{1}{x} = -\frac{5}{2}$

$\Rightarrow \frac{x^2+1}{x} = \frac{-5}{2}$

$\Rightarrow 2x^2 + 2 + 5x = 0$

$\Rightarrow 2x^2 + 5x + 2 = 0$

$\Rightarrow (x + 2)(2x + 1) = 0$

$\Rightarrow x = -2, \frac{-1}{2}$

$\therefore$ The roots are $3, \frac{1}{3}, -2, \frac{-1}{2}$

8. Let the pump A can fill the tank in x minutes, and the pump B can fill. the tank in y minutes In 1 minute A can fill $\frac{1}{x}$ units and in 1 minute B can fill $\frac{1}{y}$ units.

$\therefore \frac{1}{x} + \frac{1}{y}$ = 10

and $\frac{1}{x} - \frac{1}{y}$ = 30

Put $\frac{1}{x}$ = a and $\frac{1}{y}$ = b

$\Rightarrow$ a + b = $\frac{1}{10}$ .....(1)

and a - b = $\frac{1}{30}$ .....(2)

$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$ = -1-1= -2

$\Delta_1 = \begin{vmatrix} \frac{1}{10} & 1 \\ \frac{1}{30} & -1 \end{vmatrix} = \frac{-1}{10} - \frac{1}{30}$

$= \frac{-3-1}{30} = \frac{-4}{30} = \frac{-2}{15}$

$\Delta_2 = \begin{vmatrix} 1 & \frac{1}{10} \\ 1 & \frac{1}{30} \end{vmatrix} = \frac{1}{30} - \frac{1}{10} = \frac{1-3}{30}$

$= \frac{-2}{30} = \frac{-1}{15}$

$\therefore$ a = $\frac{\Delta_1}{\Delta} = \frac{\frac{-2}{15}}{-2} = \frac{1}{15} \Rightarrow \frac{1}{x} = \frac{1}{15} \Rightarrow$= x = 15

b = $\frac{\Delta_2}{\Delta} = \frac{\frac{-1}{15}}{-2} = \frac{1}{30} \Rightarrow \frac{1}{y} = \frac{1}{30} \Rightarrow$y = 30

Hence the pump A can fill the tank in 15 minutes and the pump B can fill the tank in 30 minutes.

9. Now, $tan^{-1}\left(\frac{d}{1+a_1a_2}\right) = tan^{-1}\frac{a_n-a_1}{1+a_1a_n} = tan^{-1}a_2 - tan^{-1}a_1$

Similarly, $tan^{-1}\left(\frac{d}{1+a_1a_2}\right) = tan^{-1}\left(\frac{a_n-a_1}{1+a_1a_n}\right) = tan^{-1}a_3 - tan^{-1}a_2$

Continuing inductively, we get

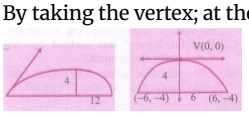
$tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right) = tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_na_{n-1}}\right) = tan^{-1}a_n - tan^{-1}a_{n-1}$

Adding vertically, we get

$tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots + tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right) tan[tan^{-1}a_n - tan^{-1}a_1]$

$tan\left[tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots + tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right] = tan\left[tan^{-1}a_n - tan^{-1}a_1\right] = tan\left[tan^{-1}\left(\frac{a_n-a_1}{1+a_1a_n}\right)\right] = \frac{a_n-a_1}{1+a_1a_n}$

10. By taking the vertex; at the origin, the parabola is open downward.



Its equation is x^2 = -4ay

It passes through (6, -4)

$\therefore$ 36 = -4a(-4) $\Rightarrow$ 4a = - $\frac{36}{4}$ = 9

$\therefore$ (1) becomes, x^2 = -9y

To find the slope at (-6, -4)

Differentiating (1) with respect to 'x' we get,

2x = $-9\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{9}$

At (-6, -4), $\frac{dy}{dx} = -2\frac{(-6)}{9} = \frac{12}{9} = \frac{4}{3}$

$\therefore tan\theta = \frac{4}{3} \Rightarrow \theta = tan^{-1}\left(\frac{4}{3}\right)$

$\therefore$ The angle of projection is $tan^{-1}\left(\frac{4}{3}\right)$

11. We observe that the required plane is parallel to the vectors $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{c} = \hat{i} + \hat{j} - \hat{k}$ and passing through the point(0,1,-5) with position vector $\vec{a}$. We observe that $\vec{b}$ is not parallel to $\vec{c}$.

Then the vector equation of the plane in non-parametric form is given by $(\vec{r} - \vec{a}).(\vec{b} \times \vec{c}) = 0$ ..(1)

Substituting $\vec{a} = \hat{j} - 5\hat{k}$ and $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix} = -9\hat{i} + 8\hat{j} - \hat{k}$ in equation (1), we get

$(\vec{r}(-\hat{j} - 5\hat{k})).(-9\hat{i} + 8\hat{j} - \hat{k}) = 0$, which implies that

$\vec{r}(-9\hat{i} + 8\hat{j} - \hat{k}) = 13$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector of an arbitrary point on the plane, then from the above equation, we get the Cartesian equation of the plane as -9x + 8y - z = 13 or 9x - 8y + z + 13 = 0

12. The table for the operation x_{11} is as follows.

QB365 QUESTION BANK 365 Study Material

x_{11}	1	3	4	5	9
1	**1**	3	4	5	9
3	3	**9**	1	4	5
4	4	1	**5**	9	3
5	5	4	9	**3**	1
9	9	5	3	1	**4**

Following the same kind of procedure as explained in the previous example, a brief outline of the process of verification of the properties of $\times_{11}$ on A is given below.

(i) Since each box has an unique element of A, $\times_{11}$ is a binary operation on A.

(ii) The entries are symmetrical about the main diagonal. Hence $\times_{11}$ has commutative property.

(iii) As usual, the associative property can be seen to be true.

(iv) The entries of both the row and column headed by the element 1 are identical. Hence 1 is the identity element.

(v) Since the identity 1 exists in each row and each column, the existence of inverse property is assured for $\times_{11}$. The inverse of 1 is 1, that of 3 is 4, that of 4 is 3, 5 is 9, and, that of 9 is 5.

13.

p	q	p ∧ q	p ∨ q	~p ∨ q	(p ∧ q) ∧ ~(p ∨ q)
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

The statement (p ∧ q) ∧ ~(p ∨ q)

14.

p	q	r	~ q	~q V r	p→(¬qVr)	~p	~pV(~qVr)
T	T	T	F	T	T	F	T
T	T	F	F	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	T	T	F	T
F	T	T	F	T	T	T	T
F	T	F	F	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

QB 365 TM Study Material
QUESTION BANK 365

From the table, it is clear that the column of p→(¬q V ~r) and ~pV(~q V r) are identical

∴ p→(¬q V ~r) ≡ ~pV(~q V r)

Hence proved.

Answer any-7

15. Since $2-\sqrt{3}i$ is a root of the required polynomial equation with real coefficients, $2+\sqrt{3}i$ is also a root. Hence the sum of the roots is 4 and the product of the roots is 7. Thus $x^2-4x+7=0$ is the required monic polynomial equation.

16.

$|z+3+4i| \leq |z|+|3+4i| = 2+5 = 7$

$|z+3+4i| \leq 7$ ..... (1)

$|z+3+4i| \geq ||z|-|3+4i|| = |2-5| = 3$

$|z+3+4i| \geq 3$ ..... (2)

From (1) and (2) we get, $3 \leq |z+3+4i| \leq 7$

17. The matrix form of the system is AX = B , where $A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

We find $|A| = \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = 10 - 6 = 4 \neq 0$. So, A^{-1} exists and $A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

Then, applying the formula $X = A^{-1}B$, we get

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix} = \begin{bmatrix} \frac{-4}{4} \\ \frac{16}{4} \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$.

So the solution is (x = -1, y = 4).

18. We get $[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} [\vec{a}, \vec{b}, \vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$

19. Given $2\cos\alpha = x + \frac{1}{x}$

$\Rightarrow 2\cos\alpha = \frac{x^2+1}{x}$

$\Rightarrow x^2+1 = 2x\cos\alpha$

$\Rightarrow x^2-2x\cos\alpha+1 = 0$

$\Rightarrow \frac{2\cos\alpha \pm \sqrt{(-2\cos\alpha)^2-4(1)(1)}}{2}$

$= \frac{2\cos\alpha \pm \sqrt{4\cos^2\alpha-4}}{2}$ $\left[\because \frac{b\pm\sqrt{b^2-4ac}}{2a}\right]$

$= \frac{2\cos\alpha \pm \sqrt{-\sin^2\alpha}}{2}$

$= \frac{2\cos\alpha \pm i\sin\alpha}{2}$ [$\because \sin^2\alpha+\cos^2\alpha = 1$]

$\Rightarrow x^2 = \cos\alpha \pm \sin\alpha$

Also, $2\cos\beta = y+\frac{1}{y}$

$\Rightarrow 2\cos\beta = \frac{y^2+1}{y}$

$\Rightarrow y^2-2y\cos\beta+1 = 0$

$\Rightarrow \frac{2\cos\beta \pm \sqrt{(-2\cos^2\beta^2-4(1)(1)}}{2}$

$= \frac{2\cos\beta \pm \sqrt{4\cos^2\beta-4}}{2} = \frac{2\cos\beta \pm 2i\sin\beta}{2}$

$\Rightarrow y = \cos\beta \pm i\sin\beta$

$x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

$x^m y^n = (\cos\alpha + i\sin m\alpha)(\cos n\beta + i\sin n\beta)$

$\cos(m\alpha+n\beta)+i\sin(m\alpha+n\beta)$

$\frac{1}{x^m y^n} = \cos(m\alpha+n\beta)-i\sin(m\alpha+n\beta)$

$$\therefore x^m y^n + \frac{1}{x^m y^n} = \cos(m\alpha+n\beta)+i\sin(\cancel{m\alpha+n\beta}) + \cos(m\alpha+n\beta)-i\sin(\cancel{m\alpha+n\beta})$$

$= 2\cos(m\alpha+n\beta)$

20. Equation of given plane is

$\vec{r}.\left(6\hat{i}+3\hat{j}+2\hat{k}\right) = 8$

$\therefore \vec{n}_1 = 6\hat{i}+3\hat{j}+2\hat{k}$

and the line is $\vec{r} = \left(2\hat{i}-\hat{j}+\hat{k}\right)+t\left(\hat{i}+2\hat{j}-2\hat{k}\right)$

$\therefore \vec{b} = \hat{i}+2\hat{j}-2\hat{k}$

Angle between a line and a plane is

$sin\theta = \frac{|\vec{b}.\vec{n}|}{|\vec{b}||\vec{n}|} = \frac{|6(1)+3(2)+2(-2)|}{\sqrt{1^2+2^2+(-2)^2}.\sqrt{6^2+3^2+2^2}}$

$= \frac{6+6-4}{\sqrt{9}.\sqrt{36+9+4}} = \frac{8}{\sqrt{9}.\sqrt{49}} = \frac{8}{3(7)} = \frac{8}{21}$

$\therefore \theta = sin^{-1}\left(\frac{8}{21}\right)$

21. $cot^{-1}(1)+sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-sec^{-1}(-\sqrt{2})$

Let $\Rightarrow cot\,x = 1 \Rightarrow tan\,x = 1 \Rightarrow tan\,x = tan\frac{\pi}{4}$

$\Rightarrow x = \frac{\pi}{4}$

$sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = y \Rightarrow sin\,y = \frac{-\sqrt{3}}{2} = -sin\frac{\pi}{3}$

$\Rightarrow sin\,y = sin\left(\frac{-\pi}{3}\right)$

$\Rightarrow y = \frac{-\pi}{3}$

$sec^{-1}\left(-\sqrt{2}\right) = z$

$\Rightarrow sec\,z = -\sqrt{2} \Rightarrow cosz = \frac{-1}{\sqrt{2}}$

$\Rightarrow cos\,z = -cos\frac{\pi}{4}$

$\Rightarrow cos\,z = cos\left(\pi-\frac{\pi}{4}\right)$

$\Rightarrow cos\,z = cos\left(\frac{3\pi}{4}\right)$

$\Rightarrow z = \frac{3\pi}{4}$

$\therefore cot^{-1}(1)+sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)-sec^{-1}\left(\frac{\sqrt{3}}{2}\right)-sec^{-1}\left(-\sqrt{2}\right)$

$= \frac{\pi}{4}-\frac{\pi}{3}-\frac{3\pi}{4}$

$= \frac{3\pi-4\pi-9\pi}{12} = \frac{-10^5\pi}{12_6}$

$= -\frac{5\pi}{6}$

$\therefore cot^{-1}(1)+sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-sec^{-1}(-\sqrt{2}) = -\frac{5\pi}{6}$

22. Truth Table for ~(p ∧ ~q)

p	q	~q	p ∧ ~q	~(p ∧ ~q)
T	T	F	F	T
T	F	T	T	F
F	T	T	F	T
F	F	T	F	T

23. Truth Table for (p V q) ∧ ~q

p	q	p V q	~q	(p V q) ∧ ~q
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

24. Truth Table for (~p ⟶ r) ∧ (p ↔ q)

QB 365 Study Material
QUESTION BANK 365

p	q	r	~ p	~ p → r	p ↔ q	(~p → r) ∧ (p ↔ q)
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	F	F

Answer any-7

25. We first find adj A. By definition, we get adj A = $\begin{bmatrix} +M_{11} & -M_{12} \\ -M_{21} & +M_{22} \end{bmatrix}^T = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T = \begin{bmatrix} d & -c \\ -c & a \end{bmatrix}$.
Since A is non-singular, |A| = ad - bc ≠ 0.
As $A^{-1} = \frac{1}{|A|}$ adj A, we get $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

26. We know that $\sin^{-1}$: [-1, 1] → $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$is given by
$\sin^{-1}x = y$ if and only if x = sin y for $-1 \le x \le$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. Thus,
$Sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$, Since $\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$and $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

27. Let A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. Then, $A^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
So, we get
$AA^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$
Similarly, we get $A^TA = I_2$. Hence $AA^T = A^TA = I_2 \Rightarrow$ A is orthogonal.

28. $\overline{(2-i)^{12} + (2+i)^{12}} = \overline{(2-i)^{12}} + \overline{(2+i)^{12}}$
$\left(2+i\sqrt{3}\right)^{10} - \left(2-i\sqrt{3}\right)^{10}$
Now $\bar{z} = (2+\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$
$\bar{z} = \overline{(2+i\sqrt{3})^{10}} - (2-i\sqrt{3})^{10}$
$[\because \overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}]$
$= (2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$
$= -\left[(2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}\right]$
$\therefore \bar{z} = -z \Rightarrow$ z is purely imaginary
Hence $(2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$is purely imaginary

29. Given ends of diameter are (3, 4)(2, -7)
∴ Equation of the circle is
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$\Rightarrow (x - 3)(x - 2) + (y - 4)(y + 7) = 0$
$\Rightarrow x^2 - 2x - 3x + 6 + y^2 + 7y - 4y - 28 = 0$
$\Rightarrow x^2 + y^2 - 5x + 3y - 22 = 0$

30. Given |z| = 3
Show that $7 \le |z+6-8i| \le 13$
$|z+6-8i| \le |z| + |6-8i|$
[Triangle law of inequality]
$\le 3\sqrt{6^2 + (-8)^2} \le 3 + \sqrt{36+64} \le +\sqrt{1w}$
$|z+6-8i| \le 3+10 \le 13$ ..... (1)
Also $||z+6-8i| \ge |x| - |6-8i| \ge |3 - \sqrt{6^2 + (-8)^2}|$
$\ge |3 - \sqrt{36+84}| \ge |3-10| \ge |-7| \ge 7$ ..... (2)
From (1) and (2) we get,
$7 \le |z+6-8i| \le 13$.

31. Since the given three vectors are coplanar, we have $\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & m & 4 \end{vmatrix} = 0 \Rightarrow m = -3$

32. Given lines are $\vec{r} = (4\hat{i} - \hat{j}) + t(\hat{i} + 2\hat{j} - 2\hat{k})$ $[\vec{r} = \vec{a} + t\vec{b}]$
$\therefore \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$
and $\vec{r} = (\hat{i} - 2\hat{j} + 4\hat{k}) + s(-\hat{i} - 2\hat{j} + 2\hat{k})$
$\therefore \vec{d} = -\hat{i} - 2\hat{j} + 2\hat{k}$
Let θ be the angle between the given lines
Then $\cos\theta = \frac{\vec{b}.\vec{d}}{|\vec{b}||\vec{d}|}$
$= \frac{(\hat{i}+2\hat{j}-2\hat{k}).(-\hat{i}-2\hat{j}+2\hat{k})}{\sqrt{1^2+2^2+(-2)^2}.\sqrt{(-1)^2+(-2)^2+(2)^2}}$
$= \frac{-1-4-4}{\sqrt{9}.\sqrt{9}} = \frac{-9}{9} = -1$
$\therefore \cos\theta = -1$

QB 365 TM Study Material
QUESTION BANK 365

⇒ cos-1(-1)

⇒ θ = 0

33. $Re\,(z) = \frac{z+\bar{z}}{2}$ and Im$(z) = \frac{z-\bar{z}}{2i}$

Let z = x+iy where x is the Re(x) and y is the Im (z)

Then $\bar{z}$ = x-iy

z+$\bar{z}$ = x + iy + x - iy = 2x

$\therefore \frac{z+\bar{z}}{2}$ = x

$\frac{z+\bar{z}}{2}$ = Re(z)

Also z-$\bar{z}$ = x+iy-(x-iy)

= x+iy-x+iy = 2iy

$\frac{z-\bar{z}}{2i}$ = y

$\therefore \frac{z-\bar{z}}{2i}$ = Im(z)

Hence proved

34. $tan^{-1}\left(tan\frac{3\pi}{5}\right)$

Let us find $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$ such that tan θ = tan$\frac{3\pi}{5}$

Since the tangent function has period $\pi, tan\frac{3\pi}{5} = tan(\frac{3\pi}{5} - \pi) = tan(-\frac{2\pi}{5})$

Therefore, $tan^{-1}\left(tan\frac{3\pi}{5}\right) = tan^{-1}\left(tan\left(-\frac{2\pi}{5}\right)\right) = -\frac{2\pi}{5}, since \frac{-2\pi}{5} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$

Answer all

35. (b) $\frac{-1}{i+2}$

36. (d) $\sqrt{5}+2$

37. (b) $-\frac{3}{2}+2i$

38. (c) n complex roots

39. (a) 0

40. (b) $\frac{\pi}{6}$

41. Comparing the two given lines with

$\vec{r} = \vec{a} + t\vec{b}, \vec{r} = \vec{c} + s\vec{d}$

we have, $\vec{a} = -\hat{-}1 - 3\hat{j} - 5\hat{k}, \vec{b} = 3\hat{i} + 5\hat{j} + 7\hat{k}, \vec{c} = 2\hat{i} + 4\hat{j} + 6\hat{k}$ and $\vec{d} = \hat{i} + 4\hat{j} + 7\hat{k}$

We know that the two given lines are coplar, if $(\vec{c} - \vec{a}).(\vec{b} \times \vec{d})$=0

Here, $\vec{b} \times \vec{d} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 7\hat{i} - 14\hat{j} + 7\hat{k}$ and $\vec{c} - \vec{a} = 3\hat{i} + 7\hat{j} + 11\hat{k}$

Then, $(\vec{c} - \vec{a}).(\vec{b} \times \vec{d}) = (3\hat{i} + 7\hat{j} + 11\hat{k})(7\hat{i} - 14\hat{j} + 7\hat{k})$

Therefore the two given lines are coplanar.Then we find the non parametric form of vector equation of the plane containing the two given coplanar lines.

We know that the plane containing the two given coplanar lines is

$(\vec{r} - \vec{a}).(\vec{b} \times \vec{d})$=0

which implies that $(\vec{r} - (-\hat{i} - 3\hat{j} - 5\hat{k})).(7\hat{i} - 14\hat{j} + 7\hat{k})$=0. Thus, the required non-parametric vector equation of the plane containing the two given coplanar lines is $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k})$=0.

QB 365 TM Study Material
QUESTION BANK 365

42. (c) $\frac{\pi}{2} - x$

43. (a) [1, 2]

44. (c) $\frac{2}{\sqrt{3}}$

45. (a) (4, 7)

46. (b) $2(a^2+b^2)$

47. (a) 2ab

48. (c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}]$ = 0

49. (a) $\frac{\pi}{6}$

50. (a) 0°

51. (c) I_3

52. (d) -1

53. (a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$

54. (a) $e^{(\Delta_2 / \Delta_1)}, e^{(\Delta_3 / \Delta_1)}$