QB365

Important Questions - Continuity and Differentiability

12th Standard CBSE

Maths

Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Examine the continuity of the function f (x) = x^2+5 at x=-1 **1**

2) Examine the continuity of the function f (x)=$\frac{1}{x+3}, \quad x \quad \varepsilon \quad R.$ **1**

3) Discuss the continuity of the function defined by f(x) =$\frac{1}{x}, \quad x \neq 0$ **1**

4) Find the point of discontinuity if any for the function f(x)=$\frac{1}{x-5}$ **1**

5) Verify MVT for the following : f (x) = $(x-1)^{2/3}$, in [0, 2]. **1**

Section-B

6) If y= $\tan^{-1}\sqrt{\frac{1-x}{1+x}} find \frac{dy}{dx}$ **2**

7) If y = $\tan^{-1}\sqrt{\frac{sinx}{1+cosx}}, find \frac{dy}{dx}$ **2**

8) If y = $a^x + x^a + x^x + a^a$, find dy/dx **2**

9) If x = $\frac{at}{1+t^2}, y = \frac{at^2}{1+t^2}, find \frac{dy}{dx} at$ t = 2 **2**

10) If e^y (x+1) = 1, show that dy/dx = $-e^y$ **2**

11) $y = tan^{-1}\frac{5x}{1-6x^2}$,\(-\frac { 1 }{ \sqrt { 6 } } . then prove that $\frac{dy}{dx} = \frac{2}{1+4x^2} + \frac{3}{1+9x^3}$. **2**

Section-C

12) Show that the function f (x)= $\begin{cases} x^3+3 \quad , & if \quad x \neq 0 \\ 1 \quad , & if \quad x = 0 \end{cases}$ is not continuous at x=0. **3**

13) Check the continuity of the function f given by: **3**

f(x)=2x+3 at x=1.

14) Discuss the continuity of the function f given by: **3**

$f(x) = |x| at \quad x = 0.$

15) Discuss the continuity of the function f defined by: **3**

$f(x) = x^3 + x^2 - 1$

Section-D

16) Find the value of 'a' for which the function 'f' defined as: **3**

$$f(x) = \begin{cases} a \quad sin\frac{\pi}{2}(x+), & x \leq 0 \\ \frac{tan \quad x - sin \quad x}{x^3}, & x > 0 \end{cases}$$

is continuous at x=0.

17) If $x^{16}y^9 = \left(x^2+y\right)^{17}$, prove that $\frac{dy}{dx} = \frac{2y}{x}$ **3**

18) Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ if $x = ae^{\theta}(sin\theta - cos\theta)$ and $y = ae^{\theta}(sin\theta + cos\theta)$ **3**

Section-E

19) Find the value of k, so that the function $f(x) = \begin{cases} kx^2, & if \quad x \geq 1 \\ 4 \quad , & if \quad x < 1 \end{cases}$ is continuous at x=1. **4**

20) Discuss the continuity of the following function at x=0 : f (x) = $f(x) = \begin{cases} \frac{x^4+2x^3+x^2}{tan^{-1}x} \quad , & x \neq 0 \\ 0 \quad , & x = 0 \end{cases}$. **4**

21) Discuss the continuity of the function f (x)=| x | + | x-1 | at x=1. **4**

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Section-A

1) Hence, continuous at x =-1. **1**

2) For x=-3 function is not defined. Hence, not continuous for $x \quad \varepsilon \quad R.$ **1**

3) Continuous at x=a ($\neq$ 0) $\in$ R. **1**

4) Function is not defined for x=5. Hence discontinuous at x=5. **1**

5) Not verified as function f (x) = $(x - 1)^{2/3}$ is not differentiable at x = 1 [0, 2]. **1**

Section-B

6) We have, y = $\tan_{-1}\sqrt{\frac{1-x}{1+x}}$ **2**

Put x = cos2θ

$\Rightarrow$ 2θ = $\cos^{-1}$x

$\Rightarrow$ θ = 1/2$\cos^{-1}$x

y = $\tan_{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$

y = $\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}$

($\because$ cos2θ = 2$\cos^2\theta$-1 = 1-2$\sin^2\theta$

y = $\tan^{-1}(\tan\theta)$

y = θ = 1/2$\cos^{-1}$x

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right) = \frac{-1}{2\sqrt{1-x^2}}$$

7) Given, y = $\tan_{-1}\sqrt{\frac{\sin x}{1+\cos x}}$ **2**

y = $\tan^{-1}\sqrt{\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}}$

y = $\tan^{-1}\left(\sqrt{\tan\frac{x}{2}}\right)$

y = $\tan^{-1}$

8) We have, y = $a^x + x^a + x^x + a^a$ **2**

Let v = x^x

log v = x log x

$\frac{1}{v}\frac{dv}{dx} = x + \frac{1}{x} + \log x$

$\frac{dv}{dx} = v|1 + \log x|$

= x^x(1+lodx)

$\frac{dy}{dx} = a^x\log a + ax^{a-1} + \frac{dv}{dx} + 0$

$\Rightarrow \frac{dy}{dx} = a^x\log a + ax^{a-1} + x^x(1 + \log x)$

9) We have, x = $\frac{at}{1+t^2}, y = \frac{at^2}{1+t^2}$ **2**

$\frac{dx}{dt} = \frac{(1+t)^2a - at(2t)}{(1+t^2)^2}$

$\Rightarrow \frac{dx}{dt} = \frac{a+at^2-2at^2}{(1+t^2)^2}$

= $\frac{a-at^2}{(1+t^2)^2}$

and $\frac{dy}{dt} = \frac{(1+t^2)^2 2at - at^2(2t)}{(1+t^2)^2}$

$\Rightarrow \frac{dy}{dt} = \frac{2at+2at^3-2at^3}{(1+t^2)^2} = \frac{2at}{(1+t^2)^2}$

$\therefore \frac{dy}{dx} = \frac{2at}{(1+t^2)^2} \times \frac{(1+t^2)^2}{a-at^2}$

$= \frac{2at}{a-at^2} = \frac{2t}{1-t^2}$

$\left(\frac{dy}{dx}\right)_{at t=2} = \frac{2(2)}{1-4}$

= -4/3

10) On differentiating e^y (x+1) = 1 **2**

e^y+(x+1)e^ydy/dx = 0

$\Rightarrow e^y + \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -e^y$

11) $y = tan^{-1}\frac{3x+2x}{1-3x2x}$ 2

$=tan^{-1}3x + tan^{-1}2x$

$\Rightarrow \frac{dy}{dx} = \frac{3}{1+9x^3} + \frac{2}{1+4x^2}$

Section-C

12) $\lim_{x \to 0}f(x) = \lim_{x \to 0}\left(x^3 + 3\right) = 0 + 3 = 3f(0) = 1 Thus \quad \lim_{x \to 0}f(x) \neq f(0) Hence, \quad f \quad is \quad not \quad continuous \quad at \quad x = 0.$ 3

13) $\lim_{x \to 1}f(x) = \lim_{x \to 1}(2x + 3) = 2(1) + 3 = 5f(1) = 2(1) + 3 = 5. Thus \quad \lim_{x \to 1}f(x) = f(1) Hence, \quad f \quad is \quad continuous \quad at \quad x = 1.$ 3

14) $By \quad definition, \quad f(x) = \begin{cases} -x, if & x < 0 \\ x, \quad if & x \geq 0. \end{cases}$ 3

$\lim_{x \to 0^-}f(x) = \lim_{x \to 0^-}(-x) = 0$

$\lim_{x \to 0^+}f(x) = \lim_{x \to 0^-}(x) = 0$

$Also, \quad f(0) = 0$

$Thus \quad \lim_{x \to 0^-}f(x) = \lim_{x \to 0^+}f(x) = f(0)$

$Hence, \quad 'f' \quad is \quad continuous \quad at \quad x = 0.$

15) $We \quad have: f(x) = x^3 + x^2 - 1$ 3

Which is polynomial function

$and \quad D_f = R$

$Let \quad c \epsilon D_f$

$Then \quad \lim_{x \to c}f(x) = \lim_{x \to c}(x^3 + x^2 - 1)$

$= c^3 + c^2 - 1 = f(c)$

$\Rightarrow \quad f \quad is \quad continuous \quad at \quad x = c.$

But c is arbitrary.

Hence, f is continuous at each of its domains.

Section-D

16) $\lim_{x \to 0^-}f(x) = \lim_{x \to 0^-}a \quad sin\frac{\pi}{2}(x + 1)$ 3

$= \lim_{h \to 0}a \quad sin\frac{\pi}{2}(0 - h + 1)$

$= a \quad sin\frac{\pi}{2}(0 - 0 + 1) = a \quad sin\frac{\pi}{2}$

$= a.1 = a$

$\lim_{x \to 0^+}f(x) = \lim_{x \to 0^+}\frac{tan \quad x - sin \quad x}{x^3}$

$= \lim_{h \to 0}\frac{tan(0+h) - sin(0+h)}{(0+h)^3}$

$= \lim_{h \to 0}\frac{tanh - sinh}{h^3}$

$= \lim_{h \to 0}\frac{sinh}{h}\frac{1 - cosh}{h^2}.\frac{1}{cos \quad h}$

$= \lim_{h \to 0}\frac{sinh}{h} \lim_{h \to 0}\frac{2sin^2h/2}{h^2}.\frac{1}{cos \quad h}$

$= \lim_{h \to 0}\left(\frac{sinh/2}{h/2}\right)^2\frac{1}{cos \quad 0}$

$= 1.\frac{1}{2}(1)^2\frac{1}{1} = \frac{1}{2}$

Also f(0)= $= a \quad sin\frac{\pi}{2}(0 + 1) = a \quad sin\frac{\pi}{2} = a(1) = a$

For continuity, $\lim_{x \to 0^-}f(x) = \lim_{x \to 0^+}f(x) = f(0)a = \frac{1}{2} = a$

Hence, $a = \frac{1}{2}$

17) 3

We have: $x^{16}y^9 = \left(x^2 + y\right)^{17},$

Taking logs.,

$log(x^{16}y^9) = log\left(x^2 + y\right)^{17} \quad 16logx + 9logy = 17log\left(x^2 + y\right)\frac{16}{x} + \frac{9}{y}.\frac{dy}{dx} = 17\frac{1}{x^2+y}\left[2x + \frac{dy}{dx}\right] \qquad = \left[\frac{9}{y} - \frac{17}{x^2+y}\right]\frac{dy}{dx} = \frac{34x}{x^2+y} - \frac{16}{x} = \frac{9x^2+9y-17y}{y\left(x^2+y\right)}\frac{dy}{dx} = \frac{34x^2-16x^2-16y}{x\left(x^2+y\right)} \quad = \frac{1}{y}\left(9x^2 - 8y\right)$

18) We have: $x = ae^\theta(sin\theta - cos\theta) \qquad y = ae^\theta(sin\theta + cos\theta)$ 3

$\frac{dx}{d\theta} = ae^\theta(cos\theta + sin\theta) + ae^\theta(sin\theta - cos\theta) = 2ae^\theta sin\theta\frac{dy}{d\theta} = ae^\theta(cos\theta - sin\theta) + ae^\theta(sin\theta + cos\theta) = 2ae^\theta cos\theta\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2ae^\theta cos\theta}{2ae^\theta sin\theta} = \frac{cos\theta}{sin\theta} = cot\theta Hence, \quad \frac{dy}{dx} = cot\frac{\pi}{4} = 1$

Section-E

19) k = 4 4

20) 0=0, true. Hence, continuous. 4

21) | x |, | x-1| are continuous function and sum is also continuous. 4