QB365 Student Learing Managemnet System
Exam Name: Pre Quarterly - Practice
Std: 12th Standard - Maths
Chapter: Applications of Vector Algebra, Inverse Trigonometric Functions, Differentials and Partial Derivatives

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Questions:

5 Marks

1. If G is the centroid of a ΔABC, prove that (area of ΔGAB) = (area of ΔGBC) = (area of ΔGCA) = $\frac{1}{3}$ (area of ΔABC)
2. Using vector method, prove that cos(α − β) = cos α cos β +sin α sin β
3. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and 0 < x, y, z < 1, show that $x^2 + y^2 + z^2 + 2xyz = 1$
4. Solve $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$
5. A straight line passes through the point (1, 2, -3) and parallel to $4\hat{i} + 5\hat{j} - 7\hat{k}$. Find
   (i) vector equation in parametric form
   (ii) vector equation in non-parametric form
   (iii) Cartesian equations of the straight line.
6. Find the parametric form of vector equation of the straight line passing through (-1, 2,1) and parallel to the straight line $\vec{r} = (2\hat{i} + 3\hat{j} - \hat{k}) + t(\hat{i} - 2\hat{j} + \hat{k})$ and hence find the shortest distance between the lines.
7. Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2, 1), (1, -2, 3) and parallel to the straight line passing through the points (2, 1, -3) and (-1, 5, -8)
8. Find the value of $tan\left[\frac{1}{2}sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$
9. The relation between the number of words y a person learns in x hours is given by $y = 52\sqrt{x}$, $0, \le x \le 9$. What is the approximate number of words learned when x changes from
10. For each of the following functions find the f_x, f_y, and show that $f_{xy} = f_{yx}$
    $f(x, y) = \tan^{-1}(x/y)$
11. Let g(x, y) = x^2 - yx + sin(x+y), x(t) = e^{3t}, y(t) = t^2, t ∈ R. Find $\frac{dg}{dt}$
12. If w (x, y, z) = $x^2 + y^2 + y^2$, x = e^t, y = e^t sin t, z = e^t cos t, find $\frac{dw}{dt}$
13. Let U(x, y) = e^x sin y, where x = st^2, y = s^2 t, s, t ∈ R. Find $\frac{\partial U}{\partial s}$, $\frac{\partial U}{\partial t}$ and evaluate them at s = t = 1.
14. If the position vectors of three points A, B and, C arc respectively $\hat{i} + 2\hat{j} + 3\hat{k}$, $4\hat{i} + \hat{j} + 5\hat{k}$ and $7(\hat{i} + \hat{k})$. Find $\overrightarrow{AB} \times \overrightarrow{AC}$. Interprer the result geometrically.

3 Marks

15. With usual notations, in any triangle ABC, prove the following by vector method.
    (i) $a^2 = b^2 + c^2 - 2bc \cos A$
    (ii) $b^2 = c^2 + a^2 - 2ca \cos B$
    (iii) $c^2 = a^2 + b^2 - 2ab \cos C$
16. Find the domain of the following
    $f(x) = sin^{-1}\left(\frac{x^2+1}{2x}\right)$
17. Prove by vector method that if a line is drawn from the centre of a circle to the midpoint of a chord, then the line is perpendicular to the chord.
18. Prove that $\tan\left(sin^{-1}x\right) = \frac{x}{\sqrt{1-x^2}} for |x| < 1$
19. If the straight line joining the points (2, 1, 4) and (a-1, 4, -1) is parallel to the line joining the points (0, 2, b -1) and (5, 3, -2), find the values of a and b.
20. Prove that $cos^{-1}\left(\frac{4}{5}\right) + tan^{-1}\left(\frac{3}{5}\right) = tan^{-1}\left(\frac{27}{11}\right)$
21. Find the partial derivatives of the following functions at the indicated point
    h (x, y, z) = x sin (xy) + z^2x, $\left(2, \frac{\pi}{4}, 1\right)$
22. If u(x, y) = $\frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$
23. Evaluate : $\lim_{(x,y,z)\to(-1,0,4)} \frac{x^2 - ze^{xy}}{6x+2y-2z}$
24. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$

2 Marks

25. Find the principal value of $\text{cosec}^{-1}(-1)$
26. The volume of the parallelepiped whose coterminus edges are $7\hat{i} + \lambda\hat{j} - 3\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-3\hat{i} + 7\hat{j} + 5\hat{k}$ is 90 cubic units. Find the value of λ.
27. If $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}, \vec{c} = y\hat{i} + x\hat{j} + (1+x+y)\hat{k}$ show that $[\vec{a}, \vec{b}, \vec{c}]$ depends on neither x nor y.
28. Find the period and amplitude of
    y = -sin($\frac{1}{3}x$)
29. Find the principal value of $sin^{-1}\left(sin\left(-\frac{\pi}{3}\right)\right)$
30. Find the principal value of $\sin^{-1}(-1)$.
31. Let g(x) = x^2 + sin x. Calculate the differential dg.
32. Find df for f(x) = x^2 + 3x and evaluate it for
    x = 2 and dx = 0.1
33. If w=xye^{xy} find $\frac{\partial^2 u}{\partial x \partial y}$

34. Find the shortest distance between the skew lines $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} - \hat{j} - \hat{k})$

Multiple Choice Question

35. If $\cot^{-1} x = \frac{2\pi}{5}$ for some x $\in$ R, the value of $\tan^{-1}$ x is

(a) $-\frac{\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{5}$

36. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a) $\frac{1}{2}\, cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2} sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2} tan^{-1}\left(\frac{3}{5}\right)$ (d) $tan^{-1}\left(\frac{1}{2}\right)$

37. If $\cot^{-1}$ 2 and $\cot^{-1}$ 3 are two angles of a triangle, then the third angle is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

38. $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$. Then x is a root of the equation

(a) x^2–x–6 = 0 (b) x^2–x–12 = 0 (c) x^2+x–12 = 0 (d) x^2+x–6 = 0

39. sin ($\tan^{-1}$x), |x| < 1 is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

40. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

(a) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ (b) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ (c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ (d) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

41. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

(a) $|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ (b) $\frac{1}{3}|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ (c) 1 (d) -1

42. The volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}, \hat{i} + \hat{j} + \pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{\pi}{4}$

43. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j}, \vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda + \mu$ is

(a) 0 (b) 1 (c) 6 (d) 3

44. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}]$ = 3, then $\{[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}]\}^2$ is equal to

(a) 81 (b) 9 (c) 27 (d) 18

45. If the planes $\vec{r}.(2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r}.(4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ are parallel, then the value of λ and μ are

(a) $\frac{1}{2}, -2$ (b) $-\frac{1}{2}, 2$ (c) $-\frac{1}{2}, -2$ (d) $\frac{1}{2}, 2$

46. If $\theta = sin^{-1}\left(sin(-60^0)\right)$ then one of the possible values of θ is _____

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{-2\pi}{3}$

47. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

48. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d) 31

49. If $f(x) = \frac{x}{x+1}$, then its differential is given by

(a) $\frac{-1}{(x+1)^2}dx$ (b) $\frac{1}{(x+1)^2}dx$ (c) $\frac{1}{x+1}dx$ (d) $\frac{-1}{x+1}dx$

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QUESTION BANK 365

50. If f(x,y, z) = xy +yz +zx, then $f_x - f_z$ is equal to

(a) z - x (b) y - z (c) x - z (d) y - x

51. If f(x, y) = $2x^2$ - 3xy + 5y + 7 then f(0, 0) and f(1, 1) is _____

(a) 7, 11 (b) 11, 7 (c) 0, 7 (d) 1, 0

52. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ then x is _____

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $-\frac{1}{2}$ (d) None of these

53. If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ for non-coplanar vectors $\vec{a}, \vec{b}, \vec{c}$ then _____

(a) $\vec{a}$ parallel to $\vec{b}$ (b) $\vec{b}$ parallel to $\vec{c}$ (c) $\vec{c}$ parallel to $\vec{a}$ (d) $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

54. The vector $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{a})$ is _____

(a) perpendicular to $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ (b) parallel to the vectors $(\vec{a} \times \vec{b})$ and $(\vec{c} \times \vec{d})$ (c) parallel to the line of intersection of the plane containing $\vec{a}$ and $\vec{b}$ and the plane containing $\vec{c}$ and $\vec{d}$
(d) perpendicular to the line of intersection of the plane containing $\vec{a}$ and $\vec{b}$ and the plane containing $\vec{c}$ and $\vec{d}$