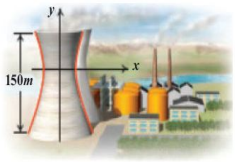
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Questions:

PART-D

ANSWER ALL OF THE FOLLOWING QUESTIONS :-

1. Find the equation of the circle passing through the points (1, 1), (2, -1) and (3, 2) .
2. Show that the line x-y+4 = 0 is a tangent to the ellipse $x^2+3y^2 = 12$. Also find the coordinates of the point of contact.
3. A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre, on either sides.
4. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150 m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



5. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :
   $18x^2+12y^2-144x+48y+120 = 0$
6. Evaluate $\int_1^4 (2x^2 + 3)\, dx$, as the limit of a sum
7. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function
   $\begin{cases} k & 200 \le x \le 600 \\ 0 & otherwise \end{cases}$
   Find
   (i) the value of k
   (ii) the distribution function
   (iii) the probability that daily sales will fall between 300 litres and 500 litres?
8. Evaluate the following integrals using properties of integration:
   $\int_0^{\pi} x\left[sin^2(sinx) + cos^2(cosx)\right]dx$
9. A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

   Find
   (i) P(2 < X < 6)
   (ii) P(2 ≤ X < 5)
   (iii) P(X ≤4)
   (iv) P(3 < X)
10. Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.
11. Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs. 20 for each black ball selected and we lose Rs. 10 for each white ball selected. Find the expected winning amount and variance
12. On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that
    (i) two products are defective
    (ii) at most one product is defective
    (iii) at least two products are defective.
13. Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.
14. Prove that $\int_0^{\pi/4} \log(\sin x + \cos x)dx = \frac{\pi}{8}\log 2.$

PART-C

ANSWER ANY 7 QUESTIONS. Q. NO :40 COMPULSORY :-

15. Find the equation of the parabola with focus $\left(-\sqrt{2}, 0\right)$ and directrix x = $\sqrt{2}$.
16. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.
17. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.
18. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{sec\,x\,tan\,x}{1+sec^2x}dx$
19. The probability density function of X is given by $f(x) = \begin{cases} kxe^{-2x} & for x > 0 \\ 0 & for \quad x \le 0 \end{cases}$ Find the value of k.
20. Evaluate $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$.

21. If μ and σ^2 are the mean and variance of the discrete random variable X, and E(X + 3) =10 and $E(X + 3)^2 = 116$, find μ and σ^2

22. If X~ B(n, p) such that 4P(X = 4) = P(X = 2) and n = 6. Find the distribution, mean and standard deviation of X.

23. Find the volume of a sphere of radius a.

24. Evaluate :

$\int_0^1 |5x-3|dx$

PART-B

ANSWER ANY 7 QUESTIONS. Q. NO :30 COMPULSORY :-

25. Obtain the equation of the circle for which (3, 4) and (2, -7) are the ends of a diameter.

26. Find centre and radius of the following circles.
$2x^2+2y^2-6x+4y+2 = 0$

27. Evaluate: $\int_{\frac{\pi}{2}}^{\pi} x \cos x \, dx$.

28. Evaluate the following
$\int_0^{\pi/2} sin^{10}x \quad dx$

29. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function $f(x) = \begin{cases} 3e^{-3x} & x > 0 \\ 0 & elsewhere \end{cases}$
Find the expected life of this electronic equipment.

30. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

31. Evaluate the following:
$\int_0^{\infty} x^5 e^{-3x} dx$

32. A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.

33. Find the equation of the parabola. if the curve ie open leftward, vertex is (2,1) and passing through the point (1, 3)

34. Prove that $\int_0^{\frac{\pi}{2}} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx = \frac{\pi}{4}$

PART-A

MULTIPLE CHOICE QUESTIONS :-

35. If P(x, y) be any point on $16x^2 + 25y^2 = 400$ with foci F_1 (3, 0) and F_2 (-3, 0) then $PF_1 + PF_2$ is
(a) 8 (b) 6 (c) 10 (d) 12

36. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$
(a) $4(a^2+b^2)$ (b) $2(a^2+b^2)$ (c) a^2+b^2 (d) $\frac{1}{2}(a^2+b^2)$

37. If x + y = k is a normal to the parabola $y^2 = 12x$, then the value of k is
(a) 3 (b) -1 (c) 1 (d) 9

38. An ellipse has OB as semi minor axes, F and F′ its foci and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is
(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

39. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are (11, 2), the coordinates of the other end are
(a) (-5, 2) (b) (-3, 2) (c) (5, -2) (d) (-2, 5)

40. The number of normals that can be drawn from a point to the parabola $y^2 = 4ax$ is _____
(a) 3 (b) 2 (c) 0 (d) 1

41. A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is
(a) 1 (b) 2 (c) 3 (d) 4

42. A random variable X has binomial distribution with n = 25 and p = 0.8 then standard deviation of X is
(a) 6 (b) 4 (c) 3 (d) 2

43. If the function $f(x) = \frac{1}{12}$ for a < x < b, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?
(a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

44. If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is
(a) 1 (b) 2 (c) 3 (d) 4

45. If in 6 trials, X is a binomial variable which follows the relation 9P(X = 4) = P(X = 2), then the probability of success is
(a) 0.125 (b) 0.25 (c) 0.375 (d) 0.75

46. The area between $y^2 = 4x$ and its latus rectum is
(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{5}{3}$

47. The value of $\int_0^1 x(1-x)^{99} dx$ is
(a) $\frac{1}{11000}$ (b) $\frac{1}{10100}$ (c) $\frac{1}{10010}$ (d) $\frac{1}{10001}$

48. The value of $\int_0^{\infty} e^{-3x} x^2 dx$ is
(a) $\frac{7}{27}$ (b) $\frac{5}{27}$ (c) $\frac{4}{27}$ (d) $\frac{2}{27}$

49. The volume of solid of revolution of the region bounded by $y^2 = x(a - x)$ about x-axis is
(a) πa^3 (b) $\frac{\pi a^3}{4}$ (c) $\frac{\pi a^3}{5}$ (d) $\frac{\pi a^3}{6}$

50. The value of $\int_0^a \left(\sqrt{a^2-x^2}\right)^3 dx$ is
(a) $\frac{\pi a^3}{16}$ (b) $\frac{3\pi a^4}{16}$ (c) $\frac{3\pi a^2}{8}$ (d) $\frac{3\pi a^4}{8}$

51. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^2x \, cos \, x \, dx$ is
(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 0 (d) $\frac{2}{3}$

52. The value of $\int_0^{\frac{\pi}{3}} tanx \, dx$ _____

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(a) -log 2 (b) log 2 (c) -log 3 (d) log 3

53. Which one of the following is not true in the case of dicrete random variable X?

(a) 0<= F(x) <= 1, for all x ϵ R (b) $lim_{x\to\infty}F(x) = F(\infty) = 1$ (c) F(x) is a real valued decreasing function (d) $lim_{x\to\infty}F(x) = F(\infty) = 0$

54. The centre of the hyperbola $\frac{(x-1)^2}{16} - \frac{(y+1)^2}{25} = 1$ is

(a) $(\frac{1}{2}, -\frac{1}{2})$ (b) (1,1) (c) (1, -1) (d) (0, 0)

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