RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.8

Straight lines Ex 23.8 Q1

The equation of line through $(1,2)$ and making an angle of 60^0 with the x-axis is

$$\frac{x-1}{\cos 60^0} = \frac{y-2}{\sin 60^0} = r$$

$$\frac{x-1}{\frac{1}{2}} = \frac{y-2}{\frac{\sqrt{3}}{2}} = r$$

Where r is the distance of any point on the line from $A(1,2)$.

The coordinates of P on the line are

$$\left(1+\frac{1}{2}r, 2+\frac{\sqrt{3}}{2}r\right)$$

and

P lies on $x+y=6$

$$\therefore 1+\frac{r}{2}+2+\frac{\sqrt{3}r}{2} = 6$$

$$\text{or } r = \frac{6}{1+\sqrt{3}} = 3\left(\sqrt{3}-1\right)$$

Hence length $AP = 3\left(\sqrt{3}-1\right)$

Straight lines Ex 23.8 Q2

The equation of line is

$$\frac{x-3}{\cos\frac{\pi}{6}} = \frac{y-4}{\sin\frac{\pi}{6}} = \pm r$$

$$\text{or } x = \pm\frac{\sqrt{3}}{2}r + 3 \text{ and } y = \pm\frac{1}{2}r + 4$$

$$Q\left(\pm\frac{\sqrt{3}r}{2}+3,\ \pm\frac{r}{2}+4\right) \text{ lie in } 12x+5y+10=0$$

$$\therefore 12\left(\pm\frac{\sqrt{3}r}{2}+3\right)+5\left(\pm\frac{r}{2}+4\right)+10=0$$

$$\pm\frac{12\sqrt{3}r}{2}+36\pm\frac{5r}{2}+20+10=0$$

$$r=\frac{\pm 132}{5+12\sqrt{3}}$$

Hence, length PQ is $\dfrac{132}{12\sqrt{3}+5}$

Straight lines Ex 23.8 Q3

The equation of line is

$$\frac{x-2}{\cos\alpha}=\frac{y-1}{\sin\alpha}=r$$

$$\Rightarrow\frac{x-2}{\frac{1}{\sqrt{2}}}=\frac{y-1}{\frac{1}{\sqrt{2}}}=r$$

$$\text{or } x=\frac{1}{\sqrt{2}}r+2,\ y=\frac{1}{\sqrt{2}}r+1$$

$$B\left(\frac{r}{\sqrt{2}}+2,\ \frac{r}{\sqrt{2}}+1\right) \text{ lie on } x+2y+1=0$$

$$\therefore \frac{r}{\sqrt{2}} + 2 + \frac{2r}{\sqrt{2}} + 2 + 1 = 0$$

$$\frac{3r}{\sqrt{2}} = \pm 5$$

$$r = \frac{5\sqrt{2}}{3}$$

The length AB is $\frac{5\sqrt{2}}{3}$ units

Straight lines Ex 23.8 Q4

The required line is parallel to $3x-4y+1=0$

$$\therefore \text{Slope of the line} = \text{slope of } 3x - 4y + 1 = \frac{-3}{-4}$$

$$\tan\alpha = \frac{3}{4}$$

$$\Rightarrow \sin\alpha = \frac{3}{5} \text{ and } \cos\alpha \frac{4}{5}$$

The equation of line is

$$\frac{x+4}{\cos\alpha} + \frac{y+1}{\sin\alpha} = r$$

$$\Rightarrow \frac{x-4}{\frac{4}{5}} + \frac{y+1}{\frac{3}{5}} = \pm 5$$

$$\Rightarrow x = 8 \text{ and } y = 2$$

or

$$x = 0 \text{ and } y = -4$$

$\therefore$ $(8, 2)$ and $(0, -4)$ are coordinates of two points on the line which are at a distance of 5 units from $(4, 1)$

Straight lines Ex 23.8 Q5

The equation of line is

$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = \pm r$$

or

$$x = x_1 \pm r\cos\theta \text{ and } y = y_1 \pm r\sin\theta$$

$$Q\left(x_1 \pm r\cos\theta,\ y_1 \pm r\sin\theta\right) \text{ lie in } ax + by + c = 0$$

$$\Rightarrow a\left(x_1 + r\cos\theta\right) + b\left(y_1 \pm r\sin\theta\right) + c = 0$$

$$\Rightarrow \pm r\left(a\cos\theta + b\sin\theta\right) = -c - ax_1 - by_1$$

$$\Rightarrow -r = \left|\frac{ax_1 + by_1 + c}{a\cos\theta + b\sin\theta}\right|$$

Straight lines Ex 23.8 Q6

Equation of line is

$$\frac{x-2}{\cos 45^0} = \frac{y-3}{\sin 45^0} = r$$

$$x = \frac{r}{\sqrt{2}} + 2 \quad \text{and} \quad y = \frac{r}{\sqrt{2}} + 3$$

$$P\left(\frac{r}{\sqrt{2}} + 2, \frac{r}{\sqrt{2}} + 3\right) \text{ lie on } 2x - 3y + 9 = 0$$

$$\therefore 2\left(\frac{r + 2\sqrt{2}}{\sqrt{2}}\right) - 3\left(\frac{r + 3\sqrt{2}}{\sqrt{2}}\right) + 9 = 0$$

$$\Rightarrow 2r + 4\sqrt{2} - 3r - 9\sqrt{2} + 9\sqrt{2} = 0$$

$$\Rightarrow r = 4\sqrt{2}$$

$\therefore$ The point $(2, 3)$ is at a distance of $4\sqrt{2}$ from $2x - 3y + 9 = 0$

Straight lines Ex 23.8 Q7

Equation of the required line is

$$\frac{x-3}{\cos\alpha} = \frac{y-5}{\sin\alpha} = r \quad ---(1)$$

$$\tan\alpha = \frac{1}{2} \quad \Rightarrow \cos\alpha = \frac{2}{\sqrt{5}} \text{ and } \sin\alpha = \frac{1}{\sqrt{5}}$$

$\therefore$ equation is

$$\frac{x-3}{\frac{2}{\sqrt{5}}} = \frac{y-5}{\frac{1}{5}} = r$$

$$\text{or } x = \frac{2}{\sqrt{5}}r + 3, y = \frac{1}{\sqrt{5}}r + 5$$

$$P\left(\frac{2r}{\sqrt{5}} + 3, \frac{r}{\sqrt{5}} + 5\right) \text{ lie on } 2x + 3y = 14$$

$$\therefore \frac{4r}{\sqrt{5}} + 6 + \frac{3r}{\sqrt{5}} + 15 = 1 \pm 14$$

$$\frac{7r}{\sqrt{5}} = \pm 17$$

$r = \pm\sqrt{5}$

$r = \sqrt{5} \quad (r \neq -\sqrt{5})$

$\therefore$ Distance of $(3,5)$ from $2x + 3y = 14$ is $\sqrt{5}$ units

Straight lines Ex 23.8 Q8

Slope of the line $= \tan\alpha = \frac{3}{4}$

$\therefore \sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$

$\therefore$ Equation of line is

$$\frac{x-2}{\cos\alpha} = \frac{y-5}{\sin\alpha} = r$$

$$\Rightarrow \frac{x-2}{\frac{4}{5}} = \frac{y-5}{\frac{3}{5}} = r$$

or $x = \frac{4r}{5} + 2$ and $y = \frac{3r}{5} + 5$

then $P\left(\frac{4r}{5} + 2, \frac{3r}{5}5\right)$ lie on $3x + y + 4 = 0$

$$\therefore 3\left(\frac{4r}{5} + 2\right) + \left(\frac{3r}{5} + 5\right) + 4 = 0$$

$$\frac{15}{5}r = \pm 15$$

$$r = \pm\frac{15 \times 5}{15}$$

$= 5$ units