RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.1

The Plane 29.1 Q1(i)

Given three points are,

$(2,1,0),(3,-2,-2)$ and $(3,1,7)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = (x-2)(-21-0)-(y-1)(7+2)+z(0+3)=0$$

$$= -21x+42-9y+9+3z=0$$

$$= -21x-9y+3z+51=0$$

Dividing by -3, we get

Equation of plane, $7x+3y-z-17=0$

The Plane 29.1 Q1(ii)

Given points are,
$(-5,0,-6),(-3,10,-9)$ and $(-2,6,-6)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x+5 & y-0 & z+6 \\ -3+5 & 10-0 & -9+6 \\ -2+5 & 6-0 & -6+6 \end{vmatrix} = 0$$

$$\begin{vmatrix} x+5 & y & z+6 \\ 2 & 10 & -3 \\ 3 & 6 & 0 \end{vmatrix} = 0$$

$$(x+5)(0+18)-y(0+9)+(z+6)(12-30)=0$$
$$(x+5)(18)-y(9)+(z+6)(-18)=0$$
$$18x+90-9y-18z-108=0$$

Dividing by 9, we get

Equation of plane, $2x-y-2z-2=0$

The Plane 29.1 Q1(iii)

Given three points are,
$(1,1,1),(1,-1,2)$ and $(-2,-2,2)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 1-1 & -1-1 & 2-1 \\ -2-1 & -2-1 & 2-1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y-1 & z-1 \\ 0 & -2 & 1 \\ -3 & -3 & 1 \end{vmatrix} = 0$$

$$(x-1)(-2+3)-(y-1)(0+3)+(z-1)(0-6)=0$$
$$(x-1)(1)-(y-1)(3)+(z-1)(-6)=0$$
$$x-1-3y+3-6z+6=0$$

$$x-3y-6z+8=0$$

Equation of plane is, $x-3y-6z+8=0$

The Plane 29.1 Q1(iv)

Given points are,
$(2,3,4),(-3,5,1)$ and $(4,-1,2)$

We know that, equation of plane passing through three points are given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -3-2 & 5-3 & 1-4 \\ 4-2 & -1-3 & 2-4 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-3 & z-4 \\ -5 & 2 & -3 \\ 2 & -4 & -2 \end{vmatrix} = 0$$

$$(x-2)(-4-12)-(y-3)(10+6)+(z-4)(20-4)=0$$
$$(x-2)(-16)-(y-3)(16)+(z-4)(16)=0$$
$$-16x+32-16y+48+16z-64=0$$

$$-16x-16y+16z+16=0$$

Dividing by (-16), we get,

Equation of plane, $x+y-z-1=0$

The Plane 29.1 Q1(v)

Given points are,
$(0,-1,0),(3,3,0)$ and $(1,1,1)$

We know that, equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 3-0 & 3+1 & 0-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z \\ 3 & 4 & 0 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$x(4-0)-(y+1)(3-0)+z(6-4)=0$$
$$4x-(y+1)(3)+z(2)=0$$
$$4x-3y-3+2z=0$$

$$4x-3y+2z-3=0$$

Equation of plane is, $4x-3y+2z-3=0$

The Plane 29.1 Q2

We have to prove that points
$(0,-1,-1),(4,5,1),(3,9,4)$ and $(-4,4,4)$ are coplanar.

First we shall find the equation of plane passing through three points:
$(0,-1,1),(4,5,1)$ and $(3,9,4)$

We know that equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y+1 & z+1 \\ 4-0 & 5+1 & 1+1 \\ 3-0 & 9+1 & 4+1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z+1 \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix} = 0$$

$$x(30-20)-(y+1)(20-6)+(z+1)(40-18)=0$$
$$10x-(y+1)(14)+(z+1)(22)=0$$
$$10x-14y-14+22z+22=0$$

$$10x-14y+22z+8=0$$

Dividing by 2, we get

$$5x-7y+11z+4=0 \qquad ---(i)$$

Now, for the fourth point $(-4,4,4)$ put $x=-4$, $y=4$, $z=4$ in equation (i),

$$5(-4)-7(4)+11(4)+4=0$$
$$-20-28+44+4=0$$
$$-48+48=0$$

$$0=0$$
$$LHS = RHS$$

Since, fourth point satisfies the equation of plane passing
through three points
So, all four points are collinear

Equation of common plane is, $5x-7y+11z+4=0$

The Plane 29.1 Q3(i)

Given, four points are
$(0,-1,0),(2,1,-1),(1,1,1)$ and $(3,3,0)$.

Now, first we find the equation of plane passing through three points:
$(0,-1,0),(2,1,-1),(1,1,1)$

We know that equation of plane passing through three points is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y+1 & z-0 \\ 2-0 & 1+1 & -1-0 \\ 1-0 & 1+1 & 1-0 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y+1 & z \\ 2 & 2 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$x(2+2)-(y+1)(2+1)+z(4-2)=0$$
$$x(4)-(y+1)(3)+z(2)=0$$
$$4x-3y-3+2z=0$$

$$4x-3y+2z-3=0 \qquad ---(i)$$

Put, $x=3$, $y=3$, $z=0$ in equation (i), we get

$$4x-3y+2z-3=0$$
$$4(3)-3(3)+2(0)-3=0$$
$$12-9+0-3=0$$
$$12-12=0$$

$$0=0$$
$$LHS = RHS$$

Since, fourth point satisfies the equation of plane passing through three points,
Hence, four points are coplanar

The Plane 29.1 Q3(ii)

Given, four points are
$(0,4,3),(-1,-5,-3),(-2,-2,1)$ and $(1,1,-1)$

First we shall find the equation of plane passing through three points:
$(0,4,3),(-1,-5,-3),(-2,-2,1)$

We know that, equation of plane passing through three given points is,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-0 & y-4 & z-3 \\ -1-0 & -5-4 & -3-3 \\ -2-0 & -2-4 & 1-3 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y-4 & z-3 \\ -1 & -9 & -6 \\ -2 & -6 & -2 \end{vmatrix} = 0$$

$$x(18-36)-(y-4)(2-12)+(z-3)(6-18)=0$$
$$x(-18)-(y-4)(-10)+(z-3)(-12)=0$$
$$-18x+10y-40-12z+36=0$$

$$-18x+10y-12z-4=0 \qquad ---(i)$$

Put, $x=1$, $y=1$, $z=-1$ in equation (i),

$$-18(1)+10(1)-12(-1)-4=0$$
$$-18+10+12-4=0$$
$$-22+22=0$$

$$0=0$$
$$LHS = RHS$$

So, fourth point $(1,1,-1)$ satisfies the equation of plane passing through three points,
Hence, four points are coplanar