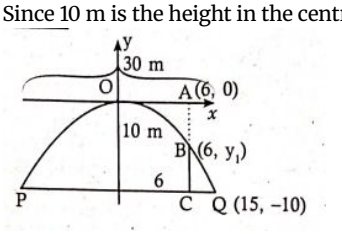
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Answers Key:

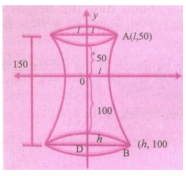
PART-D

ANSWER ALL OF THE FOLLOWING QUESTIONS :-

1. Let the general equation of the circle be
   $x^2 + y^2 + 2gx + 2fy + c = 0$ ..... (1)
   It passes through points (1, 1), (2, -1) and(3, 2) .
   Therefore,
   $2g+2f+c = -2$ ..... (2)
   $4g-2f+c = -5$ .....(3)
   $6g+4f+c = -13$ ..... (4)
   (2) – (3) gives $-2g + 4f = 3$ ... (5)
   (4) – (3) gives $2g + 6f = -8$ ....(6)
   (5) + (6) gives $f = -\frac{1}{2}$
   Substituting $f = -\frac{1}{2}$ in (6), $g = -\frac{5}{2}$
   Substituting $f = -\frac{1}{2}$ and $g = -\frac{5}{2}$ in (2), $c = 4$.
   Therefore the required equation of the circle is
   $x^2 + y^2 + 2\left(-\frac{5}{2}\right)x + 2\left(-\frac{1}{2}\right)y + 4 = 0$ and $x^2 + y^2 - 5x - y + 4 = 0$.
2. $x^2+3y^2 = 12$
   $\div 12$ we get, $\frac{x^2}{12} + \frac{y^2}{4} = 1$
   $\therefore a^2 = 12, b^2 = 4$
   The line x-y+ 4 = 0 can be rewritten as y = x+4.
   $\therefore m = 1, c = 4$
   The condition for y = mx + 4 to be a tangent to the ellipse is $c^2 = a^2m^2 + b^2$
   $\therefore (4)^2 = 12(1)^2 + 4$
   $\Rightarrow 16 = 12+4$
   $\Rightarrow 16 = 16$
   Since the condition is satisfied, the line x - y + 4 = 0 is a tangent to the ellipse $x^2 + 3y^2 = 12$.
   Also, the point of contact is $\left(-\frac{a^2m}{c}, \frac{b^2}{c}\right)$
   $\Rightarrow \left(-\frac{12(1)}{4}, \frac{4}{4}\right) \Rightarrow (-3, 1)$
   $\therefore$The point of contact is (-3, 1).
3. Let the parabolic bridge to a open downward parabola.
   Then its equation is $x^2 = -4ay$ .....(1)
   Since 10 m is the height in the centre (0, 0) and AD = 30 m.



   $\therefore$ A is the point (15, -10) which is in the IV quadrant A(15, -10) lies in (1)
   $15^2 = -4a(-10)$
   $\Rightarrow \frac{225}{10} = 4a \Rightarrow \frac{45}{2}$
   (1) becomes $x^2 = \frac{-45}{2}y$ .....(2)
   To find the height ofthe arch at x = 6 m, the point B(6, $-y_1$) lies in (2)
   $6^2 = \frac{-45}{2}(-y_1)$
   $\Rightarrow 36 = \frac{-45y_1}{2} \Rightarrow \frac{36\times 2}{45} = y_1$
   $\Rightarrow y_1 = \frac{8}{5} = 1.6$
   $\therefore$ Height of the arch on either sides
   = 10-1.6 = 8.4m
4. The cross section of a nuclear cooling tower is in the shape of a hyperbola.



   GIven OC = $\frac{1}{2}$ OD and CD = 150 m

Its equation is OC = 50 m & OD = 100 m

$\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$ .....(1)

Let I be the radius of the top of the tower

∴ A(l, 50) is a point on the hyperbola

∴ $\frac{l^2}{30^2}\frac{50^2}{44^2} = 1$

⇒ $\frac{l^2}{30^2} = 1 + \frac{50^2}{44^2} = \frac{44^2+50^2}{44^2}$

⇒ $l^2 = \frac{30^2}{44^2}$ (1936 + 2500)

⇒ $l^2 = \frac{30}{40}\sqrt{4436} = \frac{30}{44}(66.60)$

⇒ $\frac{30}{40}\sqrt{4436} = \frac{30}{44}$ = 45.41 m

Radius of the top of the tower is 45.41 m.

Let h be the radius of the base of the tower.

∴ B(h, 100) is a point on the hyperbola

∴ (1) becomes

$\frac{h^2}{30^2} - \frac{100^2}{44^2} = 1 \Rightarrow \frac{h^2}{30^2} = 1 + \frac{100^2}{44^2} = \frac{44^2+100^2}{44^2}$

⇒ $h^2 = \frac{30^2}{44^2}$ (1936 + 10000)

⇒ $h^2 = \frac{30}{44}\sqrt{11936} = \frac{30}{44}(109.25)$

⇒ $h = \frac{3277.5}{44}$ = 74.48 m.

Radius of the base of the tower is 74.48 m.

Diameter of the base = 148.96 m

Diameter of the top and base of the tower are 90.82 m and 148.96 m.

5. $18x^2 + 12y^2 - 144x + 48y + 120 = 0$

Given equation is

$18x^2 + 12y^2 - 144x + 48y + 120 = 0$

$18x^2 - 144x + 12y^2 + 48y = -120$

⇒ $18(x^2 - 8x) + 12(y^2 + 4y) = -120$

⇒ $18(x^2-8x+16-16)+ 12(y^2 +4y+4-4) =-120$

$18(x - 4)^2 - 288 + 12 (y + 2)^2 - 48 = -120$

⇒ $18(x - 4)^2 + 12(y + 2)^2 = -120 + 288 + 48$

⇒ $18(x - 4)^2 + 12(y + 2)^2 = 216$

Dividing by 216 we get,

$\frac{18(x-4)^2}{216} + \frac{12(y+2)^2}{216} = 1$

⇒ $\frac{(x-4)^2}{12} + \frac{(y+2)^2}{18} = 1$

This is an equation of the ellipse with major axis parallel to y-axis,

∴ $a^2 = 18, b^2 = 12$

∴ $c^2 = a^2 - b^2 = 18 - 12 = 6 \Rightarrow c = \sqrt{6}$

e = $\sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{12}{18}} = \sqrt{\frac{18-12}{18}}$

= $\sqrt{\frac{6}{18}_3} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$

(a) Center is (4, -2)

⇒ h = 4, k = -2

(b) Vertices are (h, k-a), (h, k + a)

⇒ $(4, -2 - 3\sqrt{2}), (4, -2 + 3\sqrt{2})$

[∴ $a^2 = 18 \Rightarrow a = \sqrt{18} = 3\sqrt{2}$]

(c) Foci are (h, k - c), (h, k + c)

⇒ $(4, -2 - \sqrt{6}), (4, -2 + \sqrt{6})$

(d) Equation of directrices are $y + 2 = \pm\frac{a}{e}$

⇒ $y + 2 = \pm\frac{a}{e}$

⇒ $y - 2 = \pm\frac{3\sqrt{2}}{\frac{\sqrt{2}}{\sqrt{3}}} = \pm 3\sqrt{2} \times \sqrt{3} = \pm 3\sqrt{6}$

⇒ $y + 2 = \pm 3\sqrt{6}, y + 2 = -3\sqrt{6}$

⇒ $y = -2 + 3\sqrt{6}$ and $y = -2 - 3\sqrt{6}$

6. We use the formula

$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{r=1}^{n} f\left(a + (b-a)\frac{r}{n}\right)$

Here $f(x) = 2x^2+3$, a = 1 and b = 4

So, we get

$f\left(a + (b-a)\frac{r}{n}\right) = f\left(1 + (4-1)\frac{r}{n}\right) = f\left(1 + \frac{3r}{n}\right) = 2\left(1 + \frac{3r}{n}\right)^2 + 3 = 5 + \frac{18r^2}{n^2} + \frac{12r}{n}$ Hence, we get

$\int_1^4 (2x^2 + 3)dx = \lim_{n\to\infty} \frac{3}{n} \sum_{r=1}^{n} \left(5 + \frac{18r^2}{n^2} + \frac{12r}{n}\right) = \lim_{n\to\infty} \left[\frac{15}{n} \sum_{r=1}^{n} 1 + \frac{54}{n^2} \sum_{r=1}^{n} r^2 + \frac{36}{n^2} \sum_{r=1}^{n} r\right]$

$= \lim_{n\to\infty} \left[\frac{15}{n}n + \frac{54}{n^3}(1^2 + 2^2 + \ldots + n^2) + \frac{36}{n^2}(1 + 2 + \ldots + n)\right]$

$= \lim_{n\to\infty} \left[15 + \frac{54}{n^3}\frac{n(n+1)(2n+1)}{6} + \frac{36}{n^2}\frac{n(n+1)}{2}\right]$

$= \lim_{n\to\infty} \left[15 + 9\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right) + 18\left(1 + \frac{1}{n}\right)\right]$

= 15+9(1+ 0)(2 + 0) +18(1+ 0) = 51.

7. Given $\begin{cases} k & 200 \le x \le 600 \\ 0 & otherwise \end{cases}$

(i) Since f{x) is a probability density function

$\int_{-\infty}^{\infty} f(x)d = 1 \Rightarrow \int_{200}^{600} kda = 1$

$\Rightarrow k[x]_{200}^{600} = 1 \Rightarrow k(600 - 200) = 1$

400 k = 1

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$\Rightarrow k = \frac{1}{400}$

(ii) The distribution function

= $\int_{-\infty}^{x} f(u)du$

Case 1: x < 200

$F(x) = \int_{-\infty}^{u} du = 0$

Case 1: x < 200 ≤ x ≤ 600

$\int_{-\infty}^{x} f(u)du$

$F(x) = \int_{-\infty}^{200} f(u)du = + \int_{200}^{x} f(u)du$

= $= \frac{1}{400}(x - 200) = \frac{x}{400} = \frac{1}{2}$

Case 3: x > 600

$F(x) = \int_{-\infty}^{u} du = 0$

$$f(x) = \begin{cases} 0, & x < 200 \\ \frac{x}{400} - \frac{1}{2}, & 200 \le x \le 600 \\ 0, & x > 600 \end{cases}$$

(iii) P(300 < x < 500)

= $\int_{300}^{500} kdx = \frac{1}{400}[x]_{300}^{500}$

= $\frac{1}{400}[500 - 300] = \frac{200}{400} = \frac{1}{2}$

8. $Let\ I = \int_0^{\pi} x\left[sin^2(sinx) + cos^2(cosx)\right]dx \quad \ldots(1)$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$I = \int_0^{\pi} (\pi - x)[sin^2(sin(\pi - x)) + cos^2(cos(\pi - x)]dx$

$= \int_0^{\pi} (\pi - x)[sin^2(sinx) + cos^2(-cosx)]dx$

$= \int_0^{\pi} (\pi - x)[sin^2(sin \quad x) + cos^2)(cosx)]dx$

$[\because cos(-x) = cos \quad x]$

$= \int_0^{\pi} \pi[sin^2(sinx) + cos^2(cosx)]dx$

$- \int_0^{\pi} x[sin^2(sinx) + cos^2(cosx)]dx$

$= \int_0^{\pi} \pi[sin^2(sinx) + cos^2(cosx)]dx - I[from \quad (1)]$

$2I = \pi \int_0^{\pi} [sin^2(sinx) + cos^2(cosx)]dx$

$I = \pi \int_0^{\pi/2} [sin^2(sin \quad x) + cos^2(cos \quad x)]dx \quad \ldots(2)$

$I = \pi \int_0^{\pi/2} [sin^2(sin\frac{\pi}{2} - x) + cos^2(cos\frac{\pi}{2} - x)]dx$

$I = \pi \int_0^{\pi/2} [sin^2(cosx) + cos^2(sin \quad x)]dx]. \ldots(3)$

Adding (2) and (3) we get,

$2I = \pi \int_0^{\pi/2} [sin^2(sinx) + cos^2(cosx)]dx + sin^2(cosx) + cos^2(sin \quad x)]dx$

$I = \pi \int_0^{\pi/2} (1 + 1)dx \ [\because sin^2\theta + cos^2\theta = 1]$

$= \pi(2)[x]_0^{\frac{\pi}{2}} = 2\pi\left[\frac{\pi}{2} - 0\right] = \pi^2$

$\therefore I = \frac{\pi^2}{2}$

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9. Since the given function is a probability mass function, the total probability is one. That is $\sum_x f(x) = 1$

From the given data k + 2k + 6k + 5k + 6k +10k+1

$30k = 1 \Rightarrow k = \frac{1}{30}$

Therefore the probability mass function is

x	1	2	3	4	5	6
f(x)	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{5}{30}$	$\frac{6}{30}$	$\frac{10}{30}$

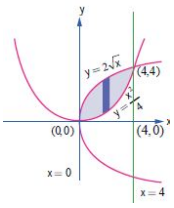
(i) P(2 < X < 6) = f(3)+ f(4)+ f(5) = $\frac{6}{30} + \frac{5}{30} + \frac{6}{30} = \frac{17}{30}$

(ii) P(2≤X≤5) = f(2)+f(3)+f(4) = $\frac{2}{30} + \frac{6}{30} + \frac{5}{30} = \frac{13}{30}$

(iii) P(2≤4) = f(1)+f(2)+f(3)+f(4) = $\frac{1}{30} + \frac{2}{30} + \frac{6}{30} + \frac{5}{30} = \frac{14}{30}$

(iv) P(3>X) = f(4)+f(5)+f(6) = $\frac{5}{30} + \frac{6}{30} + \frac{10}{30} = \frac{21}{30}$

10. First, we get the points of intersection of the parabolas. For this, we solve y^2 x = 4 and x^2 y = 4 simultaneously Eliminating y between them, we get x^4 = 64x and so x = 0 and x = 4. Then the points of intersection are (0, 0) and (4, 4). The required region is sketched.



Viewing in the direction of y -axis, the equation of the upper boundary is y = $2\sqrt{x}$ for $0 \le x \le 4$ and the equation of the lower boundary is $y = \frac{x^2}{4}$ for $0 \le x \le 4$. So, the required area Δ is

$A = \int_0^4 (y_U - Y_L)dx = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx = \left[2\left(\frac{2x^{3/2}}{3}\right) - \frac{x^3}{12}\right]_0^4 = \left[2\left(\frac{2\times 8}{3}\right) - \frac{64}{12}\right] - 0 = \frac{16}{3}$

11. Let X denote the winning amount. The possible events of selection are

(i) both balls are black, or

(ii) one white and one black or

(iii) both are white

Therefore X is a random variable that can be defined as

X (both are black balls) = Rs. 2(20) = Rs. 40

X (one black and one white ball) = Rs. 20 – Rs. 10 = Rs. 10

X (both are white balls) = (Rs. 20) = - Rs. 20

Therefore X takes on the values 40,10 and -20
Total number of balls n = 12
Total number of ways of selecting 2 balls = $\binom{12}{2} = \frac{12\times11}{1\times2} = 66$
Number of ways of selecting 2 black balls = $\binom{4}{2} = 6$
Number of ways of selecting one black ball and one white ball = $\binom{8}{1}\binom{4}{1} = 32$
Number of ways of selecting 2 white balls = $\binom{8}{2} = 28$

Values of Random Variable X	40	10	-20	Total
Number of elements in inverse images	6	32	28	66

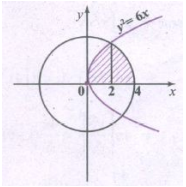
Probability mass function is

X	40	10	-20	Total
f (x)	$\frac{6}{66}$	$\frac{32}{66}$	$\frac{28}{66}$	1

Mean :
$E(X)\Sigma xf(x) = 40.\left(\frac{6}{66}\right) + 10.\left(\frac{32}{66}\right) + (-20).\left(\frac{28}{66}\right) = \frac{4000}{11}$
That is expected winning amount is 0
Variance :
$\Sigma x^2 = \Sigma x^2 f(x) = 40^2.\left(\frac{6}{66}\right) + 10^2.\left(\frac{32}{66}\right) + (-20)^2.\left(\frac{28}{66}\right) = \frac{4000}{11}$
$(E(X))^2 = 0^2 = 0$
This gives $V(X) = E(X^2) - (E(X))^2) = \frac{4000}{11} - 0 = \frac{4000}{11}$
Therefore E(X) = 0 and $V(x) = \frac{4000}{11}$

12. Given that n = 6
Probability for selecting a defective product is $\frac{20}{100}$ that is $p = \frac{1}{5}$
Since X denotes the number defective products, X can take on the values 0,1,2,...,6
The probability for defective (success) is $p - \frac{1}{5}$ and for failure $q = 1 - p = \frac{4}{5}$, and n = 6
Therefore X follows a binomial distribution denoted by $X \sim B\left(6, \frac{1}{5}\right)$
This gives $f(x) = \binom{6}{x}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{6-x}$, x = 0,1,2,...,6,
(i) Probability for two defective products is
$P(X = 2) = f(2) = \binom{6}{x}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{6-x} = 15\left(\frac{4^4}{5^6}\right)$
(ii) Probability for at most one defective products is
P(X ≤1) = P(X = 0) + P(X = 1)
$-\binom{6}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{6-0} + \binom{6}{1}\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^{6-1}$
$-\left(\frac{4}{5}\right)^6 + (6)\left(\frac{4^2}{5^2}\right) = 2\left(\frac{4}{5}\right)^2$
Probability for at most one defective products is $2\left(\frac{4}{5}\right)^5$
(iii) Probability for at least two defective products is
P(X≥2)-1-P(X<2) = 1-P(X≤1) = $1 - 2\left(\frac{4}{5}\right)^5$
Probability for at least two defective products is $1 - 2\left(\frac{4}{5}\right)^5$

13. Equation of the given circle is $x^2 + y^2 = 16$ ...(1)
and the parabola is $y^2 = 6x$. ...(2)
Substituting (2) in (1) we get,
$x^2 + 6x - 16 = 0 \Rightarrow (x + 8)(x - 2) = 0$
$\Rightarrow (x-4)(x-2) = 0 \Rightarrow x = 2, 4$
$\Rightarrow x = -8, 2$



$\therefore$ Required area = 2
$\int_0^2 Area\ below\ the\ parabola + \int_2^4 Area\ below\ the\ circle$
$= 2\left[\int_0^2 \sqrt{6x}dx + \int_2^4 \sqrt{16 - x^2}dx\right]$
$= 2\left[\left(\frac{\sqrt{6}.x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2 + \left(\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}sin^{-1}\left(\frac{x}{4}\right)\right)_0^4\right]$
$= 2\left[\frac{2}{3}\sqrt{6}.2\sqrt{2} + 8sin^{-1}\left(\frac{1}{2}\right)\right]$
$= 2\left[\frac{4\sqrt{12}}{3} + 8\left(\frac{\pi}{2}\right) - \sqrt{12} - 8\left(\frac{\pi}{6}\right)\right]$
$= 2\left[\frac{4\sqrt{12}}{3} + 4\pi - 2\sqrt{3} - \frac{4\pi}{3}\right]$
$= 2\left[\frac{8\sqrt{3}-6\sqrt{3}}{3} + \frac{12\pi-4\pi}{3}\right]$

$= \left[\frac{2\sqrt{3}}{3} + \frac{8\pi}{3}\right] = 2 \times \frac{2}{3}\left[\sqrt{3} + 4\pi\right]$

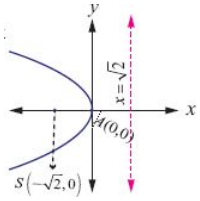
$= \frac{4}{3}\left[4\pi + \sqrt{3}\right]$ sq.units

14.

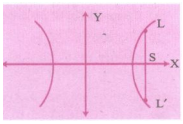
PART-C

ANSWER ANY 7 QUESTIONS. Q. NO :40 COMPULSORY :-

15. Parabola is open left and axis of symmetry as x-axis and vertex (0, 0)
Then the equation of the required parabola is
$(y - 0)^2 = -4\sqrt{2}(x - 0)$
$y^2 = -4\sqrt{2}x$



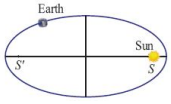
16. The latus rectum LL′ of the hyperbola passes through S(ae, 0)



∴ L is (ae, y_1)
Substituting L in $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,
Substituting L in $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
$\frac{a^2e^2}{a^2} - \frac{y_1^2}{b^2} = 1 \Rightarrow e^2 - \frac{y_1^2}{b^2} = 1$
$\Rightarrow e^2 - 1 = \frac{y_1^2}{b^2}\ [b^2 = a^2(e^2 - 1)$
$\Rightarrow y_1^2 = b^2(e^2 - 1) \Rightarrow \frac{b^2}{a^2} = e^2 - 1]$
$\Rightarrow {y_1}^2 = b^2\left(\frac{b^2}{a^2}\right) \Rightarrow {y_1}^2 = \frac{b^4}{a^2}$
$\Rightarrow y_1 = \pm\frac{b^2}{a}$
∴ End points oflatus rectum Land L′ are
$\left(ae, \frac{b^2}{a}\right)$ and $\left(ae, -\frac{b^2}{a}\right)$
Hence, the length of latus rectum LL′ = $\frac{b^2}{a} + \frac{b^2}{a} = \frac{2b^2}{a}$ units.
Hence proved.

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17. AS = 94.5×10^6 km,
SA′ = 152×10^6 km
a+c = 152×10^6
a-c = 94.5×10^6
Subtracting 2c = $57.5 \times 10^6 = 575 \times 10^5$ km
Distance of the Sun from the other focus is SS′ = 575 × 105 km.



18. Let I = $\int_0^{\frac{\pi}{3}} \frac{sec\,x\,tan\,x}{1+sec^2x}dx$
Put sec x = u. Then, sec x tan x dx = du.
When x = 0, u = sec0 = 1. When x = $\frac{\pi}{3}$, $u = sec\frac{\pi}{2} = 2$
$\therefore I = \int_1^2 \frac{du}{1+u^2} = [tan^{-1}u]_1^2 = tan^{-1}1 = tan^{-1}(2) - \frac{\pi}{4}$

19. Given $f(x) = \begin{cases} kxe^{-2x} & for x > 0 \\ 0 \quad for & x \le 0 \end{cases}$
Since the given function is a probability density function
$\int_{-\infty}^{\infty} f(x)dx = 1$
$\Rightarrow k\int_0^{\infty} xe^{-2x}dx = 1$
$\Rightarrow k\frac{1!}{(2)^2} = 1$
$[\int_0^{\infty} x^n e^{-ax} = \frac{n!}{a^{n+1}}$, Here a = 2, n = 1]
$\Rightarrow \frac{k}{4} = 1 \Rightarrow k = 4$

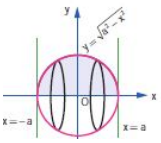
20. Let us put I = $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}$ dx --- (1)
Applying the formula $\int_b^a$ f(x) dx = $\int_b^a$ f(a+b -x) dx, we get
I = $\int_2^3 \frac{\sqrt{(2+3-x)}}{\sqrt{5-(2+3-x)}+\sqrt{(2+3-x)}}$ dx = $\int_2^3 \frac{\sqrt{(5-x)}}{\sqrt{x}+\sqrt{(5-x}}$ dx -- (2)
Adding (1) and (2), we get
2I = $\int_2^3 \frac{\sqrt{x}+\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}$ dx = $\int_2^3$ dx = $[x]_2^3$ = 3 - 2 = 1
Hence, we get I = $\frac{1}{2}$

21. Given E(X + 3) = 10
E(aX + b) = aE(X) + b
⇒ E(X) + 3 = 10

E(X) + 3 = 10
⇒ E(X) = 7
⇒μ = 7 ...(1)
$E(X + 3)^2 = 116$
$E(X^2 + 6x + 9)$ 116
$E(X^2) + 6E(X) + 9 = 116$ [∴ E(9) = 9]
$E(X^2) + 6(7) + 9 = 116$
$E(X^2) + 116 - 42 - 9$ 116 - 51
$E(X^2) = 65$ ...(2)
$Var(X) = E(X^2) - [E(X)^2]$
$65 - 7^2 = 65 - 49 = 16$
∴μ = 7 and $\sigma^2 = 16$.

22. X B(n, p)
Given 4P(X = 4) = P(X = 2) and n = 6.
$4. [{}^6C_4p^4 (1 - p)^2] = {}^6C_2p^2 q^4$
$\Rightarrow 4p^2 = q^2$
$\Rightarrow 4(1-q^2) = q^2$
$4(1-2q+q^2) = q^2$
$\Rightarrow 3q^2 - 8q + 4 = 0$
$\Rightarrow (q - 2)(3q - 2) = 0$
$-q = \frac{2}{3} \quad (q \neq 2)$
$p = 1 - q = \frac{1}{3}$
Distribution
$P(X = x) = nC_xp^x (1- p)^{n-x}$, x = 0, 1,2, ... n
(i) $P(X = x) = {}^6 C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{6-x}$, x = 0,1,2..n
(ii) $mean = np = 6 \times \frac{1}{3} = 2$
(iii) standard deviation = $\sqrt{npq} = \sqrt{2 \times \frac{2}{3}}$
= $\frac{2}{\sqrt{3}}$

23. By revolving the upper semicircular region enclosed between the circle $x^2 + y^2 = a^2$ and the x-axis, we get a sphere of radius a.



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The boundaries of the region are y = $\sqrt{a^2 - x^2}$ x-axis, the lines x = -a and x = a. Hence, the volume of the sphere is given by
$v = \pi \int_{-a}^{a} y^2 dx = \pi \int_{-a}^{a} (a^2 - x^2) dx$
$= 2\pi \int_0^a (a^2 - x^2) dx$ since the integrand (a^2-x^2) is an even function
$= 2\pi \left(a^2 x - \frac{x^3}{3}\right)_0^a = 2\pi \left(a^3 - \frac{a^3}{3}\right) = \frac{4}{3}\pi a^3$

24.

PART-B

ANSWER ANY 7 QUESTIONS. Q. NO :30 COMPULSORY :-

25. Given ends of diameter are (3, 4)(2, -7)
∴ Equation of the circle is
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
$\Rightarrow (x - 3)(x - 2) + (y - 4)(y + 7) = 0$
$\Rightarrow x^2 - 2x - 3x + 6 + y^2 + 7y - 4y - 28 = 0$
$\Rightarrow x^2 + y^2 - 5x + 3y - 22 = 0$

26. Equation of the circle is
$2x^2 + 2y^2 - 6x + 4y + 2 = 0$
Dividing by 2, we get
$x^2 + y^2 - 3x + 2y + 1 = 0$
Here $2g = -3 \Rightarrow g = \frac{-3}{2}$
$2f = 2 \Rightarrow f = 1$
and c = 1
∴ Centre is (-g, -f) = $\left(\frac{3}{2}, -1\right)$
and r = $\sqrt{g^2 + f^2 - c} = \sqrt{\left(\frac{3}{2}\right)^2 + 1^2 - 1}$
= $\sqrt{\frac{9}{4}} = \frac{3}{2}$ units.

27. Let f (x) = x cos x
Then f (-x) = (-x) cos(-x) = -x cos x = - f (x).
So f (x) = x cos x is an odd function.
Hence, applying the property, for odd function f(x), $\int_{-a}^{a} f(x)dx = 0$
∴ we get $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x dx = 0$

28.

$I_n = \int_0^{\pi/2} sin^n x = \frac{n-1}{n} I_{n-2}, n \geq 2$

$Let \quad I_{10} = \int_0^{\pi/2} sin^{10} x dx = \frac{9}{10} I_8$

$= \frac{9}{10} \times \frac{7}{8} \times I_6 = \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times I_4$

$= \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times \frac{3}{4} I_2$

$= \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}$

$= \frac{315}{1280} \times \frac{\pi}{2} = \frac{63\pi}{256(2)} = \frac{63\pi}{512}$

29. Given $f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & elsewhere \end{cases}$

$E(X) = \int_0^\infty x.\, f(x) dx = \int_0^\infty x.3.\, e^{-3x} dx$

$= 3 \int_0^\infty x.\, e^{-3x} dx \left[\therefore \int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \right]$

$= 3 \times \frac{1!}{3^2} = \frac{3}{9} = \frac{1}{3}$

∴ Expected life of the electronic equipment is = $\frac{1}{3}$

30. Given $p = \frac{3}{4}$

n = 5

$P(X = x) = nC_x p^x (1-p)^{n-x}$

$P(X = 3) = 5C_3 \left(\frac{3}{4}\right)^3 \left(1 - \frac{3}{4}\right)^2$

$P(X = 3) = 5C_2 \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right)^2$ [$\because nCr = nC_{n-r}$]

$P(X = 3) = \frac{5 \times 4}{2 \times 1} \times \frac{3 \times 3 \times 3}{4 \times 4 \times 4 \times 4 \times 4}$

$= \frac{135}{512}$

31. $\because \int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

$n = 5, \quad a = 3$

$= \frac{5!}{3^6}$

32. $S = \begin{vmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{vmatrix}$

Let X be a random variable whose values x are the number of fours.

The sample space S is given in the table.

It can also be written as

S = {(i, j)} , where i = 1, 2, 3, 6 and j = 1, 2, 3, 6

Therefore X takes on the values of 0, 1 and 2.

We observe that

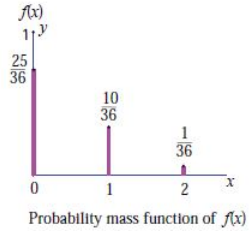
(i) X = 0, if (i, j) for $i \neq 4, j \neq 4$,

(ii) X = 1, if (1, 4), (2, 4), (3, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)

(iii) X = 2, if (4, 4) ,

Therefore,

Values of the Random Variable X	0	1	2	Toatal
Number of elements in inverse images	25	10	1	36



Probability mass function of $f(x)$

The probabilities are

$f(0) = P(X = 0) \frac{25}{36}$

$f(1) = P(X = 1) = \frac{10}{36}$

and $f(20 = P(X = 2) = \frac{1}{36}$

Clearly the function f(x) satisfies the conditions

(i) f (x) ≥ 0, for x = 0, 1, 2 and

(ii) $\sum_x f(x) = \sum_{x=0}^{x=-2} f(x) = f(0) + f(1) + f(2) = 1$

$= \frac{25}{36} + \frac{10}{36} + \frac{1}{36} = 1$

The probability mass function is presented as

x	0	1	2
f(x)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

(or)

$f(x) = \begin{cases} \frac{25}{36} & for\ x = 0 \\ \frac{10}{36} & for\ x = 1 \\ \frac{1}{36} & for\ x = 2 \end{cases}$

33.

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Since the curve is open leftward, the required equation of the parabola is

$(y-k)^2 = -4a(x-h)$

Given vertex (h, k) = (2, 1)

$\therefore (y-1)^2 = -4a(x-2)$ .....(2)

Since this pass through (1, 3) we get

$(3-1)^2 = -4a(1-2)$

$4 = -4a(-1)$

$a = 1$

$\therefore (1) \Rightarrow (y-1)^2 = -4(x-2)$

which is required equation of the parabola

34. Let I $= \int_0^{\frac{\pi}{2}} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx$ .....(1)

$I = \int_0^{\frac{\pi}{2}} \frac{f(\sin(\frac{\pi}{2}-x))}{f(\sin(\frac{\pi}{2}-x))+f(\cos(\frac{\pi}{2}-x))} dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$I = \int_0^{\frac{\pi}{2}} \frac{f(\cos x)}{f(\cos x)+f(\sin x)} dx$ .....(2)

From (1) and (2) we get,

$2I = \int_0^{\frac{\pi}{2}} \frac{f(\sin x)+f(\cos x)}{f(\sin x)+f(\cos x)} dx$

$= \int_0^{\pi/2} dx = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2}$

$\therefore I = \frac{\pi}{4}$

Hence proved.

PART-A

MULTIPLE CHOICE QUESTIONS :-

35. (c) 10
36. (b) $2(a^2+b^2)$
37. (d) 9
38. (a) $\frac{1}{\sqrt{2}}$
39. (b) (-3, 2)
40. (a) 3
41. (d) 4
42. (d) 2
43. (d) 16 and 24
44. (a) 1
45. (b) 0.25
46. (c) $\frac{8}{3}$
47. (b) $\frac{1}{10100}$
48. (d) $\frac{2}{27}$
49. (d) $\frac{\pi a^3}{6}$
50. (b) $\frac{3\pi a^4}{16}$
51. (d) $\frac{2}{3}$
52. (b) log 2
53. (c) F(x) is a real valued decreasing function
54. (c) (1, -1)

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