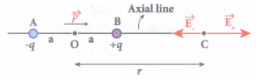
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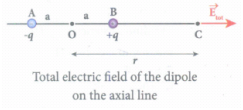
Answers Key:

Answer the following questions in detail.

1. **Case (i): Electric field due to an electric dipole at points on the axial line:**
Consider an electric dipole placed on the x-axis as shown in Figure. A point C is located at a distance of r from the midpoint O (of the dipole) along the axial line.



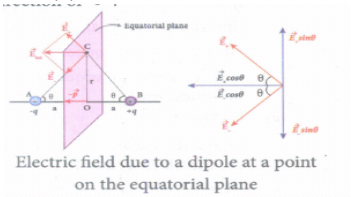
The electric field at a point C due to +q is
$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$ along BC
Since the electric dipole moment vector $\vec{p}$ is from -q to +q and is directed along BC, the above equation is rewritten as
$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p}$ ....(1)
When $\vec{p}$ is the electric dipole moment unit vector from -q to +q. The electric field at a point C due to -q is
$\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}$ ....(2)
Since +q is located closer to the point C than -q, $\vec{E}_+$ is stronger than $\vec{E}_-$. Therefore, the length of the $\vec{E}_+$ vector is drawn larger than that of $\vec{E}_-$ vector.
The total electric field at point C is calculated using the superposition principle of the electric field.
$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-$
$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}\hat{p}$
$\vec{E}_{tot} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)\hat{p}$ ....(3)
$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}q\left(\frac{4ra}{(r^2-a^2)^2}\right)\hat{p}$ ...(4)
Note that the total electric field is along $\vec{E}_+$, since +q is closer to C than -q.
If the point C is very far away from the dipole then (r >> a). Under this limit the term $(r^2 - a^2) \approx r^2$.
Substituting this into equation (4), we get
$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4aq}{r^3}\right)\hat{p}(r >> a)$
$since \quad 2aq\hat{p} = \vec{p}$
$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{r^3}(r >> a)$ ...(5)
The direction $\vec{E}_{tot}$ is shown in Figure.



Total electric field of the dipole on the axial line

NOTE: If the point C is chosen on the left side of the dipole, the total electric field is still in the direction of $\vec{p}$.

Case (ii) Electic field due to an electric dipole at a point on the equatorial plane:



Electric field due to a dipole at a point on the equatorial plane

Consider a point C at a distance r from the midpoint O of the dipole on the equatorial plane. Since the point C is equidistant from +q and -q, the magnitude of the electric fields of +q and -q are the same. The direction of $\vec{E}_+$ is along BC and the direction of $\vec{E}_-$ is along CA. $\vec{E}_+$ and $\vec{E}_-$ are resolved into two components; one component parallel to the dipole axis and the other perpendicular to it. The perpendicular components $|\vec{E}_+|$ sinθ and $|\vec{E}_-|$ sinθ are equal in magnitude and oppositely directed and cancel each other. The magnitude of the total electric field at point C is the sum of the parallel components of $\vec{E}_+$ and $\vec{E}_-$ and its direction is along $-\hat{p}$ as shown in the Figure.
$\vec{E}_{tot} = -|\vec{E}_+|cos\theta\hat{p} - |\vec{E}_-|cos\theta\hat{p}$ ...(6)
The magnitudes $\vec{E}_+$ and $\vec{E}_-$ are the same and given by,
$|\vec{E}_+| = |\vec{E}_-| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}$ ...(7)
By substituting equation (7) into equation (6), we get
$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{2qcos\theta}{(r^2+a^2)}\hat{p}$. . . . (8)
$= -\frac{1}{4\pi\varepsilon_0}\frac{2qa}{(r^2+a^2)^{\frac{3}{2}}}\hat{p}$
Since $cos\theta = \frac{a}{\sqrt{r^2+a^2}}$
$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{(r^2+a^2)^{\frac{3}{2}}}$
Since $\vec{p}$ = 2qa$\hat{p}$ ...(9)
At very large distances (r >> a), the equation (9) becomes
$\vec{E}_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{r^3}(r >> a)$ ...(10)
Negative sign shows that direction of Electric field is opposite to the direction of dipole moment vector.

2. **Electrostatic potential at a point due to an electric dipole :**
(i) Consider two equal and opposite charges separated by a small distance 2a as shown in Figure. The point P is located at a distance r from the

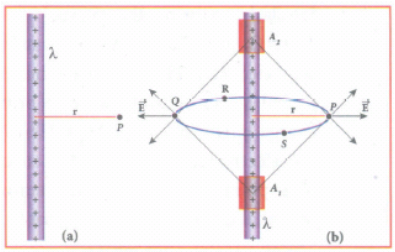
midpoint 'O' of the dipole. Let θ be the angle between the line OP and dipole axis AB.



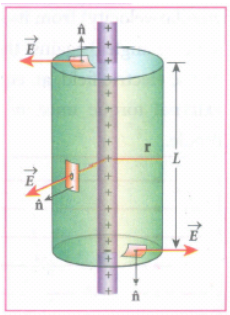
(ii) Let r_1 be the distance of point P from +q and r_2 be the distance of point P from -q.
Potential at P due to charge +q= $\frac{1}{4\pi\varepsilon_0}\frac{q}{r_1}$
Potential at P due to charge -q = $-\frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}$
Total potential at the point P,
$V = \frac{1}{4\pi\varepsilon_0} q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$ ....(1)
(iii) Suppose if the point P is far away from the dipole, such that (r >> a), then equation (1) can be expressed in terms of r.
By the cosine law for triangle BOP,
$r_1^2 = r^2 + a^2 - 2racos\theta$
$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}cos\theta\right)$
Since the point P is very far from dipole, then(r >> a). Then term $\frac{a^2}{r^2}$ is very small and can be neglected. Therefore
$r_1^2 = r^2\left(1 - 2a\frac{cos\theta}{r}\right)$
$(or) r_1 = r\left(1 - \frac{2a}{r}cos\theta\right)^{\frac{1}{2}}$
$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}cos\theta\right)^{-\frac{1}{2}}$
iv) Since $\frac{a}{r}$ << 1, we can use binomial theorem and retain the terms up to first order.
$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}cos\theta\right) \ldots (2)$
Similarly applying the cosine law for triangle AOP,
$r_2^2 = r^2 + a^2 - 2ra\cos(180-\theta)$
Since $\cos(180 - \theta) = -\cos\theta$ we get
$r_2^2 = r^2 + a^2 + 2ra\cos\theta$
Neglecting the term $\frac{a^2}{r^2}$ because (r >> a)
$r_2^2 = r^2\left(1 + \frac{2acos\theta}{r}\right)$
$r_2 = r\left(1 + \frac{2acos\theta}{r}\right)^{\frac{1}{2}}$
Using Binomial theorem, we get
$\frac{1}{r_2} = \frac{1}{r}\left(1 - a\frac{cos\theta}{r}\right) \quad \ldots (3)$
Substituting equation (3) and (2) in equation (1),
$V = \frac{1}{4\pi\varepsilon_0} q\left(\frac{1}{r}\left(1 + a\frac{cos\theta}{r}\right) - \frac{1}{r}\left(1 - a\frac{cos\theta}{r}\right)\right)$
$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}\left(1 + a\frac{cos\theta}{r} - 1 + a\frac{cos\theta}{r}\right)\right)$
$V = \frac{1}{4\pi\varepsilon_0}\frac{2aq}{r^2}cos\theta$
v) But the electric dipole moment p = 2qa and we get,
$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{pcos\theta}{r^2}\right)$
Now we can write $p\cos\theta = \vec{p}.\hat{r}$ where $\hat{r}$ is the unit vector from the point O to point P. Hence the electric potential at a point P due to an electric dipole is given by
$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\hat{r}}{r^2} (r >> a) \quad \ldots (4)$
Equation (4) is valid for distances very large compared to the size of the dipole. But for a point dipole, the equation (4) is valid for any distance.
Special cases:
Case (i) : If the point P lies on the axial line of the dipole on the side of +q, then θ = 0. Then the electric potential becomes
$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \quad \ldots (5)$
Case (ii) : If the point P lies on the axial line of the dipole on the side of -q, then θ = 180°, then
$V = -\frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \quad \ldots (6)$
Case (iii) : If the point P lies on the equatorial line of the dipole, then θ = 90°. Hence,
V = 0 .....(7)

3. (i) Consider an infinitely long straight wire having uniform linear charge density λ(charge per unit length).
(ii) Let P be a point located at a perpendicular distance r from the wire.
(iii) The electric field at the point P can be found using Gauss law. we choose two small charges elements A_1 and A_2 on the wire which are at equal distance from the point P.
(iv) The resultant electric field due to these two charge elements points radially away from the charged wire and the magnitude of electric field is same at all points on the circle of radius r.
This is shown in the Figure(b). From this property, the charged wire possesses a cylindrical symmetry.
(v) Let us choose a cylindrical Gaussian surface of radius r and length L .
The total electric flux through this closed surface is calculated as follows:
$\Phi_E = \oint \vec{E} \cdot d\vec{A}$
$= \int_{\text{Curved surface}} \vec{E} \cdot d\vec{A} + \int_{\text{top suffface}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom surface}} \vec{E} \cdot d\vec{A}$ .....(1)
It is seen from Figure that for the curved surface, $\vec{E}$ is parallel to $\vec{A}$ and $\vec{E} \cdot d\vec{A} = EdA$. For the top and bottom surfaces, $\vec{E}$ is perpendicular to $\vec{A}$and $\vec{E} \cdot d\vec{A} = 0$
Substituting these values in the equation (1) and applying Gauss law to the cylindrical surface, we have
$\Phi_E = \int_{\text{Curved surface}} EdA = \frac{Q_{encl}}{\varepsilon_o}$ .....(2)

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Electric field due to infinite long charged wire



Cylindrical Gaussian surface

Since the magnitude of the electric field for the entire curved surface is constant, E is taken out of the integration and Q_{encl} is given by Q_{encl} = λL where, λ is linear charge density (charge per unit length).

$E\int_{Curved\ surface} dA = \frac{\lambda L}{\varepsilon_0}$ ....(3)

Here, = $\int_{Curved\ surface} dA$ total area of the curved surface = 2πrL

Substituting this in equation (3), we get

E.2πrL = $\frac{\lambda L}{\varepsilon_0}$

$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$ ...(4)

In vector form, $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}\hat{r}$ ...(5)

The electric field due to the infinite charged wire depends on $\frac{1}{r}$ rather than $\frac{1}{r^2}$ for a point charge.

Equation (5) indicates that the electric field is always along the perpendicular direction ($\hat{r}$) to wire. In fact, if λ > 0 then $\vec{E}$points perpendicular outward ($\hat{r}$) from the wire and if λ < 0, then $\vec{E}$ points perpendicular inward ($-\hat{r}$).

4. **Electric field due to charged infinite plane sheet:**

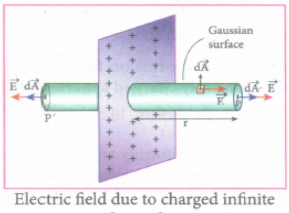
(i) Consider an infinite plane sheet of charges with uniform surface charge density σ. (Charge per unit area). Let P be a point at a distance of r from the sheet as shown in the Figure.

(ii) Since the plane is infinitely large, the electric field should be same at all points equidistant from the plane and radially directed outward at all points. A cylindrical-shaped Gaussian surface of length 2r and two flat surfaces is chosen such that the infinite plane sheet passes perpendicularly through the middle part of the Gaussian surface.

Total electric flux linked with the cylindrical surface,

$\phi_E = \int \vec{E}.d\vec{A}$

$= \int_{Curved\ surface} \vec{E}.d\vec{A} + \int_P \vec{E}.d\vec{A} + \int_{P'} \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ ...(1)



Electric field due to charged infinite planar sheet

(iii) The electric field is perpendicular to the area element at all points on the curved surface and is parallel to the surface areas at P and P ′ (Figure). Then, applying Gauss's law.

$\phi_E = \int_p EdA + \int_{p'} EdA = \frac{Q_{encl}}{\varepsilon_0}$ ...(2)

Since the magnitude of the electric field at these two equal flat surfaces is uniform, E is taken out of the integration and Q_{ncel} is given by Q_{encl} = σA, we get

$2E\int_p dA = \frac{\sigma A}{\varepsilon_0}$

The total area of surface either at P or P′

$\int_p dA = A$

Hence $2EA = \frac{\sigma A}{\varepsilon_0}$ or $E = \frac{\sigma}{2\varepsilon_0}$ ...(3)

In vector $\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$ ...(4)

(iv) Here $\hat{n}$ is the outward unit vector normal to the plane. Note that the electric field due to an infinite plane sheet of charge depends on the surface charge density and is independent of the distance r.

(v) The electric field will be the same at any point farther away from the charged plane.

(vi) Equation (4) implies that if σ > 0 the electric field at any point P is outward perpendicular $\hat{n}$ to the plane and if σ < 0 the electric field points inward perpendicularly ($-\hat{n}$) to the plane.

5. **Electric field due to a uniformly charged spherical shell :**

Consider a uniformly charged spherical shell of radius R carrying total charge Q as shown in Figure. The electric field at points outside and inside the sphere is found using Gauss law.

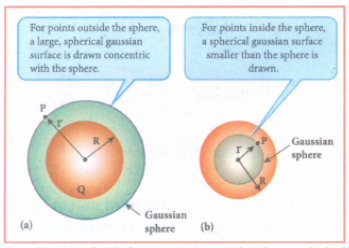
Case (a): At a point outside the shell (r > R):

(i) Let us choose a point P outside the shell at a distance r from the centre.

(ii) The charge is uniformly distributed on the surface of the sphere (spherical symmetry).

(iii) Hence the electric field must point radially outward if Q > 0 and point radially inward if Q < 0. So, a spherical Gaussian surface of radius r is chosen and the total charge enclosed by this Gaussian surface is Q. Applying Gauss law,

$\oint_{Gaussian\ surface} \vec{E}.d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$ .... (1)



The electric field due to a charged spherical shell

(iv) The electric field $\vec{E}$ and $d\vec{A}$ point in the same direction (outward normal) at all the points on the Gaussian surface. The magnitude of $\vec{E}$ is also the same at all points due to the spherical symmetry of the charge distribution.

$E \oint_{Gaussian\ surface} dA = \frac{Q}{\varepsilon_0}$.... (2)

But $\oint_{Curved\ surface} dA$ = total area of Gaussian surface = $4\pi r^2$.

Substituting this in equation (2),

$E.4\pi r^2 = \frac{Q}{\varepsilon_0}$

$E.4\pi r^2 = \frac{Q}{\varepsilon_0} (or)$ $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

In vector form $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$ .... (3)

(v) The electric field is radially outward if Q > 0 and radially inward if Q < 0. From equation (3), the electric field at a point outside the shell will be same as if the entire charge Q is concentrated at the center of the spherical shell.

Case (b): At a point on the surface of the spherical shell (r = R):

The electrical field at points on the spherical shell (r = R) is given by

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{r}$ ... (4).

Case (c): At a point inside the spherical shell (r < R):

(vi) Consider a point P inside the shell at a distance r from the centre. A Gaussian sphere of radius r is constructed as shown in the figure. Applying Gauss law,

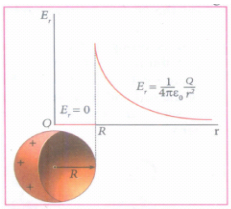
$\oint_{Gaussian\ surface} \vec{E}.d\vec{A} = \frac{Q}{\varepsilon_0}$

$E.4\pi r^2 = \frac{Q}{\varepsilon_0}$ .....(5)

(vii) Since Gaussian surface encloses no charge, Q = 0. The cquation (5) becomes

E = 0 (r < R) .....(6)

(viii) The electric field due to the uniformly charged spherical shell is zero at all points inside the shell. A graph is plotted between the electric field and radial distaance.



Electric field versus distance for a spherical shell of radius R

6. **Effect of dielectrics in capacitors:**

Suppose dielectrics like mica, glass or paper are introduced between the plates, then the capacitance of the capacitor is altered. The dielectric can be inserted into the plates in two different ways.

(i) when the capacitor is disconnected from the battery.

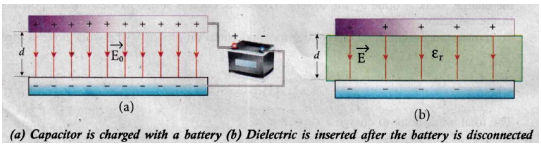
(ii) when the capacitor is connected to the battery.

(i) When the capacitor is disconnected from the battery

Consider a capacitor with two parallel plates each of cross-sectional area A and are separated by a distance d. The capacitor is charged by a battery of voltage V_o and the charge stored is Q_o. The capacitance of the capacitor without the dielectric is,

$C_0 = \frac{Q_0}{V_0}$ ....(i)

The battery is then disconnected from the capacitor and the dielectric is inserted between the plates. This is shown in Figure.



(a) Capacitor is charged with a battery (b) Dielectric is inserted after the battery is disconnected

The introduction of dielectric between the plates will decrease the electric field. Experimentally it is found that the modified electric field is given by,

$E = \frac{E_0}{\varepsilon_r}$ ....(2)

Here E_o is the electric field inside the capacitors when there is no dielectric and ε_r is the relative permittivity of the dielectric or simply known as the dielectric constant. Since $\varepsilon_r > 1$, the electric field $E < E_o$.

As a result, the electrostatic potential difference between the plates (V = Ed) is also reduced. But at the same time, the charge Q_o will remain constant once the battery is disconnected.

Hence the new potential difference is

$V = Ed = \frac{E_0}{\varepsilon_r} d = \frac{V_0}{\varepsilon_r}$ .....(3)

We know that capacitance is inversely proportional to the potential difference. Therefore as V decreases, C increases.

Thus new capacitance in the presence of a dielectric is

$C = \frac{Q_0}{V} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0$ .....(4)

Since $\varepsilon_r > 1$, we have $C > C_o$. Thus insertion of the dielectric increases the capacitance.

We know that, $C_o = \frac{\varepsilon_o A}{d}$ .....(5)

Equation (4) $\Rightarrow C = \frac{\varepsilon_r \varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$ .....(6)

where $\varepsilon = \varepsilon_r \varepsilon_o$ is the permittivity of the dielectric medium.

The energy stored in the capacitor before the insertion of a dielectric is given by,

$U_0 = \frac{1}{2} \frac{Q_0^2}{C_0}$ .....(7)

After the dielectric is inserted, the charge Q_o remains constant but the capacitance is increased. As a result, the stored energy is decreased.

$U = \frac{1}{2} \frac{Q_0^2}{C} = \frac{1}{2} \frac{Q_0^2}{\varepsilon_r C_0} = \frac{U_0}{\varepsilon_r}$ .....(8)

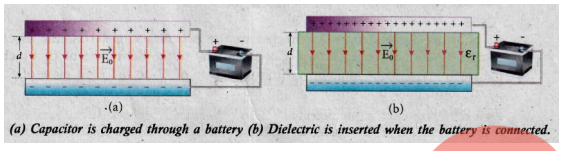
Since $\varepsilon_r > 1$we get $U < U_o$. There is a decrease in energy because, when the dielectric is inserted, the capacitor spends some energy in pulling the dielectric inside.

(ii) When the battery remains connected to the capacitor:

When the battery of voltage v_o remains connected to the capacitor and the dielectric is inserted into the capacitor, then

(a) The potential difference v_o across the plates remains constant.

(b) The charge stored in the capacitor is increased by a factor ε_r. (Experimentally found).



(a) Capacitor is charged through a battery (b) Dielectric is inserted when the battery is connected.

$Q = \varepsilon_r Q_0$ .....(1)

Due to this increased charge, the capacitance is also increased. The new capacitance is,

$C = \frac{Q}{V_0} = \varepsilon_r \frac{Q_0}{V_0} = \varepsilon_r C_0$ .....(2)

However the reason for the increase in capacitance in this case where the battery remains connected is different from the case when the battery is disconnected before introducing the dielectric.

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The energy stored in the capacitor before the insertion of a dielectric is given by,

$U_0 = \frac{1}{2} C_0 V_0^2$ ....(4)

After the dielectric is inserted, the capacitance is increased; hence the stored energy is also increased.

$U = \frac{1}{2} C V_0^2 = \frac{1}{2} \varepsilon_r C_0 V_0^2 = \varepsilon_r U_0$ .....(5)

Since $\varepsilon_r > 1$ we have $U > U_o$.

Note: Here we have not used the expression $U_o = \frac{1}{2} \frac{Q_0^2}{C_0}$because here, both charge and capacitance are changed, whereas in equation (4), V_o remains constant.

Since voltage between the capacitor V_o is constant, the electric field between the plates also remains constant .The energy density is given by,

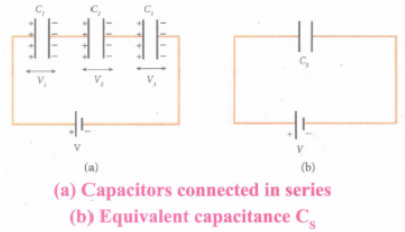
$u = \frac{1}{2} \varepsilon E_0^2$ ..(6)

where ε is the permittivity of the given dielectric material.

7. **(a) Capacitor in series**

(i) Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series with a battery of voltage V as shown in the Figure (a).

(ii) As soon as the battery is connected to the capacitors in series, the electrons of charge -Q are transferred from negative terminal to the right plate of C_3 which pushes the electrons of same amount -Q from left plate of C_3 to the right plate of C_2 due to electrostatic induction.



(a) Capacitors connected in series
(b) Equivalent capacitance C_S

(iii) Similarly, the left plate of C_2 pushes the charges of -Q to the right plate of C_1 which induces the positive charge +Q on the left plate of C_1.

(iv) At the same time, electrons of charge -Q are transferred from left plate of C_1 to positive terminal of the battery.

(v) By these processes, each capacitor stores the same amount of charge Q.

(vi) The capacitances of the capacitors are in general different so that the voltage across each capacitor is also different and are denoted as V_1, V_2 and V_3 respectively.

(vii) The sum voltage across capacitor must be equal to the voltage of the battery.

$V = V_1 + V_2 + V_3$ ....(1)

Since, Q = CV

We have $V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$= Q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \ldots (2)$

(viii) If three capacitors in series are considered to form an equivalent single capacitor C_s shown in Figure (b), then we have $V = \frac{Q}{C_s}$. Substituting this expression into equation (2), we get

$\frac{Q}{C_s} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

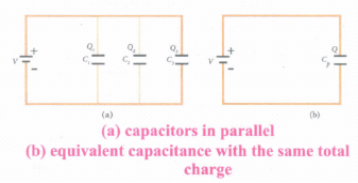
$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ ...(3)

(ix) Thus, the inverse of the equivalent capacitance C_s of three capacitors connected in series is equal to the sum of the inverses of each capacitance. This equivalent capacitance C_s, is always less than the smallest individual capacitance in the series.

(b) Capacitor in parallel

i) Consider three capacitors of capacitance C_1, C_2 and C_3 connected in parallel with a battery of voltage V as shown in Figure (a).

ii) Since corresponding sides of the capacitors are connected to the same positive and negative terminals of the battery, the voltage across each capacitor is equal to the battery's voltage.



(a) capacitors in parallel
(b) equivalent capacitance with the same total charge

iii) Since capacitance of the capacitors is different, the charge stored in each capacitor is not the same. Let the charge stored in the three capacitors be Q_1, Q_2, and Q_3 respectively.

iv) According to the law of conservation of total charge, the sum of these three charges is equal to the charge Q transferred by the battery,

$Q = Q_1 + Q_2 + Q_3$ ...(4)

Now, since Q = CV, we have

$Q = C_1V + C_2V + C_3V$ ...(5)

(v) If these three capacitors are considered to form a single capacitance C, which stores the total charge Q as shown in the Figure (b), then we can write $Q = C_pV$. Substituting this in equation (2), we get

$C_pV = C_1V + C_2V + C_3V$

$C_p = C_1 + C_2 + C_3$ ...(6)

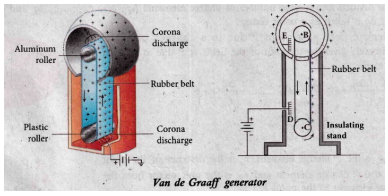
vi) Thus, the equivalent capacitance of capacitors connected in parallel is equal to the sum of the individual capacitances.

vii) The equivalent capacitance C_p, in a parallel connection is always greater than the largest individual capacitance. In a parallel connection, it is equivalent as area of each capacitance adds to give more effective area such that total capacitance increases.

8. In the year 1929, Robert Van de Graaff designed a machine which produces a large amount of electrostatic potential difference, up to several million volts (10^7 V).

Principle:

Electrostatic induction and Action at points.



Van de Graaff generator

Construction:

(i) A large hollow spherical conductor is fixed on the insulating stand as shown in Figure. A pulley B is mounted at the center of the hollow sphere and another pulley C is fixed at the bottom. A belt made up of insulating materials like silk or rubber runs over both pulleys. The pulley C is driven continuously by the electric motor. Two comb-shaped metallic conductors E and D are fixed near the pulleys.

(ii) The comb D is maintained at a positive potential of 10^4 V by a power supply. The upper comb E is connected to the inner side of the hollow metal sphere

Working:

(i) Because of the high electric field near comb D, air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D.

(ii) The positive charges stick to the belt and move up. When the positive charges reach the comb E, a large amount of negative and positive charges are induced on either side of comb E due to electrostatic induction.

(iii) As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere. Since the sphere is a conductor, the positive charges are distributed uniformly. on the outer surface of the hollow sphere.

(iv) At the same time the negative charges nullify the positive Charges in the belt due to corona discharge before it passes over the pulley.

(v) When the belt descends, it has almost no net charge. At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the outer surface of the sphere.

(vi) This process continues until the outer surface produces the potential difference of the order of 10^7 which is the limiting value. We cannot store charges beyond this limit since the extra charge starts leaking to the surroundings due to ionization of air. The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

(vii) The high voltage produced in this Van de Graaff generator is used to accelerate positive ions (protons and deuterons) for nuclear disintegrations and other applications.

Write the answer any 4 questions.Q. No.17 Compulsory.

9. **(a)** The capacitance of the capacitor is

$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times25\times10^{-4}}{1\times10^{-3}}$

$= 221.2 \times 10^{-13}$ F

C = 22.12 x 10^{-12}F = 22.12pF

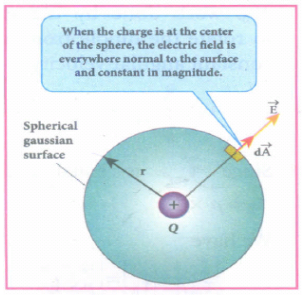
(b) The charge stored in any one of the plates is Q = CV, Then

Q = 22.12 x 10^{-12} x 10 = 221.2 x 10^{-12}C = 221.2 pC.

10. **Gauss law:**

(i) A positive point charge Q is surrounded by an imaginary sphere of radius r as shown in Figure. then the total electric flux through the closed surface of the sphere is

$\Phi_E = \oint \vec{E}.d\vec{A} = \oint EdAcos\theta$ .....(1)



(ii) The electric field of the point charge is directed radially outward at all points on the surface of the sphere. Therefore, the direction of the area element $d\vec{A}$ is along the electric field $\vec{E}$ and θ = 0°.

$\Phi_E = \oint EdA$ Since cos0° = 1 .....(2)

iii) E is uniform on the surface of the sphere,

$\Phi_E = E\oint dA$ .....(3)

Substituting for $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ in eqn (3), we get

$\therefore \phi E = 4\pi r^2 E$

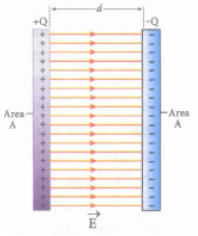
$\phi_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \times 4\pi r^2 = 4\pi\frac{1}{4\pi\varepsilon_0}Q$

$\phi_E = \frac{Q}{\varepsilon_0}$ .....(4)

The equation (4) is called as Gauss's law.

11. **Capacitance of a parallel plate capacitor:**

(i) Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d as shown in Figure.



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(ii) The electric field between two infinite parallel plates is uniform and is given by $E = \frac{\sigma}{\varepsilon_o}$ where σ is the surface charge density on the plates $\left(\sigma = \frac{Q}{A}\right)$.

iii) If the separation distance d is very much smaller than the size of the plate ($d^2 << A$), then the above result is used even for finite-sized parallel plate capacitor.

The electric field between the plates is

$E = \frac{Q}{A\varepsilon_0} \ldots (1)$

iv) Since the electric field is uniform, the electric potential between the plates having separation d is given by

$V = Ed = \frac{Qd}{A\varepsilon_0} \quad \ldots (2)$

Therefore the capacitance of the capacitor is given by

$C = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{A\varepsilon_0}\right)} = \frac{\varepsilon_0 A}{d} \quad \ldots . (3)$

(v) From equation (3), it is evident that capacitance is directly proportional to the area of cross section and is inversely proportional to the distance between the plates.

12. The superposition principle explains the interaction between multiple charges. According to this superposition principle, the total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

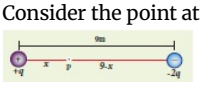
13. (a) Most people are now familiar with the digital camera. The flash which comes from the camera when we take photographs is due to the energy released from the capacitor, called a flash capacitor.

(b) During cardiac arrest, a device called heart defibrillator is used to give a sudden surge of a large amount of electrical energy to the patient's chest to retrieve the normal heart function. This defibrillator uses a capacitor of 175 μF charged to a high voltage of around 2000 V.

(c) Capacitors are used in the initial system of automobile engines to eliminate sparking.

(d) Capacitors are used to reduce power fluctuations in power supplies and to increase the efficiency of power transmission.

14. According to the superposition principle, the total electric potential at a point is equal to the sum of the potentials due to each charge at that point. Consider the point at which the total potential zero is located at a distance x from the charge +q as shown in the figure.



Since the total electric potential at P is zero,

$V_{tot} = \frac{1}{4\pi\varepsilon}\left(\frac{q}{x} - \frac{2q}{(9-x)}\right) = 0$ (or)

$\frac{q}{x} = \frac{2q}{(9-x)}$ (or)

$\frac{1}{x} = \frac{2}{(9-x)}$

Hence, x = 3 m

Write the answer any 4 questions.Q.No 12 compulsory.

15. The maximum torque experienced by the dipole is when it is aligned perpendicular to the applied field.
τ_{max} = pE sin 90^0 = 3.4 x 10^{-30} x 3 x 10^4 Nm
τ_{max} = 10.2 x 10^{-26} Nm.

16. When the water molecules are aligned in the direction of the electric field, it has minimum potential energy. The work done to rotate the dipole from θ = 0° to 90° is equal to the potential energy difference between these two configurations.
W = ΔU = U(90°) - U(0°)
From the equation U =−pE cosθ = $-\hat{p}.\hat{E}$,
we write U = − pE cosθ, Next, we calculate the work done to rotate one water molecule from θ = 0° to 90°.
For one water molecule
W = - pE cos90° + pE cos0° = pE
W= 6.3 x 10^{-30} x 3 x 10^5 = 18.9 x 10^{-25}J
For 10^{22} water molecules, the total work done is
W_{tot} = 18.9 x 10^{-25} x 10^{22} = 18.9 x 10^{-3}J

17. An equipotential surface is a surface on which all the points are at the same electric potential.

18. The alignment of the dipole moments of the permanent or induced dipoles in the direction of applied electric field is called polarisation.

19. The capacitance C of a capacitor is defined as the ratio of the magnitude of charge on either of the conductor plates to the potential difference existing between the conductors. $C = \frac{Q}{V}$
Its unit is Coulomb per volt or farad (F).

20. When an irregular shaped conductor is given positive charge, the electric field near the sharp end is very high and it ionizes the surrounding air. The positive ions are repelled at the sharp edge and negative ions are attracted towards the sharper edge. This reduces the total charge of the conductor near the sharp edge. This is called corona discharge.

Multiple Choice Question

21. (c) uniformly charged infinite plane

22. (b) 8 mC

23. The electric flux of D is less than that of C
The electric flux of C is less than that of B
The electric flux of B is less than that of A

24. Force ∝ charge
After the separation, the magnitude of charge will be increased. So the force will be more than before.

25. $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$
$i) U = \frac{1}{4\pi\varepsilon_0}\frac{Q(-Q)}{r} = \frac{1}{4\pi\varepsilon_0}[\frac{-Q^2}{r}]$
$ii) U = \frac{1}{4\pi\varepsilon_0}\frac{(-Q)(-Q)}{r} = \frac{1}{4\pi\varepsilon_0}[\frac{Q^2}{r}]$
$iii) U = \frac{1}{4\pi\varepsilon_0}\frac{Q(2Q)}{r} = \frac{1}{4\pi\varepsilon_0}[\frac{2Q^2}{r}]$
$iv) U = \frac{1}{4\pi\varepsilon_0}\frac{Q(-2Q)}{2r} = \frac{1}{4\pi\varepsilon_0}[\frac{-Q^2}{r}]$
From the values, 1 = 4 < 2 < 3

26. W = qV
V = 7 - (-4) = 11 V
q = ne = 50 x 1.6 x 10^{-19} = 8 x 10^{-18}C
W = 8 x 10^{-18} x 11 = 88 x 10^{-18} = 8.8 x 10^{-17}]

27. If voltage is increased from V to 2 V
Then Q_1 = CV
Q_2 = C(2 V) = 2 CV
∴ Q is doubled and C remains same

28. Energy density $u_E = \frac{U}{volume}$
If A′ = 2A d′ = 2d
Then V′ = 2A x 2d = 4Ad = 4V
Then volume would be increased. So, energy density will change.

29. (c) Maxwell

30. (c) Volume charge density (ρ)

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