QB365

Important Questions - Binomial Theorem

11th Standard CBSE

Mathematics Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Find the middle term(s) in the given expansion. **2**

$\left(\frac{x}{3}+9y\right)^{10}$

2) Find the middle term(s) in the given expansion. **2**

$\left(3x-\frac{x^3}{6}\right)$

3) Find the number of terms in the expansions of following expressions. **2**

$(1-z)^4$

4) Using binomial theorem, expand$(x^2+2y)^5$ **2**

5) Prove that $\sum\limits_{r=0}^{n} 3^r \quad {}^nC_r = 4^n$ **2**

Use$(1+x)^n = {}^nC_o + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + \dots + {}^nC_nx^n$ and prove the result.

6) Evaluate the following terms. **2**

7th term in the expansion of $\left(2x+\frac{y}{3}\right)^{15}$.

Section-B

7) Find the number of terms in the expansions of the following expressions. **3**

$(x+a)^{100}+(x-a)^{100}$

8) If 25^{15} is divided by 13, then find the remainder. **3**

9) Using binomial theorem, show that the expression 7^9+9^7 is divided by 64. **3**

10) Draw the shape of the hyperbola $5y^2-9x^2=36$ and find its centre, transverse axis, conjugate axis, value of c, vertices, directrices, foci, eccentricity and length of latusrectum. **3**

11) Expand the following by using binomial theorem. **3**

$\left(1-\frac{x}{3}\right)^{4/3}$ When $|x| < 3$

12) If there is a term independent of x, in the expansion of $\left(x^2+\frac{1}{x}\right)^n$ then show that the term is. $\frac{n!}{\left(\frac{n}{3}\right)!\left(\frac{2n}{3}\right)!}$ **3**

13) Find the two successive terms in the expansion of $(1+x)^{24}$, whose coefficients are in the ratio 1:4 **3**

Section-C

14) Expand $(2+3y)^{-5}$ into four terms along with the condition of validity of expansion in each case. **4**

Ascending order of y.

15) If the binomial expansion of $(c+dy)^{-2}$ is $\frac{1}{4}-3y+\dots$ then find the values of c and d. **4**

16) If x is nearly equal to 1,then prove that $\frac{ax^b-bx^a}{x^b-x^a}=\frac{1}{(1-x)}$ nearly. **4**

17) Evaluate $(1.025)^{-1/3}$ correct to three places of decimal. **4**

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Section-A

1) Here, n=10 (even) So there will be one one middle term i.e. $\left(\frac{10+2}{2}\right)$ th term or 6th term **2**

$T_6=T_{5+1}={}^{10}C_5\left(\frac{x}{3}\right)^{10-5}(9y)^5=61236\,x^5y^5$

2) **2**

Here, n=9 (odd)

So, the binomial expansion has middle terms viz.,$\left(\frac{9+1}{2}\right)$ th and $\left(\frac{9+1}{2}+1\right)$ th terms i.e. 5th term and 6th term

$T_5=T_{4+1}={}^9C_4\quad(3x)^{9-4}\quad\left(\frac{-x^3}{6}\right)^4=\frac{189}{8}x^{17}$

and $T_5=T_{4+1}={}^9C_4\quad(3x)^{9-4}\quad\left(\frac{-x^3}{6}\right)^4=\frac{189}{8}x^{17}$

3) Given expression is $(1-z)^{4.}$ Here, n= 4 **2**

$\therefore$ The number of terms in the expansion is (n+1)

i.e. 4+1 = 5

4) By binomial theorem, we have **2**

$(a+b)^n={}^nC_oa^n+{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2+\ldots+{}^nC_n(b)^n$

Here, $a=x^2$, b=2y and n = 5

$\therefore (x^2+2y)^5={}^5C_o(x^2)^5+{}^5C_1(x^2)^4(2y)+{}^5C_2(x^2)^3(2y)^2+{}^5C_3(x^2)^2(2y)^3+{}^5C_4(x^2)(2y)^4+{}^5C_5(2y)^5$

$=x^{10}+5x^8(2y)+10x^6(4y^2)+10x^4(8y^3)+5x^2(16y^4)+32y^5$

$=x^{10}+10x^8y+40x^6y^2+80x^4y^3+80x^2y^4+32y^5$

5) We have, $\sum_{r=0}^{n}\ {}^nC_r\times 3^r={}^nC_o3^o+{}^nC_13+{}^nC_23^2+{}^nC_33^3+\ldots+{}^nC_n3^n$ **2**

[on putting r=0, 1, 2,....,n]

${}^nC_o+{}^nC_13+{}^nC_23^2+{}^nC_33^3+\ldots+{}^nC_n3^n$

$=(1+3)^n\quad\left[\because(1+x)^n={}^nC_o+{}^nC_1x+{}^nC_2x^2+\ldots+{}^nC_nx^n\right]$

$=4^n$

6) The general term in the expansion of $\left(2x+\frac{y}{3}\right)^{15}$ is **2**

$T_{r+1}={}^{15}C_r(2x)^{15-r}\left(\frac{y}{3}\right)^r\left[\because T_{r+1}={}^nC_ra^{n-r}b^r\right]$

$={}^{15}C_r2^{15-r}\times3^{-r}\times x^{15-r}y^r$

For determining 7th term, put r=6, we get

$T_{6+1}={}^{15}C_62^{15-6}\times3^{-6}\times x^{15-6}y^6$

$={}^{15}C_62^9\times3^{-6}\times x^9y^6$

Section-B

7) Given expression is $(x+a)^{100}+(x-a)^{100}$ **3**

Here, n = 100, which is even.

$\therefore$ Total number of terms $=\frac{n}{2}+1=\frac{100}{2}+1=51$

8) We have,$25^{15} = (26-1)^{15}$ 3

$$= {}^{15}C_0(26)^{15} - {}^{15}C_1(26)^{14}(1)^1 + {}^{15}C_2(26)^{13}(1)^2 - \ldots\ldots - {}^{15}C_{15}$$
$$= {}^{15}C_0(26)^{15} - {}^{15}C_1(26)^{14} + \ldots\ldots -1 - 12 + 12$$
$$= \left({}^{15}C_6 2 \times 13 \times (26)^{14} - {}^{15}C_1 2 \times 13 \times (26)^{13} + \ldots\ldots 13\right) + 12$$
$$= 13\left({}^{15}C_0 \times 2 \times (26)^{14} - {}^{15}C_1 \times 2 \times (26)^{13} + \ldots\ldots -1\right) + 12$$

9) We have,7^9+9^7 = $(1+8)^7$ - $(1-8)^9$ 3

$${}^7C_0 + {}^7C_1(8) + {}^7C_2(8)^2 + {}^7C_3(8)^3 + \ldots + {}^7C_7(8)^7] - \left[{}^9C_0 - {}^9C_1(8) + {}^9C_2(8)^2 - \ldots - {}^9C_9(8)^9\right]$$
$$= \left[1 + 7 \times (8)^1 + 21 \times (8)^2 + 35 \times (8)^3 \ldots + (8)^7\right] - \left[1 - 9 \times (8)^1 + 36(8)^2 - \ldots - (8)^9\right]$$
$$= (7+9) \times (8)^1 + (21-36) \times (8)^2 + \ldots$$
$$= 2 \times 64 - 15 \times 64 + \ldots = 64(2 - 15 + ..)$$

Hence, it is clear that it is divisible by 64.

10) We have, equation of hyperbola is $5y^2-9x^2=36$ 3

It can be written as

$$\frac{y^2}{\left(\frac{6}{\sqrt{5}}\right)^2} - \frac{x^2}{(2)^2} = 1$$

On comparing with $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we get

$$a^2 = \left(\frac{6}{\sqrt{5}}\right)^2 and \quad b^2 = 2^2 \Rightarrow a = \frac{6}{\sqrt{5}} \quad and \quad b = 2$$

(ii) Centre (0,0)

(iii) Transverse axis, $2a = 2 \times \frac{6}{\sqrt{5}} = \frac{12}{\sqrt{5}}$

(iv) Conjugate axis, $2b = 2 \times 2 = 4$

(v) Value of $c = \sqrt{a^2+b^2} = \sqrt{\frac{36}{5}+4} = \sqrt{\frac{56}{5}} = \frac{2\sqrt{14}}{\sqrt{5}}$

(vi) Vertices = $(0, \pm a) = \left(0, \frac{6}{\sqrt{5}}\right)$

(vii) Directrices, $y = \pm\frac{a^2}{c} = \pm\frac{36 \times \sqrt{5}}{5 \times 2\sqrt{14}} = \pm\frac{18}{\sqrt{70}}$

(viii) Foci $(0, \pm c) = (0, \pm\frac{2\sqrt{14}}{\sqrt{5}}$

(ix) Eccentricity,

$$e = \frac{c}{a} = \frac{2\sqrt{\frac{14}{5}}}{\frac{6}{\sqrt{5}}} = \frac{\sqrt{14}}{3}$$

(x) Length of latusrectum, $\frac{2b^2}{a} = \frac{2 \times 4}{\frac{6}{\sqrt{5}}} = \frac{4\sqrt{5}}{3}$

11) $\left(1 = \frac{x}{8}\right)^{4/3} = 1 + \left(\frac{4}{3}\right)\frac{x}{8} \frac{\frac{4}{3}\left(\frac{4}{3}-1\right)}{2!}\left(\frac{x}{8}\right)^2 + \ldots$ 3

$$= 1 + \frac{x}{6} + \frac{4}{3} \times \frac{1}{3} \times \frac{1}{2} \times \frac{x^2}{64} + \ldots$$
$$= 1 + \frac{x}{6} + \frac{x^2}{288} + \ldots$$

12) General term in $\left(x^2 + \frac{1}{x}\right)^n$= ${}^nC_r$ $(x^2)^{n-r}$ $\left(\frac{1}{x}\right)^r$ 3

for term independent of x, $x^{2n-2r-r} = x^0 \Rightarrow$ r = $\frac{2}{3}n$

Required term = ${}^nC_r$ = $\frac{n!}{r!(n-r)!}$ = $\frac{n!}{\left(\frac{n}{3}\right)!\left(\frac{2n}{3}\right)!}$

13) Let two successive terms be (r+1)th and (r+1)th terms. Then, **3**

$T_{r+1}={}^{24}C_r\, x^r$ and $T_{r+2}={}^{24}C_{r+1}\, x^{r+1}$

Now, according to the given condition, we have

$\therefore \ \frac{{}^{24}C_r}{{}^{24}C_{r+1}} = \frac{1}{4}$

Ans:5th and 6th terms

Section-C

14) $(2+3y)^{-5} = 2^{-5}(1+\frac{3}{2}y)^{-5}, \quad when \quad \left|\frac{3}{2}y\right| < 1$ **4**

$= \frac{1}{32}(1+\frac{15}{2}y+\frac{135}{4}y^2-\frac{945}{8}y^3+\ldots 0, \quad when \quad |y| < \frac{2}{3}$

15) $(c+dy)^{-2} = c^{-2}(1+\frac{d}{c}y)^{-2}$ **4**

$\therefore c^{-2}[1-2\times\frac{d}{c}y+\frac{(-2)(-2-1)}{2!}\left(\frac{d}{c}y\right)^2+\ldots] = \frac{1}{4}-3y+\ldots$

$\Rightarrow c^{-2}-\frac{2d}{c^3}y+\ldots = \frac{1}{4}-3y+\ldots$

$On \quad comparing \quad first \quad term, c^{-2} = \frac{1}{4} \Rightarrow \frac{1}{c^2} = \frac{1}{4} \Rightarrow c = 2$

$On \quad comparing \quad second \quad term, \quad d = 12$

$Ans. 2, 12$

16) Let x=1+h, where h is so small that is square and higher powers can be neglected. **4**

$\therefore \frac{ax^b-bx^a}{x^b-x^a} = \frac{a(1+h)^b-b(1+h)^a}{(1+h)^b-(1+h)^a}$

$= \frac{a[1+bh+\ldots]-b[1+ah+\ldots]}{[1+bh+\ldots]-[1+ah+\ldots]}$

$[neglecting \quad h^2 \quad and \quad higher \quad powers \quad of \quad h]$

$= \frac{a(1+bh)-b(1+ah)}{(1+bh)-(1+ah)} = \frac{a-b}{(b-a)h} = -\frac{1}{h} = \frac{1}{(1-x)}$

17) $(1.025)^{-1/3} = (1+0.025)^{-1/3}$ **4**

$= 1-\left(\frac{1}{3}\right)(0.025)+\frac{\left(\frac{1}{3}\right)\left(\frac{1}{3}+1\right)}{2}(0.025)^2$

= 1 - 0.0083 + 0.000139 - ...

= 0.991