QB365

Important Questions - Principle of Mathematical Induction

11th Standard CBSE

Mathematics Reg.No. :

Time : 01:00:00 Hrs

Total Marks : 50

Section-A

1) Prove that 2+4+6+8+.....2=n(n+1). **2**

2) Prove that $1+2+2^2+...+2^n = 2^{n+1}$ 1 for all natural numbers n. **2**

3) Prove that 2^{2n} -1 is divisible by 3, for all natural numbers n. **2**

4) Prove that $\sum_{t=1}^{n-1} t(t+1) = \frac{n(n-1)(n+1)}{3}$ for all natural numbers $n \geq 2$ **2**

5) Prove by the principle of mathematical induction that $3^n > 2^n$ for all $n \in N$ **2**

6) if P(n) : "$3.5^{2n+1} + 2^{3n+1}$ is divisible by λ for all n∈N" is true, then find the value of λ **2**

Section-B

7) Prove that $2n+1 < 2^n$ for all natural numbers $n \geq 3$ by using principle of mathematical induction. **3**

8) Prove by principle of mathematical induction that, the sum of first n natural numbers is $\frac{n(n+1)}{2}$ **3**

9) Prove that $(1+x)^n \geq (1+nx)$ for all natural number n, where x>-1. **3**

10) Prove by the principle of mathematical induction that **3**

$1+3+3^2+....+3^{n-1} = \frac{3^n-1}{2}$ for all $n \in N$

11) Prove the rule of exponents $(ab)^n=a^n b^n$ by using principle of mathematical induction fo every natural number. **3**

12) Prove by the principle of mathematical induction that **3**

$$1 \times 1! + 2 \times 2! + 3 \times 3! + ... + n \times n! = (n+1)! - 1$$

for all natural numbers n.

Section-C

13) Use the principle of mathematical induction to prove that $n^3 - 7n + 3$ is divisible by 3, for all natural numbers of n. **4**

14) Prove by the principle of mathematical induction that, for all $n \in N$ 4^n when divided by 3, the remainder is always 1. **4**

15) For all positive integer n, prove that $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2}{3}n^3 - \frac{n}{105}$ is an integer **4**

16) Prove that $(2n+7)<(n+3)^2$, for all natural numbers n. **4**

17) Prove that 2n<(n+2)! for all natural numbers n. **4**

****

Section-A

1) Consider P(k):2+4+6+8+.....+2k = k(k+1) **2**

Now, P(k+1):2+4+6+.....2(k)+2(k+1)

=k(k+1)+2(k+1)=k^2 + 3k+2

=(k+1)(k+1)

2) Consider P(k) : 1+2+2^2+.... +2^k = 2^{k+1}-1 **2**

Now P(+1):1+2+2^2 +..+2^k = 2^{k+1}

=2^{k+1}-1+2^{k+1}

=$2^{(k+1)+1}$ -1

3) Consider P(k):2^{2k} -1= 3λ(say) **2**

NowP(k+1):2^{2k+1} -1 = 2^{2k}.2^2 -1

$(3\lambda+1)4-1=12\lambda+3$

$3(4\lambda+1)$, which is divisible by 3

4) Consider P(k):$\sum_{t=1}^{k-1} t(t+1)=\frac{k(k-1)(k+1)}{3}\quad k\geq 2$ **2**

P(k):1.2+2.3+3.4+....+(k-1)k=$\frac{k(k-1)(k+1)}{3}$

Now P(k+1):1.2+2.3+3.4+...+(k1)k+k(k+1)

=$\frac{k(k-1)(k+1)}{3}+k(k+1)=\frac{k(k+1)(k+2)}{3}k\geq 2$

5) **2**

Step I Let P(n) be the given statement.

i.e.P(n):$3^n>2^n$

Step II For =1,we have $3^1>2^1$

3>2, Which is true.

Thus P(1) is true.

Step III Let us assume that P(k) is true.

i.e P(k):$3^k>2^k$

Step IV Now, we shall prove the statement for n=k+1.For this,we have to show $3^{k+1}>2^{k+1}$

from Eq.(i) we have $3^k>2^k$

$3^k.3>2^k.3$ [multiplying both sides by 3]

$\Rightarrow\ 3^{k+1}>2^k.3$

$2^k.3>2^k.3\Rightarrow\ 3^k.3>2^k.2$ = 2^{k+1}

Thus ,P(K+1) is true whenever P(k) is true. Hence, by principle of mathematical induction,P(n) is true for all $n\in N$

6) Here, the given statement is true for all nϵN **2**

it is true for n=1 and n=2

For n=1, P(1) : 3.5+2^4=3x125x16

=375+16=391

and for n=2, P(2) ; 3.5^5+2^7

=3x3125+128

=9375+128=9503

Now, the HCF of 391 and 9503 is 17. So,

3.5^{2n+1}+2^{3n+1} is divisible by 17. Hence, λ is 17.

Section-B

7) **Step I** Let P(n) be the given statement. **3**

i.e P(n);$2n+1<2^n$

Step II For n = 3,we have

(2 x 3 +1)<$2^3 \Rightarrow$ 7< 8, which is true.

Thus P(1) is true.

Step III Let us assume that P(k) is true.

i.e P(k):$2k+1<2^k$

Step IV Now, we shall prove the statement for n=k+1.

for this, we have to show that $2(k+1)+1<2^{k+1}$

from Eq.(i), $2k+1<2^k$

So, (2k+1)+2<2^k+2 [adding 2 on both sides]

2k+3<2^K.2 [2^k + 2 < 2^k.2]

2k+3 < 2^{k+1} 2(k+1)+1<2^{k+1}

thus, P(k+1) is true, whenever P(k) is true.

hence, by principle of mathematical induction P(n) is true for all natural numbers, $n \geq 3$

8) **3**

Step I Let P(n) be the given statement, i.e.

$$P(n): \quad 1+2+3+\ldots\ldots+n=\frac{n(n+1)}{2}$$

Step II For n=1, we have; LHS=1 and

RHS=$\frac{1.(1+1)}{2}=\frac{1\times 2}{2}=1$

$\therefore$ LHS=RHS P(1) is true

Step III Let us assume that P(n) is true for n=k. Then we have

P(k): $1+2+3+\ldots\ldots+K=\frac{k(k+1)}{2} \qquad \ldots.(1)$

Step IV Now, we shall prove the statement for n=k+1

For this we have to show that

$1+2+\ldots+(k+1)=\frac{(k+1)(k+1+1)}{2}$

Consider, LHS=1+2+.....+k+(k+1)

$=\frac{k(k+1)}{2}+(k+1)$ [using Eq.(i)]

$=(k+1)\left(\frac{k}{2}+1\right)$ [taking common (k+1)]

$=\frac{(k+1)(k+2)}{2}=\frac{(k+1)(k+1+1)}{2}=RHS$

Thus, P(k+1) is true, whenever P(k) is true. Hence, by principle of mathematical induction, P(n) is true for all natural numbers n.

9) **3**

Step I Let P(n) be the given statement.

i.e.P(n) :$(1+x)^n \geq (1+nx)$

Step II for n=1, we have $(1+x) \geq (1+x)$, which is true.Thus P(1) is true.

Step III Let us assume that P(k) is true.

i.e P(k) : $(1+x)^k \geq (1+kx)$ ....(i)

Step IV Now, we shall prove the statement for n=k+1.For this, we have to show that

$(1+x)^{k+1} \geq (1+(k+1)x)$

from Wq (i) we have $(1+x)^k \geq (1+kx)$ ....(ii)

$\because$ x>1 $\Rightarrow$ x+1>0

So, on multiplying both sides of Eq.(ii) by (x+1), we get

$(1+x)^k(1+x) \geq (1+kx)(1+x)$

$(1+x)^{k+1} \geq 1+x+kx+kx^2$ ....(iii)

Here, k is a natural number and $x^2 \geq 0$, therefore $kx^2 \geq 0$ and so, $(1+x+kx+kx^2) \geq (1+x+kx)$

Then, from Eq(iii), we have

$(1+x)^{k+1} \geq (1+x+kx)$

or $(1+x)^{k+1} \geq [1+(1+k)x]$

Thus,P(k+1) is true whenever P(k) is true.Hence, by the principle mathematical induction, P(n) is true for all natural numbers.

10) **3**

Step I Let P(n) be the given statement.

i.e. P(n): $1+3+3^2+.....+3^{n-1} = \frac{3^n-1}{2}$

Step II For n=1, we have

LHS=3^{1-1}=3^0=1 and RHS= $\frac{3^1-1}{2} = 1$

$\therefore$ LHS=RHS

$\therefore$ P(1) is true.

Step III Let us assume that P(n) is true for n=k

Then, we have

P(k): $1+3+3^2+.....+3^{k-1} = \frac{3^k-1}{2}....(i)$

Step IV Now, we shall prove the statement for n=k+1 For this, we have to show

$1+3+3^2+.....+3^{k-1}+3^k = \frac{3^{k+1}-1}{2}$

Then, LHS=1+3+3^2+...3^{k-1}+3^k

$= \frac{3^k-1}{2} + 3^k$ [from Eq. (i)]

$= \frac{3^k-1+2.3^k}{2} = \frac{3^k(1+2)-1}{2}$

$= \frac{3^k.3-1}{2} = \frac{3^{k+1}-1}{2} = RHS$

Thus, P(k+1) is true, whenever P(k) is true. Hence, by principle of mathematical induction, P(n) is true for all $n \varepsilon N$

11) 3

Step I Let P(n) be the given statement,

i.e. P(n) : $(ab)^n=a^n\ b^n$

Step II For n=1, $(ab)^1=ab=a^1\ b^1$

So, P(1) is true.

Step III Let P(k) be true.

Then, we have $(ab)^k=a^k\ b^k$ ......(i)

Step IV Now, we shall prove the statement for n=k+1.

For this, we have to show that

$(ab)^{k+1}=a^{k+1}.b^{k+1}$

Then, LHS=$(ab)^{k+1}=(ab)^k\ (ab)$

$=(a^k\ b^k)\ (ab)$ [using Eq.(i)]

$=(a^k.a^1)\ (b^k.b^1)=a^{k+1}.b^{k+1}$

Thus, P(K+1) is also true, whenever P(k) is true. Hence, by principle of mathematical induction, P(n) is true for all $n \varepsilon N$

12) 3

Step I Let P(n) be the given statement

i.e. P(n): $1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + n \times n! = (n+1)! - 1$

Step II For n=1, we have

LHS=$1 \times 1!$=1

and RHS=(1+1)!-1=2!-1=2-1=LHS

∵ LHS=RHS

∴ P(1) is true

Step III Let us assume that P(n) is true for n=k

Then, we have

P(k): $1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + k \times k! + (k+1)! - 1 \qquad \ldots (i)$

Step IV Now, we shall prove the statement for n=k+1. For this we have to show that

$1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + k \times k! + (k+1) \times (k+1)! = (k+1+1)! - 1$

Then, LHS $= 1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + k \times k! + (k+1) \times (k+1)!$

=(k+1)!-1+(k+1)!$\times$(k+1) [from Eq.(1)]

=(k+1+1)(k+1)!-1=(k+2)(k+1)!-1

=(k+2)!-1 [∵n(n-1)!=n]

Thus, P(k+1) is true, whenever P(k) is true. Hence, by the principle of mathematical induction, P(n) is true for all natural numbers n.

Section-C

13) Consider P(k):$k^3 - 7k + 3 = 3\lambda$ 4

Now,P(k+1):$(k+1)^3 - 7(k+1) + 3$

= $k^3+3k^2+3k+1-7k-4$

= $(3\lambda - 3) + 3k^2 + 3k - 3 = 3(k^2 + k - 2)$

14) Consider P(k) : $4^k = 3\lambda + 1$ 4

Now,P(k+1):$4^{k+1} = 4^k.4 = (3\lambda + 1)4$

$= 12\lambda + 4 = 3(4\lambda + 1) + 1$

15) Consider P(k): $\frac{k^7}{7}+\frac{k^5}{5}+\frac{2}{3}k^3-\frac{k}{105}\lambda \in I$ **4**

Now,P(k+1): $\frac{(k+1)^7}{7}+\frac{(k+1)^5}{5}+\frac{2}{3}(k+1)^3-\frac{k+1}{105}$

$=\frac{1}{7}(k^7+7k^6+21k^5+35k^4+35k^3+21k^2+7k+1)+\frac{1}{5}(k^5+5k^4+10k^3+10k^2+5k)+1$

$+\frac{2}{3}(k^3+3k^2+3k+1)-\frac{k+1}{105}$

$=+\lambda+(k^6+3k^5+6k^4+7k^3+7k^2+4k)$= Integer

16) Consider P(k): $(2k+7)<(k+3)^2$ **4**

$\Rightarrow (2k+7)+2 < (k+3)^2 +2$

$\Rightarrow 2(k+1)+7 < (k+4)^2$ $[(k+3)^3+2<(k+4)^2]$

17) 2k<(k+2)!⇒2k+2(k+2)!+2] **4**

$\Rightarrow (k+1)^2 < 2^k + 2^k$ $[(2k+1)<2^k \text{ for } k\leq 3]$

$\Rightarrow (k+1)^2 < 2^{k+1}$