RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.7

Limits Ex 29.7 Q1

$$\lim_{x\to 0}\frac{\sin 3x}{5x}$$

$$=\frac{1}{5}\lim_{x\to 0}\frac{\sin 3x}{x}$$

$$=\frac{1}{5}\lim_{x\to 0}\frac{\sin 3x}{3x}\times 3$$

$$=\frac{3}{5}\lim_{3x\to 0}\frac{\sin 3x}{3x}$$

$$=\frac{3}{5}\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{3}{5}$$

Limits Ex 29.7 Q2

$$\lim_{x\to 0}\frac{\sin x^\circ}{x}$$

$$=\lim_{x\to 0}\frac{\sin\frac{x\times\pi}{180}}{x} \qquad \left[\because 1^\circ=\frac{\pi}{180}\text{ radians}\right]$$

$$=\lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{x\times\frac{\pi}{180}}\times\frac{\pi}{180}$$

$$=\frac{\pi}{180}\lim_{x\to 0}\frac{\frac{\sin\pi x}{180}}{\frac{\pi x}{180}}$$

$$=\frac{\pi}{180}\lim_{x\to 0}\frac{\frac{\sin\pi x}{180}}{\frac{\pi x}{180}}$$

$$=\frac{\pi}{180}\times 1=\frac{\pi}{180} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{\pi}{180}$$

Limits Ex 29.7 Q3

$$\lim_{x\to 0}\frac{x^2}{\sin x^2}$$

$$= \lim_{x\to 0}\frac{1}{\frac{\sin x^2}{x^2}}$$

$$= \frac{1}{\lim_{x\to 0}\frac{\sin x^2}{x^2}}$$

$$= \frac{1}{1} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= 1$$

Limits Ex 29.7 Q4

$$\lim_{x\to 0}\frac{\sin x\cos x}{3x}$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{\sin x\cos x}{x}$$

$$= \frac{1}{3}\lim_{x\to 0}\frac{\sin x}{x}\times\lim_{x\to 0}\cos x$$

$$= \frac{1}{3}\times 1\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1,\ \lim_{x\to 0}\cos x = \cos 0^\circ = 1\right]$$

$$= \frac{1}{3}$$

Limits Ex 29.7 Q5

$$\lim_{x\to 0}\frac{3\sin x-4\sin^3 x}{x}$$

$$=\lim_{x\to 0}\frac{\sin 3x}{x} \qquad \left[\because \sin 3x=3\sin x-4\sin^3 x\right]$$

$$=\lim_{x\to 0}\frac{\sin 3x}{3x}\times 3$$

$$=3\times\lim_{x\to 0}\frac{\sin 3x}{3x}$$

$$=3\times\lim_{3x\to 0}\frac{\sin 3x}{3x} \qquad \left[\because x\to 0,\ 3x\to 0\right]$$

$$=3\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=3$$

Limits Ex 29.7 Q6

$$\lim_{x\to 0}\frac{\tan 8x}{\sin 2x}$$

$$=\frac{\lim\limits_{x\to 0}\tan 8x}{\lim\limits_{x\to 0}\sin 2x}$$

$$=\frac{\lim\limits_{x\to 0}\frac{\tan 8x}{8x}\times 8x}{\lim\limits_{x\to 0}\frac{\sin 2x}{2x}\times 2x}$$

$$=\frac{\lim\limits_{8x\to 0}\frac{\tan 8x}{8x}\times\frac{8x}{2x}}{\lim\limits_{2x\to 0}\frac{\sin 2x}{2x}} \qquad \left[\begin{array}{l}\because x\to 0\\ 8x\to 0\\ 2x\to 0\end{array}\right]$$

$$= \frac{1 \times \frac{8}{2}}{1} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$$

$$= \frac{8}{2}$$

$$= 4$$

Limits Ex 29.7 Q7

$$\lim_{x \to 0} \frac{\tan mx}{\tan nx}$$

$$= \frac{\lim\limits_{x \to 0} \tan mx}{\lim\limits_{x \to 0} \tan nx}$$

$$= \frac{\lim\limits_{mx \to 0} \frac{\tan mx}{mx} \times mx}{\lim\limits_{nx \to 0} \frac{\tan nx}{nx} \times nx} \qquad [\because \text{If } x \to 0 \text{ then } mx \to 0 \text{ also } nx \to 0]$$

$$= \frac{1 \times m}{1 \times n} \qquad \left[\because \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$$

$$= \frac{m}{n}$$

Limits Ex 29.7 Q8

$$\lim_{x\to 0}\frac{\sin 5x}{\tan 3x}$$

$$=\frac{\lim\limits_{x\to 0}\sin 5x}{\lim\limits_{3x\to 0}\tan 3x}$$

$$=\frac{\lim\limits_{5x\to 0}\frac{\sin 5x}{5x}\times 5}{\lim\limits_{3x\to 0}\frac{\tan 3x}{3x}\times 3} \qquad \left[\because \text{If } x\to 0 \text{ then } 3x\to 0, 5x\to 0\right]$$

$$=\frac{5}{3}\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ also } \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{5}{3}$$

Limits Ex 29.7 Q9

$$\lim_{x\to 0}\frac{\sin x^\circ}{x^\circ}$$

$$=\lim_{x\to 0}\frac{\sin\frac{x\times\pi}{180}}{x\times\frac{\pi}{180}} \qquad \left[\because 1^\circ=\frac{\pi}{180} \text{ radians}\right]$$

$$=\lim_{\frac{\pi x}{180}\to 0}\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}} \qquad \left[\because \text{If } x\to 0 \text{ then } \frac{\pi x}{180}\to 0\right]$$

$$=1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

Limits Ex 29.7 Q10

$$\lim_{x\to 0}\frac{7x\cos x-3\sin x}{4x+\tan x}$$

$$=\lim_{x\to 0}\frac{7\cos x-\frac{3\sin x}{x}}{4+\frac{\tan x}{x}}$$

$$=\frac{\lim\limits_{x\to 0}7\cos x-\lim\limits_{x\to 0}\frac{3\sin x}{x}}{\lim\limits_{x\to 0}4+\lim\limits_{x\to 0}\frac{\tan x}{x}}$$

$$=\frac{7\times\lim\limits_{x\to 0}\cos x-3\lim\limits_{x\to 0}\frac{\sin x}{x}}{4+\lim\limits_{x\to 0}\frac{\tan x}{x}}$$

$$=\frac{7\times 1-3\times 1}{4+1}\qquad\left[\because\lim_{x\to 0}\frac{\sin x}{x}=1,\ \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{4}{5}$$

Limits Ex 29.7 Q11

$$\lim_{x\to 0}\frac{\cos ax-\cos bx}{\cos cx-\cos dx}=\lim_{x\to 0}\frac{\left(-2\sin\left(\frac{a+b}{2}\right)x\sin\left(\frac{a-b}{2}\right)x\right)}{-2\sin\left(\frac{c+d}{2}\right)x\sin\left(\frac{c-d}{2}\right)x}$$

$$=\frac{\lim\limits_{x\to 0}\sin\left(\frac{a+b}{2}\right)x\lim\limits_{x\to 0}\sin\left(\frac{a+b}{2}\right)x}{\lim\limits_{x\to 0}\sin\left(\frac{c+d}{2}\right)x\lim\limits_{x\to 0}\sin\left(\frac{c-d}{2}\right)x}$$

$$=\frac{\left(\lim\limits_{x\to 0}\frac{\sin\left(\frac{a+b}{2}\right)x}{\left(\frac{a+b}{2}\right)x}\times\left(\frac{a+b}{2}\right)x\right)\left(\lim\limits_{x\to 0}\frac{\sin\left(\frac{a-b}{2}\right)x}{\left(\frac{a-b}{2}\right)x}\times\left(\frac{a-b}{2}\right)x\right)}{\left(\lim\limits_{x\to 0}\frac{\sin\left(\frac{c+d}{2}\right)x}{\left(\frac{c+d}{2}\right)x}\times\left(\frac{c+d}{2}\right)x\right)\left(\lim\limits_{x\to 0}\frac{\sin\left(\frac{c-d}{2}\right)x}{\left(\frac{c-d}{2}\right)x}\times\left(\frac{c-d}{2}\right)x\right)}$$

$$=\frac{(a+b)(a-b)}{(c+d)(c-d)}\qquad\left[\because\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right]$$

$$=\frac{a^2-b^2}{c^2-d^2}$$