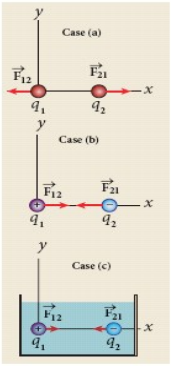
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Answers Key:

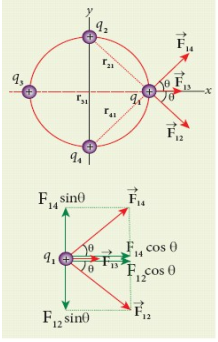
III. Three Marks:



1.

(a) q_1 = +2 μC, q_2 = +3 μC, and r = 1m. Both are positive charges. so the force will be repulsive.
Force experienced by the charge q_2 due to q_1 is given by
$\vec{F_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r_{12}}$
Here $\hat{r_{12}}$ is the unit vector from q_1 to q_2. Since q_2 is located on the right of q_1, we have
$\hat{r_{12}} = \hat{i}$, and $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ so that
$\vec{F_{21}} = \frac{9\times10^9\times2\times10^{-6}\times3\times10^{-6}}{1^2}\hat{i}$
= 54 x 10^{-3} N$\hat{i}$
According to Newton's third law, the force experienced by the charge q_1 due to q_2 is $\vec{F_{12}} = -\vec{F_{21}}$ Therefore,
$\vec{F_{12}}$= 54 x 10^{-3} N$\hat{i}$
The directions of $\vec{F_{21}}$ and $\vec{F_{12}}$ are shown in the figure (case (b)).
(b) q_1 = +2 μC, q_2 = −3 μC, and r = 1m. They are unlike charges. So the force will be attractive.
Force experienced by the charge q_2 due to q_1 is given by
$\vec{F_{21}} = \frac{9\times10^9\times(2\times10^{-6})\times(-3\times10^{-6})}{1^2}\hat{r}_{12}$
$= -54 \times 10^{-3}\ \text{N}\hat{i}\ \left(\text{Using}\hat{r_{12}} = \hat{i}\right)$
The charge q_2 will experience an attractive force towards q_1 which is in the negative x direction.
According to Newton's third law, the force experienced by the charge q_1 due to q_2 is $\vec{F_{12}} = -\vec{F_{21}}$ Therefore,
$\vec{F_{12}} = 54 \times 10^{-3}\hat{i}$ N
The directions of $\vec{F_{21}}$ and $\vec{F_{12}}$ are shown in the figure (case (b)).
(c) If these two charges are kept inside the water, then the force experienced by q_2 due to q_1
$\vec{F_{21}^W} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r_{12}}$
since ε = $\varepsilon_r\varepsilon_0$,
we have $\vec{F_{21}^W} = \frac{1}{4\pi\varepsilon_r\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r_{12}} = \frac{\vec{F_{21}}}{\varepsilon_r}$
Therefore,
$\vec{F_{21}^W} = \frac{54\times10^{-3}N}{80}\hat{i}$ = -0.675 x 10^{-3} N$\hat{i}$

2. According to the superposition principle, the total electrostatic force on charge q_1 is the vector sum of the forces due to the other charges,
$\vec{F_1^{tot}} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}}$
The following diagram shows the direction of each force on the charge q_1.



The charges q_2 and q_4 are equi-distant from q_1. As a result the strengths (magnitude) of the forces $\vec{F_{12}}$ and $\vec{F_{14}}$ are the same even though their directions are different. Therefore the vectors representing these two forces are drawn with equal lengths. But the charge q_3 is located farther compared to q_2 and q_4. Since the strength of the electrostatic force decreases as distance increases, the strength of the force $\vec{F_{13}}$ is lesser than that of forces $\vec{F_{12}}$ and $\vec{F_{14}}$. Hence the vector representing the force $\vec{F_{13}}$ is drawn with smaller length compared to that for forces $\vec{F_{12}}$ and $\vec{F_{14}}$.
From the figure, $r_{21} = \sqrt{2}$ m = r_{41} and r_{31} = 2m
The magnitudes of the forces are given by

$F_{13} = \frac{kq^2}{r_{31}^2} = \frac{9\times10^9\times10^{-12}}{4}$

$F_{13} = 2.25 \times 10^{-3}$ N

$F_{12} = \frac{kq^2}{r_{31}^2} = F_{14} = \frac{9\times10^9\times10^{-12}}{2}$

$= 4.5 \times 10^{-3}$N

From the figure, the angle $\theta = 45^0$. In terms of the components, we have

$\vec{F_{12}} = F_{12}cos\theta\hat{i} - F_{12}sin\theta\hat{j}$

$= 4.5 \times 10^{-3} \times \frac{1}{\sqrt{2}}i - 4.5 \times 10^{-3} \times \frac{1}{\sqrt{2}}\hat{j}$

$\vec{F_{13}} = F_{13}\hat{i} = 2.25 \times 10^{-3}$ N$\hat{i}$

$\vec{F_{14}} = F_{14}cos\theta\hat{i} + F_{14}sin\theta\hat{j}$

$= 4.5 \times 10^{-3} \times \frac{1}{\sqrt{2}}i + 4.5 \times 10^{-3} \times \frac{1}{\sqrt{2}}\hat{j}$

Then the total force on q_1 is,

$\vec{F_1^{tot}} = (F_{12}cos\theta\hat{i} - F_{12}sin\theta\hat{j}) + F_{13}\hat{i} + (F_{14}cos\theta\hat{i} + F_{14}sin\theta\hat{j})$

$\vec{F_1^{tot}} = (F_{12}cos\theta + F_{13} + F_{14}cos\theta)\hat{i} + (-F_{12}sin\theta + F_{14}sin\theta)\hat{j}$

Since $F_{12} = F_{14}$, the j th component is zero.

Hence we have

$\vec{F_1^{tot}} = (F_{12}cos\theta + F_{13} + F_{14}cos\theta)\hat{i}$

substituting the values in the above equation,

$\left(\frac{4.5}{\sqrt{2}} + 2.25 + \frac{4.5}{\sqrt{2}}\right) \times 10^{-3}\hat{i} = (4.5\sqrt{2} + 2.25) \times 10^{-3}\hat{i}$

$\vec{F_1^{tot}} = 8.61 \times 10^{-3}$ N$\hat{i}$

The resultant force is along the positive x-axis.

3. **Case (a)**

The magnitude of the electric field at point P is

$E_p = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times1\times10^{-6}}{4}$

$= 2.25 \times 10^3$ NC^{-1}

Since the source charge is positive, the electric field points away from the charge. So the electric field at the point P is given by

$\vec{E_p} = 2.25 \times 10^3$ NC^{-1}

For the point Q

$|\vec{E_Q}| = \frac{9\times10^9\times1\times10^{-6}}{16} = 0.56 \times 10^3$ NC^{-1}

Hence $\vec{E_Q} = 0.56 \times 10^3\hat{j}$ NC^{-1}

Case (b)

The magnitude of the electric field at point P

$\vec{E_p} = \frac{kq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{9\times10^9\times2\times10^{-6}}{4}$

$= 4.5 \times 10^3$ NC^{-1}

Since the source charge is negative, the electric field points towards the charge. So the electric field at the point P is given by

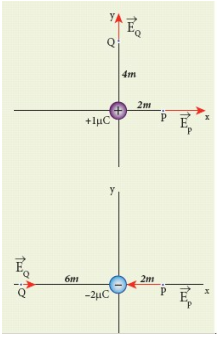
$\vec{E_p} = -4.5 \times 10^3\hat{i}$NC^{-1}

For the point Q, $|\vec{E_Q}| \frac{9\times10^9\times2\times10^{-6}}{36}$

$= 0.5 \times 10^3$ NC^{-1}

$\vec{E_Q} = 0.5 \times 10^3\hat{i}$NC^{-1}

At the point Q the electric field is directed along the positive x-axis.



4. **(a)** The capacitance of the capacitor is

$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times25\times10^{-4}}{1\times10^{-3}}$

$= 221.2 \times 10^{-13}$ F

$C = 22.12 \times 10^{-12}$F = 22.12pF

(b) The charge stored in any one of the plates is Q = CV, Then

$Q = 22.12 \times 10^{-12} \times 10 = 221.2 \times 10^{-12}$C = 221.2 pC.

5. (a) The capacitance of the capacitor in the presence of dielectric is

$C = \frac{\varepsilon_r\varepsilon_0 A}{d} = \frac{5\times8.85\times10^{-12}\times6\times10^{-4}}{6\times10^{-3}}$

$= 44.25 \times 10^{-13}$F = 4.425 pF

The stored charge is

$Q = CV = 44.25 \times 10^{-13} \times 10$

$= 442.5 \times 10^{-13}$C = 44.25pC

The stored energy is

$U = \frac{1}{2}$ CV^2 = $\frac{1}{2} \times 44.25 \times 10^{-13} \times 100$

$= 2.21 \times 10^{-10}$ J

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(b) After the removal of the dielectric, since the battery is already disconnected the total charge will not change. But the potential difference between the plates increases. As a result, the capacitance is decreased.

New capacitance is

$C_0 = \frac{C}{\varepsilon_r} = \frac{44.25\times10^{-12}}{5}$

= 0.885 x 10^{-12} F = 0.885 pF

The stored charge remains same and 44.25 pC. Hence newly stored energy is

$U_0 = \frac{Q^2}{2C_0} = \frac{Q^2\varepsilon_r}{2C} = \varepsilon_r U$

= 5 x 2.21 x 10^{-10}J = 11.05 x 10^{-10}J

The increased energy is ΔU = (11.05 - 2.21) x 10^{-10} J = 8.84 x 10^{-10} J

When the dielectric is removed, it experiences an inward pulling force due to the plates. To remove the dielectric, an external agency has to do work on the dielectric which is stored as additional energy. This is the source for the extra energy 8.84 x 10^{-10} J.

II. Two Marks:

6. By using superposition principle, the net electric field at point A is

$\vec{E_A} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1A}^2}\hat{r_{1A}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{2A}^2}\hat{r_{2A}}$

where r_{1A} and r_{2A} are the distances of point A from the two charges respectively

$\vec{E_A} = \frac{9\times10^4\times1\times10^{-6}}{(2\times10^{-3})^2}(\hat{j}) + \frac{9\times10^9\times1\times10^{-6}}{(2\times10^{-3})^2}(\hat{i})$

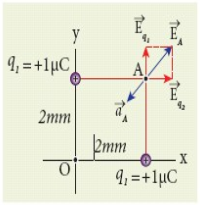
= 2.25 x $10^9\hat{j}$ + 2.25 x $10^9\hat{i}$ = 2.25 x 10^9 $(\hat{i}+\hat{j})$

The magnitude of electric field

$|\vec{E_A}| = \sqrt{(2.25\times10^9)^2 + (2.25\times10^9)^2}$

= 2.25 x $\sqrt{2}$ x 10^9 NC^{-1}

The direction of $\vec{E_A}$ is given by $\frac{\vec{E_A}}{|\vec{E_A}|} = \frac{2.25\times10^9(\hat{i}+\hat{j})}{2.25\times\sqrt{2}\times10^9} = \frac{(\hat{i}+\hat{j})}{\sqrt{2}}$, which is the unit vector along OA as shown in the figure.



The acceleration experienced by an electron placed at point A is

$\vec{a_A} = \frac{\vec{F}}{m} = \frac{q\vec{E_A}}{m}$

$= \frac{(-1.6\times10^{-19})\times(2.25\times10^9)(\hat{i}+\hat{j})}{9.1\times10^{-31}}$

= -3.95 x 10^{20} $(\hat{i}+\hat{j})$ Nkg^{-1}

The electron is accelerated in a direction exactly opposite to $\vec{E_A}$.

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7. The maximum torque experienced by the dipole is when it is aligned perpendicular to the applied field.

τ_{max} = pE sin 90^0 = 3.4 x 10^{-30} x 3 x 10^4 Nm

τ_{max} = 10.2 x 10^{-26} Nm.

8. (a) The work done to arrange the charges in the corners of the square is independent of the way they are arranged. We can follow any order.

(i) First, the charge +q is brought to the corner P. This requires no work since no charge is already present, W_P = 0

(ii) Work required to bring the charge −q to the corner Q = (-q) x potential at a point Q due to +q located at a point P

W_Q = -q x $\frac{1}{4\pi\varepsilon_0}\frac{q}{a} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}$

(iii) Work required to bring the charge +q to the corner R = q x potential at the point R due to charges at the point P and Q.

W_R = q x $\frac{1}{4\pi\varepsilon_0}\left(-\frac{q}{a} + \frac{q}{\sqrt{2}a}\right)$

$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(-1 + \frac{1}{\sqrt{2}}\right)$

(iv) Work required to bring the fourth charge −q at the position S = q × potential at the point S due the all the three charges at the point P, Q and R.

W_s = - q x $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a} + \frac{q}{a} - \frac{q}{\sqrt{2}a}\right)$

$W_s = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left(2 - \frac{1}{\sqrt{2}}\right)$

(b) Work required to bring the charge q′ to the center of the square = q′ x potential at the center point O due to all the four charges in the four corners.

The potential created by the two +q charges are canceled by the potential created by the -q charges which are located in the opposite corners. Therefore the net electric potential at the center O due to all the charges in the corners is zero.

Hence no work is required to bring any charge to the point O. Physically this implies that if any charge q′ when brought close to O, then it moves to the point O without any external force.

9. The electric field on the surface of the sphere(by Gauss law) is given by

$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

The potential on the surface of the hollow metallic sphere is given by

$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ = ER

Since $V_{max} = E_{max}R$

Here E_{max} = 3 x 10^6 Vm^{-1}. So the maximum potential difference created is given by

V_{max} = 3 x 10^6 x 0.5

= 1.5 x 10^6V (or) 1.5 million volt.

10. Charge q = 50 nC = 50 x 10^{-9} C , e = 1.6 x 10^{-19} C

No. of electrons $n = \frac{q}{e}$

$\frac{q}{e} = \frac{50\times10^{-9}}{1.6\times10^{-19}}$

= 31.25 x 10^{10} electrons

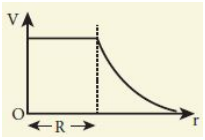
To produce 50 nC charge, number of electrons transferred = 31.25 x 10^{10} electrons

I. Multiple Choice Question:

11. The charge + q will be stable between B_1 and B_2 with respect to the displacement.
12. (c) uniformly charged infinite plane
13. (b) 8 mC
14. $\vec{E}$ = 10x$\hat{i}$, when x = 2 m

    $\vec{E}$ = 10 x 2 x $\hat{i}$ = 20$\hat{i}$

    Since, $E = \frac{-dV}{dx}$ $\therefore V = +20V$

15. (b)

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