21. Linear Differential Equations

Exercise 21

1. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx}+\frac{1}{x}.y=x^2$$

Answer

Given Differential Equation :

$\frac{dy}{dx}+\frac{1}{x}.y=x^2$ ....eq(1)

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1}+c$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and Q = x^2

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x} dx}$$

$$= e^{\log x} \text{.........} \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= x \text{.........} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$$\therefore xy = \int x^3 dx + c$$

$$\therefore xy = \frac{x^4}{4} + c \text{.........} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

$$\therefore y = \frac{x^3}{4} + \frac{c}{x}$$

2. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} + 2y = x^2$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} + 2y = x^2$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} + 2y = x^2$$

Dividing the above equation by x,

$\frac{dy}{dx} + \frac{2}{x}.y = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2}{x}$ and Q = x

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{2}{x} dx}$$

$= e^{2\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log x^2}$ ....$(\because a \log b = \log b^a)$

$= x^2$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x^2) = \int x.(x^2)dx + c$$

$$\therefore x^2 y = \int x^3 dx + c$$

$$\therefore x^2 y = \frac{x^4}{4} + c \text{ ....} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

$$\therefore y = \frac{x^2}{4} + \frac{c}{x^2}$$

3. Question

Find the general solution for each of the following differential equations.

$$2x\frac{dy}{dx} + y = 6x^3$$

Answer

Given Differential Equation :

$$2x\frac{dy}{dx} + y = 6x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$2x\frac{dy}{dx} + y = 6x^3$$

Dividing the above equation by 2x,

$\frac{dy}{dx} + \frac{1}{2x}.y = 3x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{2x}$ and Q = $3x^2$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{2x} dx}$$

$= e^{\frac{1}{2}\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log \sqrt{x}}$ ....$(\because a \log b = \log b^a)$

= $\sqrt{x}$.........$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\sqrt{x}\right) = \int 3x^2.\left(\sqrt{x}\right)dx + c$$

$$\therefore \sqrt{x}.y = \int 3x^{5/2}dx + c$$

$$\therefore \sqrt{x}.y = 3\frac{x^{7/2}}{7/2} + c \ldots\ldots\ldots \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + c\right)$$

Dividing the above equation by √x

$$\therefore y = \frac{6}{7}x^3 + \frac{c}{\sqrt{x}}$$

$$\therefore y = \frac{6}{7}x^3 + \frac{c}{\sqrt{x}}$$

4. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} + y = 3x^2 - 2, x > 0$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} + y = 3x^2 - 2$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx}+y=3x^2-2$$

Dividing the above equation by x,

$\frac{dy}{dx}+\frac{1}{x}.y=\frac{3x^2-2}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{1}{x}$ and $Q=\frac{3x^2-2}{x}$

Therefore, the integrating factor is

$$I.F.=\ e^{\int P\,dx}$$

$$=e^{\int\frac{1}{x}dx}$$

$=e^{\log x}$ ....$\left(\because \int\frac{1}{x}dx=\log x\right)$

= x.........$\left(\because a^{\log_a b}=b\right)$

General solution is

$$y.(I.F.)=\int Q.(I.F.)dx\ +c$$

$$\therefore y.(x)=\int\left(\frac{3x^2-2}{x}\right).(x)dx\ +c$$

$$\therefore xy=\int(3x^2-2)dx\ +c$$

$\therefore xy=3\frac{x^3}{3}-2x+c$ ....$\left(\because \int x^n dx=\frac{x^{n+1}}{n+1}+c\right)$

Dividing the above equation by x

$$\therefore y=x^2-2+\frac{c}{x}$$

$$\therefore y=x^2-2+\frac{c}{x}$$

5. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = 2x^3$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} - y = 2x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

The general solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = 2x^3$$

Dividing the above equation by x,

$\frac{dy}{dx} - \frac{1}{x}.y = 2x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{-1}{x}$ and Q = $2x^2$

Therefore, integrating factor is

$$I.F.=e^{\int P\,dx}$$

$$=e^{\int \frac{-1}{x}dx}$$

$$=e^{-\log x}\;\ldots\ldots\ldots\left(\because \int \frac{1}{x}dx=\log x\right)$$

$$=e^{\log\frac{1}{x}}\;\ldots\ldots\ldots(\because a\log b=\log b^a)$$

$$=\frac{1}{x}\;\ldots\ldots\ldots\left(\because a^{\log_a b}=b\right)$$

General solution is

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

$$\therefore y.\left(\frac{1}{x}\right)=\int 2x^2.\left(\frac{1}{x}\right)dx+c$$

$$\therefore \frac{y}{x}=\int 2xdx+c$$

$$\therefore \frac{y}{x}=2\frac{x^2}{2}+c\;\ldots\ldots\ldots\left(\because \int x^n dx=\frac{x^{n+1}}{n+1}+c\right)$$

Multiplying above equation by x

$$\therefore y=x^3+cx$$

$$\therefore y=x^3+cx$$

6. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx}-y=x+1$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} - y = x + 1$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = x + 1$$

Dividing above equation by x,

$\frac{dy}{dx} - \frac{1}{x}.y = \frac{x+1}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{x+1}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$= e^{\int \frac{-1}{x} dx}$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log \frac{1}{x}}$ ....$(\because a \log b = \log b^a)$

$= \frac{1}{x}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{x+1}{x}\right).\left(\frac{1}{x}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{x+1}{x^2}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{1}{x} + \frac{1}{x^2}\right)dx + c$$

$$\therefore \frac{y}{x} = \int \left(\frac{1}{x} + x^{-2}\right)dx + c$$

$$\therefore \frac{y}{x} = \log x + \frac{x^{-1}}{-1} + c \text{ ....} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} \;\&\; \int \frac{1}{x} dx = \log x\right)$$

$$\therefore \frac{y}{x} = \log x - \frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = x \log x - 1 + cx$$

$$\therefore y = x \log x - 1 + cx$$

7. Question

Find the general solution for each of the following differential equations.

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{(1+x^2)}$$

Answer

Given Differential Equation :

$$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{(1+x^2)}$$

<u>Formula</u> :

i) $\int \frac{f'(x)}{f(x)}dx = \log f(x)$

ii) $\int \frac{1}{(1+x^2)}dx = \tan^{-1}x$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.)=\int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$(1+x^2)\frac{dy}{dx}+2xy=\frac{1}{(1+x^2)}$$

Dividing above equation by $(1+x^2)$,

$\frac{dy}{dx}+\frac{2x}{(1+x^2)}.y=\frac{1}{(1+x^2)^2}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{2x}{(1+x^2)}$ and $Q=\frac{1}{(1+x^2)^2}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$= e^{\int \frac{2x}{(1+x^2)} dx}$

Let, $f(x) = (1 + x^2)$ & $f'(x) = 2x$

$= e^{\log(1+x^2)}$ ....$\left(\because \int \frac{f'(x)}{f(x)} dx = \log f(x)\right)$

$= (1 + x^2)$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(1 + x^2) = \int \frac{1}{(1 + x^2)^2}.(1 + x^2)dx + c$$

$$\therefore y.(1 + x^2) = \int \frac{1}{(1 + x^2)}dx + c$$

$\therefore y.(1 + x^2) = \tan^{-1}x + c$ ....$\left(\because \int \frac{1}{(1+x^2)} dx = \tan^{-1}x\right)$

Therefore, general solution is

$$y.(1 + x^2) = tan^{-1}x + c$$

8. Question

Find the general solution for each of the following differential equations.

$$(1 - x^2)\frac{dy}{dx} + xy = x\sqrt{1 - x^2}$$

Answer

Given Differential Equation :

$$(1 - x^2)\frac{dy}{dx} + xy = x\sqrt{1 - x^2}$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$(1 - x^2)\frac{dy}{dx} + xy = x\sqrt{1 - x^2}$$

Dividing above equation by $(1 - x^2)$,

$$\frac{dy}{dx} + \frac{x}{(1 - x^2)}.y = \frac{x\sqrt{1 - x^2}}{(1 - x^2)}$$

$\frac{dy}{dx} + \frac{x}{(1-x^2)}.y = \frac{x}{\sqrt{1-x^2}}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{x}{(1-x^2)}$ and $Q = \frac{x}{\sqrt{1-x^2}}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{x}{(1-x^2)}dx}$$

$$= e^{\frac{-1}{2}\int \frac{-2x}{(1-x^2)}dx}$$

Let $(1 - x^2) = f(x)$

Therefore $f'(x) = -2x$

$\therefore \int \frac{f'(x)}{f(x)} dx = \int \frac{-2x}{(1-x^2)} dx = \log f(x) = \log(1-x^2)$ ......eq(2)

$\therefore I.F. = e^{\frac{-1}{2}\log(1-x^2)}$

$= e^{\log(1-x^2)^{-1/2}}$ ....($\because a\log b = \log b^a$)

$= e^{\log\left(\frac{1}{\sqrt{1-x^2}}\right)}$

$= \frac{1}{\sqrt{1-x^2}}$ ....($\because a^{\log_a b} = b$)

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{\sqrt{1-x^2}}\right) = \int \left(\frac{x}{\sqrt{1-x^2}}\right).\left(\frac{1}{\sqrt{1-x^2}}\right)dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \int \frac{x}{(1-x^2)} dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-1}{2}\int \frac{-2x}{(1-x^2)} dx + c$$

$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{-1}{2}\log(1-x^2) + c$ ....from eq(2)

Multiplying above equation by $\sqrt{1-x^2}$,

$$\therefore y = \frac{-1}{2}\sqrt{1-x^2}\log(1-x^2) + c\sqrt{1-x^2}$$

9. Question

Find the general solution for each of the following differential equations.

$$(1-x^2)\frac{dy}{dx} + xy = ax$$

Answer

Given Differential Equation :

$$(1-x^2)\frac{dy}{dx} + xy = ax$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

<u>Answer</u> :

Given differential equation is

$$(1 - x^2)\frac{dy}{dx} + xy = ax$$

Dividing above equation by $(1 - x^2)$,

$\frac{dy}{dx} + \frac{x}{(1-x^2)}.y = \frac{ax}{(1-x^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{x}{(1-x^2)}$ and $Q = \frac{ax}{(1-x^2)}$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int \frac{x}{(1-x^2)}dx}$$

$$= e^{\frac{-1}{2}\int \frac{-2x}{(1-x^2)}dx}$$

Let $(1 - x^2) = f(x)$

Therefore f′(x) = -2x

$$\therefore \int \frac{f'(x)}{f(x)} dx = \int \frac{-2x}{(1-x^2)} dx = \log f(x) = \log(1-x^2)$$

$$\therefore I.F. = e^{\frac{-1}{2}\log(1-x^2)}$$

$$= e^{\log(1-x^2)^{-1/2}} \text{ ....}(\because a\log b = \log b^a)$$

$$= e^{\log\left(\frac{1}{\sqrt{1-x^2}}\right)}$$

$$= \frac{1}{\sqrt{1-x^2}} \text{ ....}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{\sqrt{1-x^2}}\right) = \int \left(\frac{ax}{(1-x^2)}\right).\left(\frac{1}{\sqrt{1-x^2}}\right) dx + c$$

$$\therefore \frac{y}{\sqrt{1-x^2}} = \int \frac{ax}{(1-x^2)^{3/2}} dx + c \text{ ......eq(2)}$$

Let

$$I = \int \frac{ax}{(1-x^2)^{3/2}} dx$$

Put $(1 - x^2) = t$

$$\therefore -2x\,dx = dt$$

$$\therefore x\,dx = \frac{-dt}{2}$$

$$\therefore I = \int \frac{a}{t^{3/2}}.\frac{-dt}{2}$$

$$\therefore I = \frac{-a}{2}\int t^{-3/2} dt$$

$$\therefore I = \frac{-a}{2}.\frac{t^{-1/2}}{-1/2}$$

$$\therefore I = a.\frac{1}{\sqrt{t}}$$

$$\therefore I = \frac{a}{\sqrt{1-x^2}}$$

Substituting I in eq(2)

$$\therefore \frac{y}{\sqrt{1-x^2}} = \frac{a}{\sqrt{1-x^2}} + c$$

Multiplying above equation by $\sqrt{1-x^2}$,

$$\therefore y = a + c\sqrt{1-x^2}$$

10. Question

Find the general solution for each of the following differential equations.

$$(x^2+1)\frac{dy}{dx} - 2xy = (x^2+1)(x^2+2)$$

Answer

Given Differential Equation :

$$(x^2+1)\frac{dy}{dx} - 2xy = (x^2+1)(x^2+2)$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log f(x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int 1 dx = x$

v) $\int \frac{1}{1+x^2} dx = \tan^{-1} x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$\mathrm{I.F.} = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(x^2+1)\frac{dy}{dx} - 2xy = (x^2+1)(x^2+2)$$

Dividing above equation by $(1 + x^2)$,

$\frac{dy}{dx} + \frac{-2x}{(1+x^2)}.y = (x^2+2)$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-2x}{(1+x^2)}$ and $Q = (x^2+2)$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{-2x}{(1+x^2)}dx}$$

$$= e^{-\int \frac{2x}{(1+x^2)}dx}$$

Let $(1 + x^2) = f(x)$

Therefore $f'(x) = 2x$

$$\therefore \int \frac{f'(x)}{f(x)}dx = \int \frac{2x}{(1+x^2)}dx = \log f(x) = \log(1+x^2)$$

$$\therefore \mathrm{I.F.} = e^{-\log(1+x^2)}$$

$= e^{\log(1+x^2)^{-1}}$ ....$(\because a\log b = \log b^a)$

$$= e^{\log\left(\frac{1}{(1+x^2)}\right)}$$

$= \frac{1}{(1+x^2)}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{(1+x^2)}\right) = \int (2+x^2).\left(\frac{1}{(1+x^2)}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \frac{2+x^2}{1+x^2}dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \frac{1+x^2+1}{1+x^2}dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \left(\frac{1+x^2}{1+x^2} + \frac{1}{1+x^2}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = \int \left(1 + \frac{1}{1+x^2}\right)dx + c$$

$$\therefore \frac{y}{(1+x^2)} = x + \tan^{-1}x + c$$

$$\text{.........}\left(\because \int 1dx = x \ \& \ \int \frac{1}{1+x^2}dx = \tan^{-1}x\right)$$

$$\therefore y = (1+x^2)(x+\tan^{-1}x+c)$$

Therefore general solution is

$$y = (1+x^2)(x+tan^{-1}x+c)$$

11. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y = 6e^x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 2y = 6e^x$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + 2y = 6e^x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2$ and $Q = 6e^x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2dx}$$

$$= e^{2\int 1dx}$$

$= e^{2x}$ ....$(\because \int 1dx = x)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(e^{2x}) = \int (6e^x).(e^{2x})dx + c$$

$$\therefore y.(e^{2x}) = 6\int e^{3x}dx + c$$

$$\therefore y.(e^{2x}) = 6\frac{e^{3x}}{3} + c \text{.........} \left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

$$\therefore y.(e^{2x}) = 2e^{3x} + c$$

Dividing above equation by (e^{2x}),

$$\therefore y = \frac{2e^{3x}}{e^{2x}} + \frac{c}{e^{2x}}$$

$$\therefore y = 2e^{(3x-2x)} + ce^{-2x}$$

$$\therefore y = 2e^{x} + ce^{-2x}$$

Therefore general solution is

$$y = 2e^{x} + ce^{-2x}$$

12. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 3y = e^{-2x}$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I. F.} = e^{\int P\, dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} + 3y = e^{-2x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 3$ and $Q = e^{-2x}$

Therefore, integrating factor is

$\text{I. F.} = e^{\int P\, dx}$

$= e^{\int 3dx}$

$= e^{3\int 1dx}$

$= e^{3x}$ ....$(\because \int 1dx = x)$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(e^{3x}) = \int (e^{-2x}).(e^{3x})dx + c$$

$$\therefore y.(e^{3x}) = \int e^{x}dx + c$$

$\therefore y.(e^{3x}) = e^{x} + c$ ....$\left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$

Dividing above equation by (e^{3x}),

$$\therefore y = \frac{e^{x}}{e^{3x}} + \frac{c}{e^{3x}}$$

$$\therefore y = e^{(x-3x)} + ce^{-3x}$$

$$\therefore y = e^{-2x} + ce^{-3x}$$

Therefore general solution is

$y = e^{-2x} + ce^{-3x}$

13. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 8y = 5e^{-3x}$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 8y = 5e^{-3x}$$

Formula :

i) $\int 1dx = x$

ii) $\int e^{kx}dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 8y = 5e^{-3x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 8$ and $Q = 5e^{-3x}$

Therefore, integrating factor is

$\text{I.F.} = e^{\int P\,dx}$

$= e^{\int 8dx}$

$= e^{8\int 1dx}$

$= e^{8x}$ ....$(\because \int 1dx = x)$

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.(e^{8x}) = \int (5e^{-3x}).(e^{8x})dx + c$$

$$\therefore y.(e^{8x}) = 5\int e^{5x}dx + c$$

$$\therefore y.(e^{8x}) = 5\frac{e^{5x}}{5} + c \text{ ....} \left(\because \int e^{kx}dx = \frac{e^{kx}}{k}\right)$$

$$\therefore y.(e^{8x}) = e^{5x} + c$$

Dividing above equation by (e^{8x}),

$$\therefore y = \frac{e^{5x}}{e^{8x}} + \frac{c}{e^{8x}}$$

$$\therefore y = e^{(5x-8x)} + ce^{-8x}$$

$$\therefore y = e^{-3x} + ce^{-8x}$$

Therefore general solution is

$$y = e^{-3x} + ce^{-8x}$$

14. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = (x-1)e^x, x > 0$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} - y = (x-1)e^x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^x \left(f(x) + f'(x)\right) dx = e^x . f(x)$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = (x-1)e^x$$

Dividing above equation by x,

$\frac{dy}{dx} - \frac{1}{x}y = \frac{(x-1)}{x}e^x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{(x-1)}{x}e^x$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int \frac{-1}{x} dx}$$

$= e^{-\log x}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log x^{-1}}$ ....$(\because a \log b = \log b^a)$

$= \frac{1}{x}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{(x-1)}{x} e^x\right).\left(\frac{1}{x}\right) dx + c$$

$\therefore \frac{y}{x} = \int \left(\frac{x-1}{x^2} e^x\right) dx + c$ ....eq(2)

Let,

$$I = \int \left(\frac{x-1}{x^2} e^x\right) dx$$

$$\therefore I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

Let $f(x) = \frac{1}{x}$ $\therefore f'(x) = \frac{-1}{x^2}$

$\therefore I = e^x . \frac{1}{x}$ ....$\left(\because \int e^x \left(f(x) + f'(x)\right) dx = e^x . f(x)\right)$

Substituting I in eq(2),

$$\therefore \frac{y}{x} = e^x . \frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = e^x + cx$$

Therefore general solution is

$$y = e^x + cx$$

15. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} - y \tan x = e^x \sec x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} - y\tan x = e^x \sec x$$

Formula :

i) $\int \tan x\, dx = \log(\sec x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^x dx = e^x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} - y\tan x = e^x \sec x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\tan x$ and $Q = e^x \sec x$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int -\tan x\, dx}$$

$= e^{-\log(\sec x)}$ ....($\because \int \tan x\,dx = \log(\sec x)$)

$= e^{\log(\sec x)^{-1}}$ ....($\because a\log b = \log b^a$)

$= e^{\log(\cos x)}$

$= \cos x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\cos x) = \int (e^x \sec x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = \int \left(e^x.\frac{1}{\cos x}\right).(\cos x)dx + c$$

$$\therefore y.(\cos x) = \int e^x dx + c$$

$\therefore y.(\cos x) = e^x + c$ ....($\because \int e^x dx = e^x$)

Therefore general solution is

$$y.(\cos x) = e^x + c$$

16. Question

Find the general solution for each of the following differential equations.

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

Answer

Given Differential Equation :

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

Formula :

i) $\int \frac{f'(x)}{f(x)} dx = \log(f(x))$

ii) $a^{\log_a b} = b$

iii) $\int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx$

iv) $\frac{d}{dx}(\log x) = \frac{1}{x}$

v) $\int \frac{1}{x} dx = \log x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

<u>Answer</u> :

Given differential equation is

$$(x\log x)\frac{dy}{dx} + y = 2\log x$$

Dividing above equation by (x.log x),

$\frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x\log x}$ and $Q = \frac{2}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dx}$$

$$= e^{\int \frac{1}{x\log x} dx}$$

$$= e^{\int \frac{1/x}{\log x} dx}$$

Let, $f(x) = \log x \therefore f'(x) = 1/x$

$\therefore$ I. F. $= e^{\log(\log x)}$ ....$\left(\because \int \frac{f'(x)}{f(x)} dx = \log(f(x))\right)$

$= \log x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\log x) = \int \left(\frac{2}{x}\log x\right) dx + c$$

$\therefore y.(\log x) = 2\int \left(\frac{1}{x}\log x\right) dx + c$ ....eq(2)

Let,

$$I = \int \frac{1}{x}.\log x\, dx$$

Let, $u = \log x$ & $v = \frac{1}{x}$

$$\therefore I = \log x \int \frac{1}{x} dx - \int \left(\frac{d}{dx}(\log x). \int \frac{1}{x} dx\right) dx$$

.........$\left(\because \int u.v\, dx = u. \int v\, dx - \int \left(\frac{du}{dx}. \int v\, dx\right) dx\right)$

$$\therefore I = \log x . \log x - \int \left(\frac{1}{x}.\log x\right) d$$

.........$\left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \text{ \& } \int \frac{1}{x} dx = \log x\right)$

$$\therefore I = (\log x)^2 - I$$

$$\therefore 2I = (\log x)^2$$

$$\therefore I = \frac{1}{2}(\log x)^2$$

Substituting I in eq(2),

$$\therefore y.(\log x) = 2.\frac{1}{2}(\log x)^2 + c$$

$$y.(\log x) = (\log x)^2 + c$$

17. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx}+y=x\log x$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx}+y=x\log x$$

Formula :

i) $\int \frac{1}{x}dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx$

iv) $\frac{d}{dx}(\log x) = \frac{1}{x}$

v) $\int x^n\,dx = \frac{x^{n+1}}{n+1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx}+y=x\log x$$

Dividing above equation by x,

$\frac{dy}{dx}+\frac{1}{x}y=\log x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = \log x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x} dx}$$

$$= e^{\log x} \ldots\ldots\ldots \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= x \ldots\ldots\ldots \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(x) = \int (x \log x)dx + c$ ....eq(2)

Let,

$$I = \int (x \log x)dx$$

Let, $u = \log x \;\&\; v = x$

$$\therefore I = \log x \int x\,dx - \int \left(\frac{d}{dx}(\log x).\int x\,dx\right)dx$$

$$\ldots\ldots\ldots \left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I = \log x.\frac{x^2}{2} - \int \left(\frac{1}{x}.\frac{x^2}{2}\right)dx$$

$$\ldots\ldots\ldots \left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \;\&\; \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore I = \log x.\frac{x^2}{2} - \frac{1}{2}\int (x)\,dx$$

$$\therefore I = \log x.\frac{x^2}{2} - \frac{1}{2}\left(\frac{x^2}{2}\right) \ldots\ldots\ldots \left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore I = \frac{x^2}{2}.\log x - \frac{x^2}{4}$$

Substituting I in eq(2),

$$\therefore xy = \frac{x^2}{2}.\log x - \frac{x^2}{4} + c$$

Multiplying above equation by 4,

$$\therefore 4xy = 2x^2.\log x - x^2 + 4c$$

Therefore general equation is

$$4xy = 2x^2.\log x - x^2 + 4c$$

18. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int u.v\, dx = u. \int v\, dx - \int \left(\frac{du}{dx}. \int v\, dx\right) dx$

v) $\frac{d}{dx}(\log x) = \frac{1}{x}$

vi) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

vii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

Dividing above equation by x,

$\frac{dy}{dx} + \frac{2}{x}y = x\log x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2}{x}$ and $Q = x\log x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{2}{x}dx}$$

$$= e^{2\int \frac{1}{x}dx}$$

$= e^{2\log x}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= e^{\log x^2}$ ....$(\because a\log b = \log b^a)$

$= x^2$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(x^2) = \int (x^2.x\log x)dx + c$$

$\therefore y.(x^2) = \int (x^3 \log x)dx + c$ ....eq(2)

Let,

$$I = \int (x^3 \log x)dx$$

Let, $u = \log x \ \& \ v = x^3$

$$\therefore I = \log x \int x^3\,dx - \int \left(\frac{d}{dx}(\log x).\int x^3\,dx\right)dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$\therefore I = \log x.\frac{x^4}{4} - \int\left(\frac{1}{x}.\frac{x^4}{4}\right)dx$$

.........$\left(\because \frac{d}{dx}(\log x) = \frac{1}{x} \ \& \ \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore I = \log x.\frac{x^4}{4} - \frac{1}{4}\int (x^3)\,dx$$

$\therefore I = \log x.\frac{x^4}{4} - \frac{1}{4}\left(\frac{x^4}{4}\right)$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore I = \frac{x^4}{4}.\log x - \frac{x^4}{16}$$

Substituting I in eq(2),

$$\therefore x^2 y = \frac{x^4}{4}.\log x - \frac{x^4}{16} + c$$

Dividing above equation by x^2,

$$\therefore y = \frac{x^2}{4}.\log x - \frac{x^2}{16} + \frac{c}{x^2}$$

$$\therefore y = \frac{x^2}{16}(4\log x - 1) + \frac{c}{x^2}$$

Therefore general equation is

$$y = \frac{x^2}{16}(4\log x - 1) + \frac{c}{x^2}$$

19. Question

Find the general solution for each of the following differential equations.

$$(1+x)\frac{dy}{dx} - y = e^{3x}(1+x)^2$$

Answer

Given Differential Equation :

$$(1+x)\frac{dy}{dx} - y = e^{3x}(1+x)^2$$

Formula :

i) $\int \frac{1}{px+q} dx = \frac{1}{p}\log(px+q)$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int e^{kx} dx = \frac{1}{k}e^{kx}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$(1+x)\frac{dy}{dx} - y = e^{3x}(1+x)^2$$

Dividing above equation by (1+x),

$\frac{dy}{dx} - \frac{1}{(1+x)}y = e^{3x}(1+x)$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{(1+x)}$ and $Q = e^{3x}(1+x)$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{(1+x)}dx}$$

$$= e^{-\int \frac{1}{(1+x)}dx}$$

$$= e^{-\log(1+x)} \text{.........}\left(\because \int \frac{1}{px+q}dx = \frac{1}{p}\log(px+q)\right)$$

$$= e^{\log\frac{1}{(1+x)}} \text{.........}(\because a\log b = \log b^a)$$

$$= \frac{1}{(1+x)} \text{.........}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{(1+x)}\right) = \int e^{3x}(1+x)\left(\frac{1}{(1+x)}\right)dx + c$$

$$\therefore y.\left(\frac{1}{(1+x)}\right) = \int e^{3x}dx + c$$

$$\therefore y.\left(\frac{1}{(1+x)}\right) = \frac{1}{3}e^{3x} + c \text{.........}\left(\because \int e^{kx}dx = \frac{1}{k}e^{kx}\right)$$

Multiplying above equation by (1+x),

$$\therefore y = \frac{1}{3}(1+x)e^{3x} + c(1+x)$$

Therefore general equation is

$$y = \frac{1}{3}(1+x)e^{3x} + c(1+x)$$

20. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + \frac{4x}{(x^2+1)}y + \frac{1}{(x^2+1)^2} = 0$$

Answer

Given Differential Equation :

$$\frac{dy}{dx}+\frac{4x}{(x^2+1)}y+\frac{1}{(1+x^2)^2}=0$$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx=\log(f(x))$

ii) $a\log b=\log b^a$

iii) $a^{\log_a b}=b$

iv) $\int 1\,dx=x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

Where, integrating factor,

$$I.F.=e^{\int P\,dx}$$

Answer :

Given differential equation is

$$\frac{dy}{dx}+\frac{4x}{(x^2+1)}y+\frac{1}{(1+x^2)^2}=0$$

$\therefore \frac{dy}{dx}+\frac{4x}{(x^2+1)}y=\frac{-1}{(1+x^2)^2}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{4x}{(x^2+1)}$ and $Q=\frac{-1}{(1+x^2)^2}$

Therefore, integrating factor is

$\text{I.F.} = e^{\int P\,dx}$

$= e^{\int \frac{4x}{(x^2+1)}dx}$

$= e^{2\int \frac{2x}{(x^2+1)}dx}$

Let, $f(x) = (x^2 + 1)$ & $f'(x) = 2x$

$\therefore \text{I.F.} = e^{2\log(x^2+1)}$ ....$\left(\because \int \frac{f'(x)}{f(x)}dx = \log\left(f(x)\right)\right)$

$= e^{\log(1+x^2)^2}$ ....$(\because a\log b = \log b^a)$

$= (1+x^2)^2$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.(1+x^2)^2 = \int \frac{-1}{(1+x^2)^2}(1+x^2)^2 dx + c$$

$$\therefore y.(1+x^2)^2 = \int -1dx + c$$

$\therefore y.(1+x^2)^2 = -x + c$ ....$(\because \int 1\,dx = x)$

Dividing above equation by $(1+x^2)^2$,

$$\therefore y = \frac{-x}{(1+x^2)^2} + \frac{c}{(1+x^2)^2}$$

Therefore general equation is

$$y = \frac{-x}{(1+x^2)^2} + \frac{c}{(1+x^2)^2}$$

21. Question

Find the general solution for each of the following differential equations.

$$(y + 3x^2)\frac{dx}{dy} = x$$

Answer

Given Differential Equation :

$$(y+3x^2)\frac{dx}{dy}=x$$

Formula :

i) $\int \frac{1}{x}dx=\log x$

ii) $a\log b=\log b^a$

iii) $a^{\log_a b}=b$

iv) $\int 1\,dx=x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx}+Py=Q$$

General solution is given by,

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

Where, integrating factor,

$$I.F.=e^{\int P\,dx}$$

Answer :

Given differential equation is

$$(y+3x^2)\frac{dx}{dy}=x$$

$$\therefore \frac{dy}{dx}=\frac{(y+3x^2)}{x}$$

$$\therefore \frac{dy}{dx}=\frac{y}{x}+3x$$

$\therefore \frac{dy}{dx}-\frac{y}{x}=3x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P = \frac{-1}{x}$ and $Q = 3x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x} dx}$$

$$= e^{-\log x} \text{.........}\left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= e^{\log\left(\frac{1}{x}\right)} \text{.........}(\because a\log b = \log b^{a})$$

$$= \frac{1}{x} \text{.........}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int 3x.\left(\frac{1}{x}\right) dx + c$$

$$\therefore \frac{y}{x} = \int 3dx + c$$

$$\therefore \frac{y}{x} = 3\int 1dx + c$$

$$\therefore \frac{y}{x} = 3x + c \text{.........}(\because \int 1\,dx = x)$$

Multiplying above equation by x,

$$\therefore y = 3x^2 + cx$$

Therefore general equation is

$$y = 3x^2 + cx$$

22. Question

Find the general solution for each of the following differential equations.

$$xdy - (y + 2x^2)dx = 0$$

Answer

Given Differential Equation :

$xdy - (y + 2x^2)dx = 0$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int 1\, dx = x$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$xdy - (y + 2x^2)dx = 0$

$\therefore xdy = (y + 2x^2)dx$

$$\therefore \frac{dy}{dx} = \frac{(y + 2x^2)}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x} + 2x$$

$\therefore \frac{dy}{dx} - \frac{y}{x} = 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = 2x$

Therefore, integrating factor is

$$\text{I. F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x} dx}$$

$$= e^{-\log x} \;......... \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= e^{\log\left(\frac{1}{x}\right)} \;......... (\because a\log b = \log b^{a})$$

$$= \frac{1}{x} \;......... \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int 2x.\left(\frac{1}{x}\right) dx + c$$

$$\therefore \frac{y}{x} = \int 2dx + c$$

$$\therefore \frac{y}{x} = 2\int 1dx + c$$

$$\therefore \frac{y}{x} = 2x + c \;......... (\because \int 1\,dx = x)$$

Multiplying above equation by x,

$$\therefore y = 2x^2 + cx$$

Therefore general equation is

$$y = 2x^2 + cx$$

23. Question

Find the general solution for each of the following differential equations.

$$xdy + (y - x^3)dx = 0$$

Answer

Given Differential Equation :

$$xdy + (y - x^3)dx = 0$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

<u>Answer</u> :

Given differential equation is

$$xdy + (y - x^3)dx = 0$$

$$\therefore xdy = -(y - x^3)dx$$

$$\therefore xdy = (x^3 - y)dx$$

$$\therefore \frac{dy}{dx} = \frac{(x^3 - y)}{x}$$

$$\therefore \frac{dy}{dx} = x^2 - \frac{y}{x}$$

$\therefore \frac{dy}{dx} + \frac{y}{x} = x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = x^2$

Therefore, integrating factor is

$\text{I. F.} = e^{\int P\,dx}$

$= e^{\int \frac{1}{x}dx}$

$= e^{\log x}$ ....$\left(\because \int \frac{1}{x}dx = \log x\right)$

$= x$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$$\therefore xy = \int x^3 dx + c$$

$\therefore xy = \frac{x^4}{4} + c$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

Dividing above equation by x,

$$\therefore y = \frac{x^3}{4} + \frac{c}{x}$$

Therefore general equation is

$$y = \frac{x^3}{4} + \frac{c}{x}$$

24. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y = \sin x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 2y = \sin x$$

Formula :

i) $\int 1\,dx = x$

ii) $\int u.v\,dx = u.\int v dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(\sin x) = \cos x$

v) $\frac{d}{dx}(\cos x) = \sin x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + 2y = \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2$ and $Q = \sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2\,dx}$$

$$= e^{2\int 1\,dx}$$

$= e^{2x}$ ....$(\because \int 1\,dx = x)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(e^{2x}) = \int \sin x.(e^{2x})dx + c$ ....eq(2)

Let,

$$I = \int \sin x.(e^{2x})dx$$

Let, u=sin x and v=e^{2x}

$$I = \sin x.\int e^{2x}dx - \int\left(\frac{d}{dx}(\sin x).\int e^{2x}\,dx\right)dx$$

.........$\left(\because \int u.v\,dx = u.\int v dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$= \sin x.\frac{e^{2x}}{2} - \int\left(\cos x.\frac{e^{2x}}{2}\right)dx$$

.........$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(\sin x) = \cos x\right)$

$$= \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\int(\cos x.e^{2x})dx$$

Again, let u=cos x and v=e^{2x}

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\int e^{2x}dx - \int\left(\frac{d}{dx}(\cos x).\int e^{2x}\,dx\right)dx\right\}$$

.........$\left(\because \int u.v\,dx = u.\int v dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2} - \int\left((-\sin x).\frac{e^{2x}}{2}\right)dx\right\}$$

.........$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(\cos x) = \sin x\right)$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2} + \int\left(\sin x.\frac{e^{2x}}{2}\right)dx\right\}$$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2} + \frac{1}{2}\int(\sin x.e^{2x})dx\right\}$$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \frac{1}{2}\left\{\cos x.\frac{e^{2x}}{2} + \frac{I}{2}\right\}$$

$$\therefore I = \sin x.\frac{e^{2x}}{2} - \cos x.\frac{e^{2x}}{4} - \frac{I}{4}$$

$$\therefore I + \frac{I}{4} = \sin x.\frac{e^{2x}}{2} - \cos x.\frac{e^{2x}}{4}$$

$$\therefore \frac{5I}{4} = \sin x.\frac{e^{2x}}{2} - \cos x.\frac{e^{2x}}{4}$$

Multiplying above equation by 4,

$$\therefore 5I = 2\sin x.e^{2x} - \cos x.e^{2x}$$

$$\therefore 5I = e^{2x}(2\sin x - \cos x)$$

$$\therefore I = \frac{e^{2x}}{5}(2\sin x - \cos x)$$

Substituting I in eq(2),

$$\therefore y.(e^{2x}) = \frac{e^{2x}}{10}(2\sin x - \cos x) + c$$

Dividing above equation by e^{2x},

$$\therefore y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$$

Therefore general equation is

$$y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$$

25. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + y = \cos x - \sin x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + y = \cos x - \sin x$$

Formula :

i) $\int 1\,dx = x$

ii) $\int e^x.\left(f(x) + f'(x)\right)dx = e^x.f(x)$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y = \cos x - \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 1$ and $Q = \cos x - \sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 1\,dx}$$

$= e^x$ ....($\because \int 1\,dx = x$)

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(e^x) = \int (\cos x - \sin x).(e^x)dx + c$$

Let, f(x)=cos x => f $'$(x) = -sin x

$\therefore y.(e^x) = (e^x).\cos x + c$

$\ldots\ldots\ldots\left(\because \int e^x.\left(f(x) + f'(x)\right)dx = e^x.f(x)\right)$

Dividing above equation by e^x,

$\therefore y = \cos x + \frac{c}{e^x}$

Therefore general equation is

$y = \cos x + ce^{-x}$

26. Question

Find the general solution for each of the following differential equations.

$$\sec x\frac{dy}{dx} - y = \sin x$$

Answer

Given Differential Equation :

$$\sec x\frac{dy}{dx} - y = \sin x$$

Formula :

i) $\int \cos x\,dx = \sin x$

ii) $\int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(kx) = k$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I. F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$$\sec x \frac{dy}{dx} - y = \sin x$$

Dividing above equation by sec x,

$$\frac{dy}{dx} - \frac{1}{\sec x} y = \frac{\sin x}{\sec x}$$

$\therefore \frac{dy}{dx} - \cos x . y = \sin x . \cos x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\cos x$ and $Q = \sin x . \cos x$

Therefore, integrating factor is

$\text{I. F.} = e^{\int P\,dx}$

$= e^{\int -\cos x\,dx}$

$= e^{-\sin x}$ ....($\because \int \cos x\,dx = \sin x$)

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$\therefore y.\left(e^{-\sin x}\right) = \int (\sin x . \cos x).\left(e^{-\sin x}\right)dx + c$ ....eq(2)

Let,

$$I = \int (\sin x . \cos x).\left(e^{-\sin x}\right)dx$$

Put sin x=t => cos x.dx=dt

$$\therefore I = \int e^{-t}.t\,dt$$

$$\therefore I = t.\int e^{-t}\,dt - \int \left(\frac{d}{dt}(t).\int e^{-t}\,dt\right) dt$$

$$\text{.........}\left(\because \int u.v\,dx = u.\int v\,dx - \int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$$

$$\therefore I = -t.e^{-t} - \int((1).(-e^{-t}))\,dt$$

$$\text{.........}\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(kx) = k\right)$$

$$\therefore I = -t.e^{-t} + (-e^{-t})\ \text{.........}\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$$

$$\therefore I = -\sin x.e^{-\sin x} - e^{-\sin x}$$

Substituting I in eq(2),

$$\therefore y.\left(e^{-\sin x}\right) = -\sin x.e^{-\sin x} - e^{-\sin x} + c$$

$$\therefore y.\left(e^{-\sin x}\right) = -e^{-\sin x}(\sin x + 1) + c$$

$$\therefore y.\left(e^{-\sin x}\right) = c - e^{-\sin x}(\sin x + 1)$$

Dividing above equation by $e^{-\sin x}$,

$$\therefore y = \frac{c}{e^{-\sin x}} - (\sin x + 1)$$

Therefore general equation is

$$y = ce^{-\sin x} - (\sin x + 1)$$

27. Question

Find the general solution for each of the following differential equations.

$$\left(1+x^2\right)\frac{dy}{dx} + 2xy = \cot x$$

Answer

Given Differential Equation :

$$(1+x^2)\frac{dy}{dx} + 2xy = \cot x$$

Formula :

i) $\int \frac{f'(x)}{f(x)}dx = \log(f(x))$

ii) $a^{\log_a b} = b$

iii) $\int \cot x \, dx = \log|\sin x|$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$$(1 + x^2)\frac{dy}{dx} + 2xy = \cot x$$

Dividing above equation by $(1+x^2)$,

$\therefore \frac{dy}{dx} + \frac{2x}{(1+x^2)}y = \frac{\cot x}{(1+x^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{2x}{(1+x^2)}$ and $Q = \frac{\cot x}{(1+x^2)}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{2x}{(1+x^2)}dx}$$

Let, $f(x) = (1+x^2) \Rightarrow f'(x) = 2x$

$= e^{\log(1+x^2)}$ ....$\left(\because \int \frac{f'(x)}{f(x)}dx = \log(f(x))\right)$

$= (1 + x^2)$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(1+x^2) = \int \frac{\cot x}{(1+x^2)}.(1+x^2)dx + c$$

$$\therefore y.(1+x^2) = \int \cot x\, dx + c$$

$$\therefore y.(1+x^2) = \log|\sin x| + c \text{ ....}(\because \int \cot x\, dx = \log|\sin x|)$$

Therefore, general solution is

$$y.(1+x^2) = \log|\sin x| + c$$

28. Question

Find the general solution for each of the following differential equations.

$$(\sin x)\frac{dy}{dx} + (\cos x)y = \cos x \sin^2 x$$

Answer

Given Differential Equation :

$$\sin x \frac{dy}{dx} + (\cos x)y = \cos x.\sin^2 x$$

Formula :

v) $\int \cot x\, dx = \log(\sin x)$

vi) $a^{\log_a b} = b$

vii) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I. F.} = e^{\int P\,dx}$

<u>Answer</u> :

Given differential equation is

$$\sin x \frac{dy}{dx} + (\cos x)y = \cos x \, . \sin^2 x$$

Dividing above equation by sin x,

$$\therefore \frac{dy}{dx} - \frac{\cos x}{\sin x} y = \frac{\cos x \, . \sin^2 x}{\sin x}$$

$\therefore \frac{dy}{dx} + (\cot x)y = \cos x \, . \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = \sin x \, . \cos x$

Therefore, integrating factor is

$\text{I. F.} = e^{\int P\,dx}$

$= e^{\int \cot x\,dx}$

$= e^{\log(\sin x)}$ ....$(\because \int \cot x\,dx = \log(\sin x))$

= sin x ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(\sin x) = \int (\sin x \, . \cos x).(\sin x)dx + c$$

$\therefore y.(\sin x) = \int (\sin^2 x . \cos x)dx + c$ ....eq(2)

Let,

$$I = \int (\sin^2 x . \cos x)dx$$

Put sin x=t => cos x.dx=dt

$$\therefore I = \int t^2 dt$$

$$\therefore I = \frac{t^3}{3} \text{ ....} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore I = \frac{\sin^3 x}{3}$$

Substituting I in eq(2),

$$\therefore y.(\sin x) = \frac{\sin^3 x}{3} + c$$

Therefore, general solution is

$$y.(\sin x) = \frac{\sin^3 x}{3} + c$$

29. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 2y(\cot x) = 3x^2 \operatorname{cosec}^2 x$$

Formula :

i) $\int \cot x \, dx = \log(\sin x)$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int x^n dx = \frac{x^{n+1}}{n+1}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y(\cot x) = 3x^2 \operatorname{cosec}^2 x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2\cot x$ and $Q = 3x^2 \operatorname{cosec}^2 x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2\cot x\,dx}$$

$= e^{2\log(\sin x)}$ ....$(\because \int \cot x\,dx = \log(\sin x))$

$= e^{\log(\sin x)^2}$ ....$(\because a\log b = \log b^a)$

$= \sin^2 x$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin^2 x) = \int (3x^2 \operatorname{cosec}^2 x).(\sin^2 x)dx + c$$

$$\therefore y.(\sin^2 x) = \int \left(3x^2 \frac{1}{\sin^2 x}\right).(\sin^2 x)dx + c$$

$$\therefore y.(\sin^2 x) = 3\int (x^2)dx + c$$

$\therefore y.(\sin^2 x) = 3\frac{x^3}{3} + c$ ....$\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$

$$\therefore y.(\sin^2 x) = x^3 + c$$

Therefore, general solution is

$$y.(sin^2 x) = x^3 + c$$

30. Question

Find the general solution for each of the following differential equations.

$$x\frac{dy}{dx} - y = 2x^2 \sec x$$

Answer

Given Differential Equation :

$$x\frac{dy}{dx} - y = 2x^2 \sec x$$

Formula :

vi) $\int \cot x \, dx = \log(\sin x)$

vii) $a \log b = \log b^a$

viii) $a^{\log_a b} = b$

ix) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

x) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$x\frac{dy}{dx} - y = 2x^2 \sec x$ ....eq(1)

Dividing above equation by x,

$$\therefore \frac{dy}{dx} - \frac{1}{x}y = 2x\sec x$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = 2x\sec x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}dx}$$

$$= e^{-\log x} \;.........\left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= e^{\log x^{-1}} \;.........(\because a\log b = \log b^a)$$

$$= \frac{1}{x} \;.........\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int (2x\sec x).\left(\frac{1}{x}\right)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = 2\int \sec x\,dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = 2\log|\sec x + \tan x| + c$$

$$.........(\because \int \sec x\,dx = \log|\sec x + \tan x|)$$

Multiplying above equation by x,

$$\therefore y = 2x\log|\sec x + \tan x| + cx$$

Therefore, general solution is

$$y = 2x\log|\sec x + \tan x| + cx$$

31. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

Formula :

i) $\int \tan x\, dx = \log|\sec x|$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $2\sin x.\cos x = \sin 2x$

v) $\int \sin x\, dx = -\cos x$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$$\frac{dy}{dx} = y\tan x - 2\sin x$$

$\frac{dy}{dx} - y\tan x = -2\sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -\tan x$ and $Q = -2\sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -\tan x\,dx}$$

$$= e^{-\log|\sec x|} \text{ ....}\left(\because \int \tan x\,dx = \log|\sec x|\right)$$

$$= e^{\log|\sec x|^{-1}} \text{ ....}\left(\because a\log b = \log b^a\right)$$

$$= e^{\log\left(\frac{1}{\sec x}\right)}$$

$$= e^{\log(\cos x)}$$

$$= \cos x \text{ ....}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\cos x) = \int (-2\sin x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = -\int (2\sin x).(\cos x)dx + c$$

$$\therefore y.(\cos x) = -\int (\sin 2x)\,dx + c \text{ ....}(\because 2\sin x.\cos x = \sin 2x)$$

$$\therefore y.(\cos x) = \frac{\cos 2x}{2} + c \text{ ....}\left(\because \int \sin x\,dx = -\cos x\right)$$

Multiplying above equation by 2,

$$\therefore 2y.(\cos x) = \cos 2x + 2c$$

$\therefore 2y.(\cos x) = \cos 2x + C$ where, C=2c

Therefore, general solution is

$$2y.(\cos x) = \cos 2x + C$$

32. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} = y\cot x = \sin 2x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + y\cot x = \sin 2x$$

Formula :

i) $\int \cot x \, dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v \, dx = u.\int v \, dx - \int \left(\frac{du}{dx}.\int v \, dx\right) dx$

iv) $\int \sin x \, dx = -\cos x$

v) $\frac{d}{dx}(\sin x) = \cos x$

vi) $2\sin x.\cos x = \sin 2x$

vii) $\cos 2x = (\cos^2 x - \sin^2 x)$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y\cot x = \sin 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = \sin 2x$

Therefore, integrating factor is

$$\text{I. F.} = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$$= e^{\log|\sin x|} \text{.........} \left(\because \int \cot x\,dx = \log|\sin x|\right)$$

$$= \sin x \text{.........} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(\sin x) = \int (\sin 2x).(\sin x)dx + c \text{.........eq(2)}$$

Let,

$$I = \int (\sin 2x).(\sin x)dx$$

Let, u=sin 2x & v=sin x

$$\therefore I = \sin 2x. \int \sin x\ dx - \int \left(\frac{d}{dt}(\sin 2x). \int \sin x\ dx\right) dx$$

$$\text{.........}\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$$

$$\therefore I = -\sin 2x.\cos x - \int ((2\cos 2x).(-\cos x))\ dx$$

$$\text{.........}\left(\because \int \sin x\,dx = -\cos x\ \&\ \frac{d}{dx}(\sin x) = \cos x\right)$$

$$\therefore I = -\sin 2x.\cos x + 2\int ((\cos 2x).(\cos x))\ dx$$

Again let, u=cos 2x & v=cos x

$$\therefore I = -\sin 2x.\cos x + 2\left\{\cos 2x. \int \cos x\ dx - \int \left(\frac{d}{dt}(\cos 2x). \int \cos x\ dx\right) dx\right\}$$

$$\text{.........}\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$$

$$\therefore I = -\sin 2x.\cos x + 2\left\{\cos 2x.\sin x - \int ((-2\sin 2x).(\sin x))\ dx\right\}$$

$$\therefore I = -\sin 2x.\cos x + 2\{\cos 2x.\sin x + 2\int((\sin 2x).(\sin x))\,dx\}$$

$$\therefore I = -\sin 2x.\cos x + 2\{\cos 2x.\sin x + 2I\}$$

$$\therefore I = -\sin 2x.\cos x + 2\cos 2x.\sin x + 4I$$

$$\therefore I - 4I = -2\sin x\cos x.\cos x + 2(\cos^2 x - \sin^2 x).\sin x$$

$$.........(\because \sin 2x = 2\sin x.\cos x \ \&\ \cos 2x = (\cos^2 x - \sin^2 x))$$

$$\therefore -3I = -2\sin x\cos^2 x + 2\sin x\cos^2 x - 2\sin^3 x$$

$$\therefore -3I = -2\sin^3 x$$

$$\therefore I = \frac{2}{3}\sin^3 x$$

Substituting I in eq(2),

$$\therefore y.(\sin x) = \frac{2}{3}\sin^3 x + c$$

Therefore, general solution is

$$y.(\sin x) = \frac{2}{3}sin^3 x + c$$

33. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 2y\tan x = \sin x$$

Formula :

i) $\int \tan x\,dx = \log|\sec x|$

ii) $a\log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int \left(\frac{-1}{x^2}\right) dx = \frac{1}{x}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y\tan x = \sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2\tan x$ and $Q = \sin x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int 2\tan x\,dx}$$

$= e^{2\log|\sec x|}$ ....$(\because \int \tan x\,dx = \log|\sec x|)$

$= e^{\log|\sec x|^2}$ ....$(\because a\log b = \log b^a)$

$= \sec^2 x$ ....$(\because a^{\log_a b} = b)$

$$= \frac{1}{\cos^2 x}$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.\left(\frac{1}{\cos^2 x}\right) = \int(\sin x).\left(\frac{1}{\cos^2 x}\right)dx + c$ ....eq(2)

Let,

$$I = \int (\sin x).\left(\frac{1}{\cos^2 x}\right) dx$$

Put, cos x=t => -sin x dx = dt

$$\therefore I = \int \left(\frac{-1}{t^2}\right) dt$$

$$\therefore I = \frac{1}{t} \text{.........} \left(\because \int \left(\frac{-1}{x^2}\right) dx = \frac{1}{x}\right)$$

$$\therefore I = \frac{1}{\cos x}$$

Substituting I in eq(2),

$$\therefore y.\left(\frac{1}{\cos^2 x}\right) = \frac{1}{\cos x} + c$$

Multiplying above equation by $\cos^2 x$,

$$\therefore y = \cos x + c(\cos^2 x)$$

Therefore, general solution is

$$y = \cos x + c(cos^2 x)$$

34. Question

Find the general solution for each of the following differential equations.

$$\frac{dy}{dx} + y \cot x = x^2 \cot x + 2x$$

Answer

Given Differential Equation :

$$\frac{dy}{dx} + y \cot x = x^2 \cot x + 2x$$

Formula :

i) $\int \cot x \, dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v \, dx = u.\int v \, dx - \int \left(\frac{du}{dx}.\int v \, dx\right) dx$

iv) $\int \cos x \, dx = \sin x$

v) $\frac{d}{dx}(x^n) = nx^{n-1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + y\cot x = x^2 \cot x + 2x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = x^2 \cot x + 2x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$= e^{\log|\sin x|}$ ....$(\because \int \cot x\,dx = \log|\sin x|)$

= sin x ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cot x + 2x).(\sin x)dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cot x . \sin x + 2x\sin x)\, dx + c$$

$$\therefore y.(\sin x) = \int \left(x^2 \frac{\cos x}{\sin x} . \sin x + 2x\sin x\right) dx + c$$

$$\therefore y.(\sin x) = \int (x^2 \cos x + 2x\sin x)\, dx + c$$

$\therefore y.(\sin x) = \int x^2 \cos x \; dx + \int 2x\sin x\, dx + c$ ....eq(2)

Let,

$$I = \int x^2 \cos x \; dx$$

Let, $u=x^2$ and v=cos x

$$\therefore I = x^2 . \int \cos x \; dx - \int \left(\frac{d}{dt}(x^2) . \int \cos x \; dx\right) dx$$

.........$\left(\because \int u.v\, dx = u. \int v\, dx - \int \left(\frac{du}{dx} . \int v\, dx\right) dx\right)$

$$\therefore I = x^2 . \sin x - \int 2x . \sin x\, dx$$

.........$\left(\because \int \cos x\, dx = \sin x \;\&\; \frac{d}{dx}(x^n) = nx^{n-1}\right)$

Substituting I in eq(2),

$$\therefore y.(\sin x) = x^2 . \sin x - \int 2x . \sin x\, dx + \int 2x\sin x\, dx + c$$

$$\therefore y.(\sin x) = x^2 . \sin x + c$$

Dividing above equation by sin x,

$$\therefore y = x^2 + \frac{c}{\sin x}$$

Therefore, general solution is

$$y = x^2 + c(cosec\; x)$$

35. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$x\frac{dy}{dx} + y = x^3$, given that y =1 when x =2

Answer

Given Differential Equation :

$$x\frac{dy}{dx} + y = x^3$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} + y = x^3$$

Dividing above equation by x,

$\therefore \frac{dy}{dx} + \frac{1}{x}.y = x^2$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{1}{x}$ and $Q = x^2$

Therefore, integrating factor is

$$\text{I.F.} = e^{\int P\,dx}$$

$$= e^{\int \frac{1}{x} dx}$$

$$= e^{\log x} \text{ ....}\left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= x \text{ ....}\left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.(x) = \int x^2.(x)dx + c$$

$$\therefore xy = \int x^3 dx + c$$

$$\therefore xy = \frac{x^4}{4} + c \text{ ....}\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

Dividing above equation by x,

$$\therefore y = \frac{x^3}{4} + \frac{c}{x}$$

Therefore general equation is

$$y = \frac{x^3}{4} + \frac{c}{x}$$

For particular solution put y=1 and x=2 in above equation,

$$\therefore 1 = \frac{2^3}{4} + \frac{c}{2}$$

$$\therefore 1 = \frac{8}{4} + \frac{c}{2}$$

$$\therefore 1 = 2 + \frac{c}{2}$$

$$\therefore \frac{c}{2} = -1$$

$$\therefore c = -2$$

Therefore, particular solution is

$y = \frac{x^3}{4} - \frac{2}{x}$

36. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + y\cot x = 4x\operatorname{cosec} x$, given that $y = 0$ when $x = \frac{\pi}{2}$.

Answer

Given Differential Equation :

$$\frac{dy}{dx} + y.\cot x = 4x\operatorname{cosec} x$$

Formula :

i) $\int \cot x\, dx = \log|\sin x|$

ii) $a^{\log_a b} = b$

iii) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

iv) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

Answer :

Given differential equation is

$\frac{dy}{dx} + y.\cot x = 4x\operatorname{cosec} x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \cot x$ and $Q = 4x \operatorname{cosec} x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \cot x\,dx}$$

$$= e^{\log|\sin x|} \text{.........}\left(\because \int \cot x\,dx = \log|\sin x|\right)$$

= sin x ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(\sin x) = \int (4x \operatorname{cosec} x).(\sin x)dx + c$$

$$\therefore y.(\sin x) = 4\int \left(x \frac{1}{\sin x}\right).(\sin x)dx + c$$

$$\therefore y.(\sin x) = 4\int (x)\,dx + c$$

$$\therefore y.(\sin x) = 4\frac{x^2}{2} + c \text{.........}\left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$\therefore y.(\sin x) = 2x^2 + c$$

Therefore general equation is

$$y.(\sin x) = 2x^2 + c$$

For particular solution put y=0 and $x = \frac{\pi}{2}$ in above equation,

$$\therefore 0 = 2\frac{\pi^2}{4} + c$$

$$\therefore 0 = \frac{\pi^2}{2} + c$$

$$\therefore c = -\frac{\pi^2}{2}$$

Therefore, particular solution is

$y.(\sin x) = 2x^2 - \frac{\pi^2}{2}$

37. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + 2xy = x$, given that y = 0 when x =0.

Answer

<u>Given Differential Equation</u> :

$$\frac{dy}{dx} + 2xy = x$$

<u>Formula</u> :

i) $\int x^n\, dx = \frac{x^{n+1}}{n+1}$

ii) $\int (e^{kx}) dx = \frac{e^{kx}}{k}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dx}$$

<u>Answer</u> :

Given differential equation is

$\frac{dy}{dx} + 2xy = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2x$ and $Q = x$

Therefore, integrating factor is

$$\mathrm{I.F.} = e^{\int P\,dx}$$

$$= e^{\int 2x\,dx}$$

$$= e^{2\frac{x^2}{2}} \text{ ....} \left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}\right)$$

$$= e^{x^2}$$

General solution is

$$y.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dx + c$$

$$\therefore y.\left(e^{x^2}\right) = \int (x).\left(e^{x^2}\right)dx + c$$

$$\therefore y.\left(e^{x^2}\right) = \frac{1}{2}\int (2x).\left(e^{x^2}\right)dx + c \text{ ....eq(2)}$$

Let,

$$I = \int (2x).\left(e^{x^2}\right)dx$$

Put, $x^2=t \Rightarrow 2x\,dx = dt$

$$\therefore I = \int (e^t)dt$$

$$\therefore I = e^t \text{ ....} \left(\because \int \left(e^{kx}\right)dx = \frac{e^{kx}}{k}\right)$$

$$\therefore I = e^{x^2}$$

Substituting I in eq(2),

$$\therefore y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} + c$$

Therefore, general solution is

$$y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} + c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0 = \frac{1}{2}.e^0 + c$$

$$\therefore 0 = \frac{1}{2} + c$$

$$\therefore c = -\frac{1}{2}$$

Substituting c in general solution,

$$y.\left(e^{x^2}\right) = \frac{1}{2}.e^{x^2} - \frac{1}{2}$$

Multiplying above equation by $\frac{2}{e^{x^2}}$

$$\therefore 2y = 1 - e^{-x^2}$$

Therefore, particular solution is

$$2y = 1 - e^{-x^2}$$

38. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + 2y = e^{-2x}\sin x$, given that y = 0, when x = 0.

Answer

Given Differential Equation :

$$\frac{dy}{dx} + 2y = e^{-2x}.\sin x$$

Formula :

i) $\int 1\,dx = x$

ii) $\int (\sin x)dx = -\cos x$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I.F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} + 2y = e^{-2x}.\sin x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = 2$ and $Q = e^{-2x}.\sin x$

Therefore, integrating factor is

$\text{I.F.} = e^{\int P\,dx}$

$= e^{\int 2\,dx}$

$= e^{2x}$ ....($\because \int 1\,dx = x$)

General solution is

$$y.(\text{I.F.}) = \int Q.(\text{I.F.})dx + c$$

$$\therefore y.(e^{2x}) = \int (e^{-2x}.\sin x).(e^{2x})dx + c$$

$$\therefore y.(e^{2x}) = \int \left(\frac{1}{e^{2x}}.\sin x\right).(e^{2x})dx + c$$

$$\therefore y.(e^{2x}) = \int (\sin x)dx + c$$

$\therefore y.(e^{2x}) = -\cos x + c$ ....($\because \int(\sin x)dx = -\cos x$)

Therefore, general solution is

$$y.(e^{2x}) = -\cos x + c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0 = -\cos 0 + c$$

$$\therefore 0 = -1 + c$$

$$\therefore c = 1$$

Substituting c in general solution,

$$y.(e^{2x}) = -\cos x + 1$$

Therefore, particular solution is

$$y.(e^{2x}) = -\cos x + 1$$

39. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$, given that y = 0 when x = 0.

Answer

Given Differential Equation :

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

Formula :

i) $\int \frac{f(x)}{f'(x)} dx = \log f(x)$

ii) $\int x^n \, dx = \frac{x^{n+1}}{n+1}$

iii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P \, dx}$$

Answer :

Given differential equation is

$$(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$$

Dividing above equation by $(1+x^2)$,

$\therefore \frac{dy}{dx}+\frac{2x}{(1+x^2)}y=\frac{4x^2}{(1+x^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx}+Py=Q$$

Where, $P=\frac{2x}{(1+x^2)}$ and $Q=\frac{4x^2}{(1+x^2)}$

Therefore, integrating factor is

$$I.F.=e^{\int P\,dx}$$

$$=e^{\int \frac{2x}{(1+x^2)}dx}$$

Let, $f(x)=(1+x^2)\ \therefore f'(x)=2x$

$$\therefore I.F.=e^{\log(1+x^2)}\ ....\left(\because \int \frac{f(x)}{f'(x)}dx=\log f(x)\right)$$

$$=(1+x^2)$$

General solution is

$$y.(I.F.)=\int Q.(I.F.)dx+c$$

$$\therefore y.(1+x^2)=\int\left(\frac{4x^2}{(1+x^2)}\right).(1+x^2)dx+c$$

$$\therefore y.(1+x^2)=4\int x^2dx+c$$

$$\therefore y.(1+x^2)=4\frac{x^3}{3}+c\ ....\left(\because \int x^n\,dx=\frac{x^{n+1}}{n+1}\right)$$

Therefore, general solution is

$$y.(1+x^2)=4\frac{x^3}{3}+c$$

For particular solution put y=0 and x=0 in above equation,

$$\therefore 0=0+c$$

$$\therefore c=0$$

Substituting c in general solution,

$$\therefore y.(1+x^2) = 4\frac{x^3}{3}$$

Dividing above equation by $(1+x^2)$,

$$\therefore y = \frac{4x^3}{3(1+x^2)}$$

Therefore, particular solution is

$$y = \frac{4x^3}{3(1+x^2)}$$

40. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$x\frac{dy}{dx} - y = \log x$, given that $y = 0$ when $x = 1$.

Answer

Given Differential Equation :

$$x\frac{dy}{dx} - y = \log x$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

v) $\int e^{kx}\, dx = \frac{e^{kx}}{k}$

vi) $\frac{d}{dx}(kx) = k$

vii) $\log 1 = 0$

viii) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

Given differential equation is

$$x\frac{dy}{dx} - y = \log x$$

Dividing above equation by x,

$$\therefore \frac{dy}{dx} - \frac{1}{x}y = \frac{\log x}{x} \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \frac{-1}{x}$ and $Q = \frac{\log x}{x}$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int \frac{-1}{x}dx}$$

$$= e^{-\log(x)} \text{ ....} \left(\because \int \frac{1}{x}dx = \log x\right)$$

$$= e^{\log x^{-1}} \text{ ....} (\because a\log b = \log b^a)$$

$$= e^{\log\left(\frac{1}{x}\right)}$$

$$= \frac{1}{x} \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.\left(\frac{1}{x}\right) = \int \left(\frac{\log x}{x}\right).\left(\frac{1}{x}\right)dx + c \text{ ....eq(2)}$$

Let,

$$I = \int \left(\frac{\log x}{x}\right).\left(\frac{1}{x}\right)dx$$

Put, log x =t => x=e^t

Therefore, (1/x) dx = dt

$$\therefore I = \int \left(\frac{t}{e^t}\right)\, dt$$

$$\therefore I = \int t.e^{-t}\, dt$$

Let, u=t and v=e^{-t}

$$\therefore I = t.\int e^{-t}\, dt - \int \left(\frac{d}{dt}(t).\int e^{-t}\, dt\right)dt$$

$$\text{.........}\left(\because \int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right)dx\right)$$

$$\therefore I = -t.e^{-t} - \int \left((1).(-e^{-t})\right)dt$$

$$\text{.........}\left(\because \int e^{kx}\, dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(kx) = k\right)$$

$$\therefore I = -t.e^{-t} - e^{-t}\ \text{.........}\left(\because \int e^{kx}\, dx = \frac{e^{kx}}{k}\right)$$

$$\therefore I = -\frac{\log x}{x} - \frac{1}{x}$$

Substituting I in eq(2),

$$\therefore y.\left(\frac{1}{x}\right) = -\frac{\log x}{x} - \frac{1}{x} + c$$

Multiplying above equation by x,

$$\therefore y = -\log x - 1 + cx$$

Therefore, general solution is

$$y = -\log x - 1 + cx$$

For particular solution put y=0 and x=1 in above equation,

$$\therefore 0 = -\log 1 - 1 + c$$

$\therefore c = 1$ ....($\because \log 1 = 0$)

Substituting c in general solution,

$\therefore y = -\log x - 1 + x$

$\therefore y = x - \log x - 1$

Therefore, particular solution is

$y = x - \log x - 1$

41. Question

Find a particular solution satisfying the given condition for each of the following differential equations.

$\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, given that y = 1 when x = 0.

Answer

Given Differential Equation :

$$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$$

Formula :

i) $\int \tan x\, dx = \log|\sec x|$

ii) $a^{\log_a b} = b$

iii) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

iv) $\int \sec x.\tan x\, dx = \sec x$

v) $\frac{d}{dx}(x^n) = nx^{n-1}$

vi) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$\text{I. F.} = e^{\int P\,dx}$

Answer :

Given differential equation is

$\frac{dy}{dx} + y\tan x = 2x + x^2 \tan x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = \tan x$ and $Q = 2x + x^2 \tan x$

Therefore, integrating factor is

$\text{I. F.} = e^{\int P\,dx}$

$= e^{\int \tan x\,dx}$

$= e^{\log|\sec x|}$ ....$(\because \int \tan x\,dx = \log|\sec x|)$

$= \sec x$ ....$(\because a^{\log_a b} = b)$

General solution is

$$y.(\text{I. F.}) = \int Q.(\text{I. F.})dx + c$$

$$\therefore y.(\sec x) = \int (2x + x^2 \tan x).(\sec x)dx + c$$

$$\therefore y.(\sec x) = \int (x^2 \tan x.\sec x + 2x\sec x)\,dx + c$$

$\therefore y.(\sec x) = \int x^2 \tan x.\sec x\;dx + \int 2x\sec x\,dx + c$ ....eq(2)

Let,

$$I = \int x^2 \tan x.\sec x\;dx$$

Let, $u=x^2$ and $v=\tan x.\sec x$

$$\therefore I = x^2.\int \sec x.\tan x\;dx - \int \left(\frac{d}{dt}(x^2).\int \sec x.\tan x\;dx\right) dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$

$$\therefore I = x^2.\sec x - \int 2x.\sec x\ dx$$

$$\text{.........}\left(\because \int \sec x.\tan x\, dx = \sec x\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$$

Substituting I in eq(2),

$$\therefore y.(\sec x) = x^2.\sec x - \int 2x.\sec x\ dx + \int 2x \sec x\, dx + c$$

$$\therefore y.(\sec x) = x^2.\sec x + c$$

$$\therefore y.\left(\frac{1}{\cos x}\right) = x^2.\left(\frac{1}{\cos x}\right) + c$$

Multiplying above equation by cos x,

$$\therefore y = x^2 + c.(\cos x)$$

Therefore, general solution is

$$y = x^2 + c.(\cos x)$$

For particular solution put y=1 and x=0 in above equation,

$$\therefore 1 = 0 + c$$

$$\therefore c = 1$$

Substituting c in general solution,

$$\therefore y = x^2 + \cos x$$

Therefore, particular solution is

$$y = x^2 + \cos x$$

42. Question

A curve passes through the origin and the slope of the tangent to the curve at any point (x,) is equal to the sum of the coordinates of the point. Find the equation of the curve.

Answer

Formula :

i) $\int 1\, dx = x$

ii) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

iii) $\int e^{kx}\, dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

The slope of the tangent to the curve $= \frac{dy}{dx}$

The slope of the tangent to the curve is equal to the sum of the coordinates of the point.

$$\therefore \frac{dy}{dx} = x + y$$

Therefore differential equation is

$$\therefore \frac{dy}{dx} = x + y$$

$\therefore \frac{dy}{dx} - y = x$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -1$ and $Q = x$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -1\,dx}$$

$= e^{-x}$ ....$(\because \int 1\,dx = x)$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$\therefore y.(e^{-x}) = \int (x).(e^{-x})dx + c$ ....eq(2)

Let,

$$I = \int (x).(e^{-x})dx$$

Let, u=x and v= e^{-x}

$$\therefore I = x.\int e^{-x}\,dx - \int \left(\frac{d}{dx}(x).\int e^{-x}\,dx\right) dx$$

.........$\left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$

$$\therefore I = -x.e^{-x} - \int (1).(-e^{-x})\,dx$$

.........$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(x^n) = nx^{n-1}\right)$

$\therefore I = -x.e^{-x} - e^{-x}$ ....$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$$\therefore y.(e^{-x}) = -x.e^{-x} - e^{-x} + c$$

Dividing above equation by e^{-x},

$$\therefore y = -x - 1 + c.e^{x}$$

Therefore, general solution is

$$y + x + 1 = c.e^{x}$$

The curve passes through origin , therefore the above equation satisfies for x=0 and y=0,

$$\therefore 0 + 0 + 1 = c.e^{0}$$

$$\therefore c = 1$$

Substituting c in general solution,

$$\therefore y + x + 1 = e^{x}$$

Therefore, equation of the curve is

$$y + x + 1 = e^{x}$$

43. Question

A curve passes through the point (0, 2) and the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5. Find the equation of the curve.

Answer

Formula :

i) $\int 1\,dx = x$

ii) $\int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right)dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dy}{dx} + Py = Q$$

General solution is given by,

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dx}$$

Answer :

The slope of the tangent to the curve $= \frac{dy}{dx}$

The sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at the given point by 5.

$$\therefore 5 + \frac{dy}{dx} = x + y$$

Therefore differential equation is

$$\therefore 5 + \frac{dy}{dx} = x + y$$

$\therefore \frac{dy}{dx} - y = x - 5$ ....eq(1)

Equation (1) is of the form

$$\frac{dy}{dx} + Py = Q$$

Where, $P = -1$ and $Q = x - 5$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dx}$$

$$= e^{\int -1\,dx}$$

$$= e^{-x} \text{ ....} (\because \int 1\,dx = x)$$

General solution is

$$y.(I.F.) = \int Q.(I.F.)dx + c$$

$$\therefore y.(e^{-x}) = \int (x-5).(e^{-x})dx + c \text{ ....eq(2)}$$

Let,

$$I = \int (x-5).(e^{-x})dx$$

Let, u=x-5 and v= e^{-x}

$$\therefore I = (x-5).\int e^{-x}\,dx - \int \left(\frac{d}{dt}(x-5).\int e^{-x}\,dx\right) dx$$

$$\text{.........} \left(\because \int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx\right)$$

$$\therefore I = -(x-5).e^{-x} - \int (1).(-e^{-x})\,dx$$

$$\text{.........} \left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k} \;\&\; \frac{d}{dx}(x^n) = nx^{n-1}\right)$$

$$\therefore I = -(x-5).e^{-x} - e^{-x} \text{ ....} \left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$$

Substituting I in eq(2),

$$\therefore y.(e^{-x}) = -(x-5).e^{-x} - e^{-x} + c$$

Dividing above equation by e^{-x},

$$\therefore y = -(x-5) - 1 + c.e^{x}$$

$\therefore y = -x + 5 - 1 + c.e^x$

$\therefore y = -x + 4 + c.e^x$

Therefore, general solution is

$y = -x + 4 + c.e^x$

The curve passes through point (0,2) , therefore the above equation satisfies for x=0 and y=2,

$\therefore 2 = -0 + 4 + c.e^0$

$\therefore c = -2$

Substituting c in general solution,

$\therefore y = -x + 4 - 2e^x$

Therefore, equation of the curve is

$y = 4 - x - 2e^x$

44. Question

Find the general solution for each of the following differential equations.

$$ydx - (x + 2y^2)dy = 0$$

Answer

Given Differential Equation :

$$ydx - (x + 2y^2)dy = 0$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

iii) $a \log b = \log b^a$

iv) $a^{\log_a b} = b$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dy}$$

<u>Answer</u> :

Given differential equation is

$$ydx - (x + 2y^2)dy = 0$$

$$\therefore ydx = (x + 2y^2)dy$$

$$\therefore \frac{dx}{dy} = \frac{(x + 2y^2)}{y}$$

$$\therefore \frac{dx}{dy} = \frac{x}{y} + 2y$$

$\therefore \frac{dx}{dy} - \frac{1}{y}.x = 2y$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{-1}{y}$ and $Q = 2y$

Therefore, integrating factor is

$$I.F. = e^{\int P\, dy}$$

$$= e^{\int \frac{-1}{y} dy}$$

$= e^{-\log y}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= e^{\log \frac{1}{y}}$ ....$(\because a \log b = \log b^a)$

$= \frac{1}{y}$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.\left(\frac{1}{y}\right) = \int (2y).\left(\frac{1}{y}\right) dy + c$$

$$\therefore \frac{x}{y} = \int (2) dy + c$$

$$\therefore \frac{x}{y} = 2y + c \;.........(\because \int 1 dx = x)$$

Multiplying above equation by y,

$$\therefore x = 2y^2 + cy$$

Therefore, general solution is

$$\therefore x = 2y^2 + cy$$

45. Question

Find the general solution for each of the following differential equations.

$$y dx + (x - y^2) dy = 0$$

Answer

Given Differential Equation :

$$y dx + (x - y^2) dy = 0$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int 1 dx = x$

iv) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.) dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dy}$$

Answer :

Given differential equation is

$$ydx + (x - y^2)dy = 0$$

$$\therefore ydx = -(x - y^2)dy$$

$$\therefore ydx = (y^2 - x)dy$$

$$\therefore \frac{dx}{dy} = \frac{(y^2 - x)}{y}$$

$$\therefore \frac{dx}{dy} = -\frac{x}{y} + y$$

$\therefore \frac{dx}{dy} + \frac{1}{y}.x = y$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{y}$ and $Q = y$

Therefore, integrating factor is

$$I.F. = e^{\int P\,dy}$$

$$= e^{\int \frac{1}{y} dy}$$

$= e^{\log y}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= y$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

$$\therefore x.(y) = \int (y).(y)dy + c$$

$$\therefore xy = \int y^2\,dy + c$$

$$\therefore xy = \frac{y^3}{3} + c \text{ ....}(\because \int 1dx = x)$$

Dividing above equation by y,

$$\therefore x = \frac{1}{3}y^2 + \frac{c}{y}$$

Therefore, general solution is

$$x = \frac{1}{3}y^2 + \frac{c}{y}$$

46. Question

Find the general solution for each of the following differential equations.

$$ydx + (x - y^2)dy = 0$$

Answer

<u>Given Differential Equation</u> :

$$ydx + (x - y^2)dy = 0$$

<u>Formula</u> :

i) $\int \frac{1}{x}dx = \log x$

ii) $a^{\log_a b} = b$

iii) $\int 1dx = x$

iv) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

<u>Answer</u> :

Given differential equation is

$\mathrm{ydx} + (\mathrm{x} - \mathrm{y}^2)\mathrm{dy} = 0$

$\therefore \mathrm{ydx} = -(\mathrm{x} - \mathrm{y}^2)\mathrm{dy}$

$\therefore \mathrm{ydx} = (\mathrm{y}^2 - \mathrm{x})\mathrm{dy}$

$\therefore \frac{\mathrm{dx}}{\mathrm{dy}} = \frac{(\mathrm{y}^2 - \mathrm{x})}{\mathrm{y}}$

$\therefore \frac{\mathrm{dx}}{\mathrm{dy}} = -\frac{\mathrm{x}}{\mathrm{y}} + \mathrm{y}$

$\therefore \frac{\mathrm{dx}}{\mathrm{dy}} + \frac{1}{\mathrm{y}}.\mathrm{x} = \mathrm{y}$ ....eq(1)

Equation (1) is of the form

$\frac{\mathrm{dx}}{\mathrm{dy}} + \mathrm{Px} = \mathrm{Q}$

Where, $\mathrm{P} = \frac{1}{\mathrm{y}}$ and $\mathrm{Q} = \mathrm{y}$

Therefore, integrating factor is

$\mathrm{I.F.} = e^{\int P\, dy}$

$= e^{\int \frac{1}{y} dy}$

$= e^{\log y}$ ....$\left(\because \int \frac{1}{x} dx = \log x\right)$

$= y$ ....$\left(\because a^{\log_a b} = b\right)$

General solution is

$x.(\mathrm{I.F.}) = \int Q.(\mathrm{I.F.})dy + c$

$\therefore x.(y) = \int (y).(y)dy + c$

$\therefore xy = \int y^2\, dy + c$

$\therefore xy = \frac{y^3}{3} + c$ ....$(\because \int 1dx = x)$

Dividing above equation by y,

$$\therefore x = \frac{1}{3}y^2 + \frac{c}{y}$$

Therefore, general solution is

$$x = \frac{1}{3}y^2 + \frac{c}{y}$$

47. Question

Find the general solution for each of the following differential equations.

$$(x + 3y^3)\frac{dy}{dx} = y, (y > 0)$$

Answer

Given Differential Equation :

$$(x + 3y^3)\frac{dy}{dx} = y$$

Formula :

i) $\int \frac{1}{x} dx = \log x$

ii) $a \log b = \log b^a$

iii) $a^{\log_a b} = b$

iv) $\int x^n dx = \frac{x^{n+1}}{n+1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\,dy}$$

Answer :

Given differential equation is

$$(x + 3y^3)\frac{dy}{dx} = y$$

$$\therefore \frac{dx}{dy} = \frac{(x + 3y^3)}{y}$$

$$\therefore \frac{dx}{dy} = \frac{x}{y} + 3y^2$$

$$\therefore \frac{dx}{dy} - \frac{1}{y}.x = 3y^2 \text{ ....eq(1)}$$

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{-1}{y}$ and $Q = 3y^2$

Therefore, integrating factor is

$$\text{I.F.} = e^{\int P\,dy}$$

$$= e^{\int \frac{-1}{y} dy}$$

$$= e^{-\log y} \text{ ....} \left(\because \int \frac{1}{x} dx = \log x\right)$$

$$= e^{\log \frac{1}{y}} \text{ ....} (\because a \log b = \log b^a)$$

$$= \frac{1}{y} \text{ ....} \left(\because a^{\log_a b} = b\right)$$

General solution is

$$x.(\text{I.F.}) = \int Q.(\text{I.F.})dy + c$$

$$\therefore x.\left(\frac{1}{y}\right) = \int (3y^2).\left(\frac{1}{y}\right)dy + c$$

$$\therefore \frac{x}{y} = 3\int (y)dy + c$$

$$\therefore \frac{x}{y} = \frac{3y^2}{2} + c \text{ ....} \left(\because \int x^n dx = \frac{x^{n+1}}{n+1}\right)$$

Multiplying above equation by y,

$$\therefore x = \frac{3}{2}y^3 + cy$$

Therefore, general solution is

$$x = \frac{3}{2}y^3 + cy$$

48. Question

Find the general solution for each of the following differential equations.

$$(x+y)\frac{dy}{dx} = 1$$

Answer

Given Differential Equation :

$$(x+y)\frac{dy}{dx} = 1$$

Formula :

i) $\int 1 dx = x$

ii) $\int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx$

iii) $\int e^{kx} dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$I.F. = e^{\int P\, dy}$$

Answer :

Given differential equation is

$$(x+y)\frac{dy}{dx}=1$$

$$\therefore \frac{dx}{dy}=x+y$$

$\therefore \frac{dx}{dy}-x=y$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy}+Px=Q$$

Where, $P=-1$ and $Q=y$

Therefore, integrating factor is

$$I.F.=\ e^{\int P\,dy}$$

$$=e^{\int -1\,dy}$$

$=e^{-y}$ ....$(\because \int 1dx=x)$

General solution is

$$x.(I.F.)=\int Q.(I.F.)dy+c$$

$\therefore x.(e^{-y})=\int (y).(e^{-y})dy+c$ ....eq(2)

Let,

$$I=\int (y).(e^{-y})dy$$

Let, u=y and v= e^{-y}

$$\therefore I=y.\int e^{-y}\,dy-\int\left(\frac{d}{dy}(y).\int e^{-y}\,dy\right)dy$$

.........$\left(\because\ \int u.v\,dx=u.\int v\,dx-\int\left(\frac{du}{dx}.\int v\,dx\right)dx\right)$

$$\therefore I=-y.e^{-y}\ -\int (1).(-e^{-y})\,dy$$

.........$\left(\because\ \int e^{kx}\,dx=\frac{e^{kx}}{k}\ \&\ \frac{d}{dx}(x^n)=nx^{n-1}\right)$

$\therefore I = -y.e^{-y} - e^{-y}$ ....$\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$

Substituting I in eq(2),

$\therefore x.(e^{-y}) = -y.e^{-y} - e^{-y} + c$

$\therefore x.(e^{-y}) + y.e^{-y} + e^{-y} = c$

$\therefore e^{-y}(x + y + 1) = c$

Therefore, general solution is

$e^{-y}(x + y + 1) = c$

49. Question

Find the general solution for each of the following differential equations.

$$(x + y + 1)\frac{dy}{dx} = 1$$

Answer

Given Differential Equation :

$$(x + y + 1)\frac{dy}{dx} = 1$$

Formula :

i) $\int 1dx = x$

ii) $\int u.v\,dx = u.\int v\,dx - \int \left(\frac{du}{dx}.\int v\,dx\right) dx$

iii) $\int e^{kx}\,dx = \frac{e^{kx}}{k}$

iv) $\frac{d}{dx}(x^n) = nx^{n-1}$

v) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$$\text{I. F.} = e^{\int P\, dy}$$

<u>Answer</u> :

Given differential equation is

$$(x + y + 1)\frac{dy}{dx} = 1$$

$$\therefore \frac{dx}{dy} = x + y + 1$$

$\therefore \frac{dx}{dy} - x = y + 1$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = -1$ and $Q = y + 1$

Therefore, integrating factor is

$$\text{I. F.} = e^{\int P\, dy}$$

$$= e^{\int -1\, dy}$$

$= e^{-y}$ ....$(\because \int 1dx = x)$

General solution is

$$x.(\text{I. F.}) = \int Q.(\text{I. F.})dy + c$$

$\therefore x.(e^{-y}) = \int (y + 1).(e^{-y})dy + c$ ....eq(2)

Let,

$$I = \int (y + 1).(e^{-y})dy$$

Let, u=y+1 and v= e^{-y}

$$\therefore I = (y + 1).\int e^{-y}\, dy - \int \left(\frac{d}{dy}(y + 1).\int e^{-y}\, dy\right) dy$$

.........$\left(\because \int u.v\, dx = u.\int v\, dx - \int \left(\frac{du}{dx}.\int v\, dx\right) dx\right)$

$$\therefore I = -(y+1).e^{-y} - \int (1).(-e^{-y})\,dy$$

$$\text{.........}\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k} \;\&\; \frac{d}{dx}(x^n) = nx^{n-1}\right)$$

$$\therefore I = -(y+1).e^{-y} - e^{-y} \text{ ....}\left(\because \int e^{kx}\,dx = \frac{e^{kx}}{k}\right)$$

Substituting I in eq(2),

$$\therefore x.(e^{-y}) = -(y+1).e^{-y} - e^{-y} + c$$

$$\therefore x.(e^{-y}) = -e^{-y}(y+1+1) + c$$

$$\therefore x.(e^{-y}) = -e^{-y}(y+2) + c$$

$$\therefore x.(e^{-y}) = c - e^{-y}(y+2)$$

Dividing above equation by e^{-y}

$$\therefore x = ce^{y} - (y+2)$$

Therefore, general solution is

$$x = ce^{y} - (y+2)$$

50. Question

Solve $(x+1)\dfrac{dy}{dx} = 2e^{-y} - 1$, given that x = 0 when y = 0.

Answer

Given Equation: $(x+1)\dfrac{dy}{dx} = 2e^{-y} - 1$

Re-arranging, we get,

$$\frac{1}{2e^{-y} - 1}dy = \frac{dx}{(x+1)}$$

$$\frac{e^{y}}{2 - e^{y}}dy = \frac{dx}{(x+1)}$$

Let $2 - e^{y} = t$

$-e^{y}dy = dt$

Therefore,

$$\frac{dt}{t} = \frac{dx}{x+1}$$

Integrating both sides, we get,

log t = log(x + 1) + C

log (2 − e^y) = log (x + 1) + C

At x = 0, y = 0.

Therefore,

log(2) = log(1) + C

Therefore,

C = log 2

Now, we have,

log (2 − e^y) − log (x + 1) − log 2 = 0

$$y = \log\left|\frac{2x+1}{x+1}\right|$$

51. Question

Solve $(1+y^2)dx + (x - e^{-\tan^{-1}y})dy = 0$, given that when y =o, then x = 0.

Answer

Given Differential Equation :

$$(1+y^2)dx + \left(x - e^{-\tan^{-1}y}\right)dy = 0$$

Formula :

i) $\int \frac{1}{(1+x^2)} dx = \tan^{-1}x$

ii) General solution :

For the differential equation in the form of

$$\frac{dx}{dy} + Px = Q$$

General solution is given by,

$$x.(I.F.) = \int Q.(I.F.)dy + c$$

Where, integrating factor,

$\text{I.F.} = e^{\int P\,dy}$

Answer :

Given differential equation is

$$(1+y^2)dx + \left(x - e^{-\tan^{-1}y}\right)dy = 0$$

$$\therefore (1+y^2)dx = -\left(x - e^{-\tan^{-1}y}\right)dy$$

$$\therefore (1+y^2)dx = \left(e^{-\tan^{-1}y} - x\right)dy$$

$$\therefore \frac{dx}{dy} = \frac{\left(e^{-\tan^{-1}y} - x\right)}{(1+y^2)}$$

$$\therefore \frac{dx}{dy} = \frac{e^{-\tan^{-1}y}}{(1+y^2)} - \frac{x}{(1+y^2)}$$

$\therefore \frac{dx}{dy} + \frac{x}{(1+y^2)} = \frac{e^{-\tan^{-1}y}}{(1+y^2)}$ ....eq(1)

Equation (1) is of the form

$$\frac{dx}{dy} + Px = Q$$

Where, $P = \frac{1}{(1+y^2)}$ and $Q = \frac{e^{-\tan^{-1}y}}{(1+y^2)}$

Therefore, integrating factor is

$$\text{I.F.} = e^{\int P\,dy}$$

$$= e^{\int \frac{1}{(1+y^2)} dy}$$

$$= e^{\tan^{-1}y} \text{ ....} \left(\because \int \frac{1}{(1+x^2)} dx = \tan^{-1}x\right)$$

General solution is

$$x.(\text{I.F.}) = \int Q.(\text{I.F.})dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \left(\frac{e^{-\tan^{-1}y}}{(1+y^2)}\right).\left(e^{\tan^{-1}y}\right)dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \left(\frac{1}{e^{\tan^{-1}y}.(1+y^2)}\right).\left(e^{\tan^{-1}y}\right)dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \int \frac{1}{(1+y^2)}dy + c$$

$$\therefore x.\left(e^{\tan^{-1}y}\right) = \tan^{-1}y + c \,\ldots\ldots\left(\because \int \frac{1}{(1+x^2)}dx = \tan^{-1}x\right)$$

Putting x=0 and y=0

$$\therefore 0 = 0 + c$$

$$\therefore c = 0$$

Therefore, general solution is

$$x.\left(e^{tan^{-1}y}\right) = tan^{-1}y$$

Objective Questions

1. Question

Mark (√) against the correct answer in the following:

The solution of the □DE $\frac{dy}{dx} = e^{x+y}$ is

A. $e^x + e^y = C$

B. $e^x - e^{-y} = C$

C. $e^x + e^{-y} = C$

D. None of these

Answer

Given, $\frac{dy}{dx} = e^{x+y}$

$$\frac{dy}{dx} = e^x e^y$$

$$e^{-y}dy = e^x dx$$

On integrating on both sides, we get

$$-e^{-y} + c_1 = e^x + c_2$$

$$e^{-y} + e^x = c$$

Conclusion: Therefore, $e^{-y} + e^{x} = c$ is the solution of $\frac{dy}{dx} = e^{x+y}$

2. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = 2^{x+y}$ is

A. $2^{x} + 2^{y} = C$

B. $2^{x} + 2^{-y} = C$

C. $2^{x} - 2^{-y} = C$

D. None of these

Answer

Given, $\frac{dy}{dx} = 2^{x+y}$

$$\frac{dy}{dx} = 2^{x}2^{y}$$

$$2^{-y}dy = 2^{x}dx$$

On integrating on both sides, we get

$$-\frac{2^{-y}}{\log 2} + c_2 = \frac{2^{x}}{\log 2} + c_2$$

$$2^{x} + 2^{-y} = c_3 \log 2$$

$$2^{x} + 2^{-y} = c$$

Conclusion: Therefore, $2^{x} + 2^{-y} = c$ is the solution of $\frac{dy}{dx} = 2^{x+y}$

3. Question

Mark (√) against the correct answer in the following:

The solution of the DE $(e^{x} + 1)y\,dy = (y+1)e^{x}dx$ is

A. $e^{y} = C(e^{x} + 1)(y + 1)$

B. $e^{y} = e^{x} + y + 1$

C. $y = (e^{x} + 1)(y + 1)$

D. None of these

Answer

$$(e^x + 1)y\,dy = (y+1)e^x dx$$

$$\frac{y\,dy}{y+1} = \frac{e^x\,dx}{(e^x+1)}$$

Let, $e^x + 1 = t$

On differentiating on both sides we get $e^x dx = dt$

Now we can write this equation as $\frac{y\,dy}{y+1} = \frac{e^x\,dx}{(e^x+1)}$

$$\frac{((y+1)-1)\,dy}{y+1} = \frac{e^x\,dx}{(e^x+1)}$$

$$\left(1 - \frac{1}{y+1}\right)dy = \frac{e^x\,dx}{(e^x+1)}$$

$$\left(1 - \frac{1}{y+1}\right)dy = \frac{dt}{t}$$

On integrating on both sides, we get

$$y - \log(y+1) = \log(e^x+1) + \log c$$

$$y = \log(y+1) + \log(e^x+1) + \log c$$

$$y = \log(y+1)\,(e^x+1)c$$

$$e^y = c(y+1)(e^x+1)$$

Conclusion: Therefore, $e^y = c(y+1)(e^x+1)$ is the solution of $(e^x+1)y\,dy = (y+1)e^x dx$

4. Question

Mark (√) against the correct answer in the following:

The solution of the $\text{DE}\,x dy + y dx = 0$ is

A. $x + y = C$

B. $xy = C$

C. $\log(x+y) = C$

D. None of these

Answer

Given xdy + ydx = 0

xdy = -ydx

$$-\frac{dy}{y} = \frac{dx}{x}$$

On integrating on both sides we get,

-log y = log x + c

log x + log y = c

log xy = c

xy = C

Conclusion: Therefore xy = c is the solution of xdy + ydx = 0

5. Question

Mark (√) against the correct answer in the following:

The solution of the $x\frac{dy}{dx} = \cot y$ is

A. $x\cos y = C$

B. $x\tan y = C$

C. $x\sec y = C$

D. None of these

Answer

Given: $x\frac{dy}{dx} = \cot y$

Separating the variables, we get,

$$\frac{dy}{\cot y} = \frac{dx}{x}$$

$$\tan y\, dy = \frac{dx}{x}$$

Integrating both sides, we get,

$$\int \tan y\, dy = \int \frac{dx}{x}$$

log sec y = log x + log c

xcosy = c

Hence, A is the correct answer.

6. Question

Mark (√) against the correct answer in the following:

The solution of the $DE \frac{dy}{dx} = \frac{(1+y^2)}{(1+x^2)}$ is.

A. (y +x)=C(1-yx)

B. (y – x) = C(1+yx)

C. y = (1+x)C

D. None of these

Answer

Given $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

On integrating on both sides, we get

$$\tan^{-1} y = \tan^{-1} x + c$$

$$\tan^{-1} y - \tan^{-1} x = c$$

$\frac{y-x}{1+yx} = c$ (since $\tan^{-1} y - \tan^{-1} x = \frac{y-x}{1+yx}$)

y-x = C(1+yx)

Conclusion: Therefore, y-x = C(1+yx) is the solution of $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

7. Question

Mark (√) against the correct answer in the following:

The solution of the $DE \frac{dy}{dx}$ =1 - x + y – xy is

A. Log (1 + y) = x - $\frac{x^2}{2}$+C

B. $e^{(1+y)} = x - \frac{x^2}{2} + C$

C. $e^y = x - \frac{x^2}{2} + C$

D. None of these

Answer

$\frac{dy}{dx} = 1 - x + y - xy$

$$\frac{dy}{dx} = 1 - x + y(1 - x)$$

$$\frac{dy}{dx} = (1 + y)(1 - x)$$

$$\frac{dy}{1 + y} = (1 - x)dx$$

On integrating on both sides, we get

$$\log(1 + y) = x - \frac{x^2}{2} + c$$

Conclusion: Therefore, $\log(1 + y) = x - \frac{x^2}{2} + c$ is the solution of $\frac{dy}{dx} = 1 - x + y - xy$

8. Question

Mark (√) against the correct answer in the following:

The solution of the $DE \frac{dy}{dx} = e^{x+y} + x^2.e^y$ is

A. $e^{x-y} + \frac{x^3}{3} + C$

B. $e^x + e^{-y} + \frac{x^3}{3} + C'$

C. $e^x - e^{-y} + \frac{x^3}{3} + C$

D. None of these

Answer

Given $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

$$(e^{-y})dy = (e^x + x^2)dx$$

On integrating on both sides, we get

$$-e^{-y} = e^{x} + \frac{x^3}{3} + C$$

$$e^{-y} + e^{x} + \frac{x^3}{3} = C$$

Conclusion: Therefore, $e^{-y} + e^{x} + \frac{x^3}{3} = C$ is the

solution of $\frac{dy}{dx} = e^{x+y} + x^2 e^{y}$

9. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is

A. $y + \sin^{-1}y = \sin^{-1}x + C$

B. $\sin^{-1}y - \sin^{-1}x = C$

C. $\sin^{-1}y + \sin^{-1}x = C$

D. None of these

Answer

Given $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

$$-\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

On integrating on both sides, we get

$$-\sin^{-1}y = \sin^{-1}x + C \text{ (As } \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C \text{)}$$

$$\sin^{-1}y + \sin^{-1}x = C$$

Conclusion: Therefore, $\sin^{-1}y + \sin^{-1}x = C$ is the

solution of $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

10. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$ is

A. $y = 2\tan\frac{x}{2} - x + C$

B. $y = \tan\frac{x}{2} - 2x + C$

C. $y = \tan x - x + C$

D. None of these

Answer

Given $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

$$\frac{dy}{dx} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$\frac{dy}{dx} = \tan^2\frac{x}{2}$$

$$dy = dx(\tan^2\frac{x}{2})$$

On integrating on both sides, we get

$$y = 2\tan\frac{x}{2} - x + C$$

Conclusion: Therefore, $y = 2\tan\frac{x}{2} - x + C$ is the solution of $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

11. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$ is

A. $y^2\,(x + 1) = C$

B. $y\,(x^2 + 1) = C$

C. $x^2\,(y + 1) = C$

D. None of these

Answer

Given $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$

$$\frac{dy}{y} = \frac{-2xdx}{(x^2+1)}$$

Let $x^2 + 1 = t$

On differentiating on both sides we get 2xdx = dt

$$\frac{dy}{y} = \frac{-dt}{t}$$

On integrating on both sides, we get

$$\log y = -\log t + C$$

$$\log y + \log t = C$$

$$\log yt = C$$

yt = C

As $t = x^2 + 1$

$$y(x^2 + 1) = C$$

Conclusion: Therefore, $y(x^2 + 1) = C$ is the solution of $\frac{dy}{dx} = \frac{-2xy}{(x^2+1)}$

12. Question

Mark (√) against the correct answer in the following:

The solution of the DE cos x (1 + cos y) dx – sin y (1 + sin x) dy = 0 is

A. 1 + sin x cos y = C

B. (1 + sin x) (1 + cos y) = C

C. sin x cos y + cos x = C

D. none of these

Answer

Given cos x (1+cos y) dx – sin y (1+sin x) dy = 0

Let 1+cos y = t and 1+sin x = u

On differentiating both equations, we get

-sin y dy = dt and cos x dx = du

Substitute this in the first equation

t du + u dt = 0

$$-\frac{du}{u} = \frac{dt}{t}$$

-log u = log t + C

log u + log t = C

log ut = C

ut = C

(1+sin x)(1+cos y) = C

Conclusion: Therefore, (1+sin x)(1+cos y) = C is the solution of cos x (1+cos y) dx – sin y (1+sin x) dy = 0

13. Question

Mark (√) against the correct answer in the following:

the solution of the DE x cos y dy = (xe^{x} log x + e^{x}) dx is

A. sin y = e^{x} log x +C

B. sin y - e^{x} log x = C

C. sin y = e^{x} (log x) +C

D. none of these

Answer

Given $x\cos y\ dy = (xe^{x}\log x + e^{x})dx$

$$\cos y\ dy = \frac{(xe^{x}\log x + e^{x})}{x}dx$$

On integrating on both sides we get

$$\sin y = \log x \int e^{x}dx - \int \frac{1}{x}\left(\int e^{x}\right)dx + \int \frac{e^{x}}{x}dx$$

$$\sin y = \log x\,(e^{x}) - \int \frac{e^{x}}{x}dx + \int \frac{e^{x}}{x}dx + C$$

$$\sin y = e^{x}\log x + C$$

Conclusion: Therefore, $\sin y = e^{x}\log x + C$ the solution of

$$x\cos y\ dy = (xe^{x}\log x + e^{x})dx$$

14. Question

Mark (√) against the correct answer in the following:

The solution of the DE $\frac{dy}{dx} + y \log y \cot x = 0$ is

A. $\cos x \log y = C$

B. $\sin x \log y = C$

C. $\log y = C \sin x$

D. none of these

Answer

Given $\frac{dy}{dx} + y \log y \cot x = 0$

$$\frac{dy}{y \log y} = -\cot x \; dx$$

Let log y = t

On differentiating we get

$$\frac{1}{y} \; dy = dt$$

$$\frac{dt}{t} = -\cot x \; dx$$

log t = -log (sin x) + C

log t + log(sin x) = C

log(tsin x) = C

tsin x = C

(log y)(sin x) = C

Conclusion: Therefore, (log y)(sin x) = C is the solution of $\frac{dy}{dx} + y \log y \cot x = 0$

15. Question

Mark (√) against the correct answer in the following:

the general solution of the DE $(1 + x^2)\, dy - xy\, dx = 0$ is

A. $y = C(1 + x^2)$

B. $y^2 = C(1 + x^2)$

C. $y\sqrt{1+x^2} = C$

D. None of these

Answer

Given $(1+x^2)dy - xy\,dx = 0$

$$\frac{dy}{y} = \frac{x}{1+x^2}dx$$

Let $1+x^2 = t$

2x dx = dt

$$\frac{dy}{y} = \frac{dt}{2t}$$

On integrating on both sides we get

$$\log y = \frac{\log t}{2} + C$$

2 log y = log t + C

$$\log y^2 = \log t + C$$

$$y^2 = (1+x^2)c$$

Conclusion: Therefore, $y^2 = (1+x^2)c$ is the solution of

$$(1+x^2)dy - xy\,dx = 0$$

16. Question

Mark (√) against the correct answer in the following:

The general solution of the DE$x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy$ =0 is

A. $\sin^{-1}x + \sin^{-1}y = C$

B. $\sqrt{1+x^2} + \sqrt{1+y^2} = C$

C. $\tan^{-1}x + \tan^{-1}y = C$

D. None of these

Answer

Given $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}\,dy = 0$

$$\frac{ydy}{\sqrt{1+y^2}} = -\frac{xdx}{\sqrt{1+x^2}}$$

Let $1+y^2 = t$ and $1+x^2 = u$

2y dy = dt and 2x dx = du

$$\frac{dt}{\sqrt{t}} = -\frac{du}{\sqrt{u}}$$

On integrating on both sides we get

$$\sqrt{t} = -\sqrt{u} + C$$

$$\sqrt{1+y^2} + \sqrt{1+x^2} = C$$

Conclusion: Therefore, $\sqrt{1+y^2} + \sqrt{1+x^2} = C$ is the

solution of $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}\,dy = 0$

17. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\log\left(\frac{dy}{dx}\right) = (ax+by)$ is

A. $\frac{-e^{-by}}{b} = \frac{e^{ax}}{a} + C$

B. $e^{ax} - e^{-by} = C$

C. $be^{ax} + ae^{by} = C$

D. None of these

Answer

Given $\log(\frac{dy}{dx}) = (ax+by)$

$$\frac{dy}{dx} = e^{ax+by}$$

$$\frac{dy}{e^{by}} = e^{ax}dx$$

On integrating on both sides we get

$$-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + C$$

Conclusion: Therefore, $-\frac{e^{-by}}{b} = \frac{e^{ax}}{a} + C$ is the solution of

$\log(\frac{dy}{dx}) = (ax + by)$

18. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} = \left(\sqrt{1-x^2}\right)\left(\sqrt{1-y^2}\right)$ is

A. $\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$

B. $2\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$

C. $2\sin^{-1} y - \sin^{-1} x = C$

D. None of these

Answer

Given $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$

$$\frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}\, dx$$

Let x = sin t

dx = cos t dt

We know $\cos t = \sqrt{1-x^2}$

On integrating on both sides we get

$$\sin^{-1} y = \frac{t}{2} + \frac{\sin 2t}{4}$$

Sin 2t = 2 sin t cost

= $2x\sqrt{1-x^2}$

$$\sin^{-1} y = \frac{\sin^{-1} x}{2} + \frac{x\sqrt{1-x^2}}{2} + C$$

$$2\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$$

Conclusion: Therefore, $2\sin^{-1} y - \sin^{-1} x = x\sqrt{1-x^2} + C$ is the solution of $\frac{dy}{dx} = (\sqrt{1-x^2})(\sqrt{1-y^2})$

19. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ is

A. $x^2 - y^2 = C_1 x$

B. $x^2 + y^2 = C_1 y$

C. $x^2 + y^2 = C_1 x$

D. None of these

Answer

Given $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{x^2v^2 - x^2}{2vx^2} = v + x\frac{dv}{dx}$$

$$\frac{v^2 - 1}{2v} - v = x\frac{dv}{dx}$$

$$\frac{-v^2 - 1}{2v} = x\frac{dv}{dx}$$

$$\frac{dx}{x} + \frac{2vdv}{v^2 + 1} = 0$$

On integrating on both sides, we get

$$\log x + \log(v^2 + 1) = c$$

$$\log(x(v^2 + 1)) = c$$

$$x\left(\frac{y^2}{x^2} + 1\right) = C$$

$$y^2 + x^2 = Cx$$

Conclusion: Therefore, $y^2 + x^2 = Cx$ is the solution of

$$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$

20. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $x^2\frac{dy}{dx}=x^2+xy+y^2$ is.

A. $\tan^{-1}\frac{y}{x}=\log x+C$

B. $\tan^{-1}\frac{x}{y}=\log x+C$

C. $\tan^{-1}\frac{y}{x}=\log y+C$

D. None of these

Answer

Given $x^2\frac{dy}{dx}=x^2+xy+y^2$

$$\frac{dy}{dx}=1+\frac{y}{x}+\frac{y^2}{x^2}$$

Let y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$1+v+v^2=v+x\frac{dv}{dx}$$

$$1+v^2=x\frac{dv}{dx}$$

$$\frac{dx}{x}=\frac{dv}{v^2+1}$$

On integrating on both sides, we get

$$\log x=\tan^{-1}v+C$$

$$\tan^{-1}\frac{y}{x}=\log x+C$$

Conclusion: Therefore, $\tan^{-1}\frac{y}{x} = \log x + C$ is the solution of

$$x^2\frac{dy}{dx} = x^2 + xy + y^2$$

21. Question

Mark (√) against the correct answer in the following:

The general solution od the DE $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$ is

A. $\sin\left(\frac{y}{x}\right) = C$

B. $\sin\left(\frac{y}{x}\right) = Cx$

C. $\sin\left(\frac{y}{x}\right) = Cy$

D. None of these

Answer

Given DE: $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$ Now,Dividing both sides by x, we get, $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ Let y = vxDifferentiating both sides,dy/dx = v + xdv/dxNow, our differential equation becomes, $v + x\frac{dv}{dx} = v + \tan v$ On separating the variables, we get, $\frac{dv}{\tan v} = \frac{dx}{x}$ Integrating both sides, we get,sinv = CxPutting the value of v we get, $\sin\left(\frac{y}{x}\right) = Cx$ Hence, B is the correct answer.

22. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $2xy\ dy + (x^2 - y^2)\ dx = 0$ is

A. $x^2 + y^2 = Cx$

B. $x^2 + y^2 = Cy$

C. $x^2 + y^2 = C$

D. None of these

Answer

Given $2xy\,dy + (x^2 - y^2)dx = 0$

$$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$$

Let y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\frac{x^2v^2-x^2}{2vx^2}=v+x\frac{dv}{dx}$$

$$\frac{v^2-1}{2v}-v=x\frac{dv}{dx}$$

$$\frac{-v^2-1}{2v}=x\frac{dv}{dx}$$

$$\frac{dx}{x}+\frac{2vdv}{v^2+1}=0$$

On integrating on both sides, we get

$$\log x+\log(v^2+1)=c$$

$$\log(x(v^2+1))=c$$

$$x\left(\frac{y^2}{x^2}+1\right)=C$$

$$y^2+x^2=Cx$$

Conclusion: Therefore, $y^2+x^2=Cx$ is the solution of

$$2xy\,dy+(x^2-y^2)dx=0$$

23. Question

Mark (√) against the correct answer in the following:

The general solution of the DE (x - y) dy + (x + y) dx is

A. $\tan^{-1}\frac{y}{x}=C\sqrt{x^2+y^2}$

B. $\tan^{-1(y-x)}=C\sqrt{x^2+y^2}$

C. $\tan^{-1}\left(\frac{y}{x}\right)=x^2+y^2+C$

D. None of these

Answer

Given (x-y)dy + (x+y) dx = 0

$$\frac{dy}{dx} = \frac{x+y}{y-x}$$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx+x}{vx-x}$$

$$v + x\frac{dv}{dx} = \frac{v+1}{v-1}$$

$$x\frac{dv}{dx} = \frac{v+1-v^2+v}{v-1}$$

$$x\frac{dv}{dx} = \frac{2v+1-v^2}{v-1}$$

Question is wrong. I think subtraction should be there instead of addition in LHS(left hand side)

24. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$ is

A. $\tan\frac{y}{2x} = Cx$

B. $\tan\frac{y}{x}Cx$

C. $\tan\frac{y}{2x} = C$

D. None of these

Answer

Given $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$

Let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v + \sin v$$

$$x\frac{dv}{dx} = \sin v$$

$$\frac{dv}{\sin v} = \frac{dx}{x}$$

$$\log\tan\frac{v}{2} = \log x + C$$

$$\tan\frac{v}{2} = Cx$$

$$\tan\frac{y}{2x} = Cx$$

Conclusion: Therefore, $\tan\frac{y}{2x} = Cx$ is the solution of $\frac{dy}{dx} = v + x\frac{dv}{dx}$

25. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} + y\tan x = \sec x$ is

A. y = sin x – C cos x

B. y = sin x + C cos x

C. y = cos x – C sin x

D. None of these

Answer

Given $\frac{dy}{dx} + y\tan x = \sec x$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \tan x dx} = e^{\log\sec x} = \sec x$

General solution $y\sec x = \int(\sec x)(\sec x)\,dx + C$

$$y\sec x = \int \sec^2 x\,dx + C$$

y sec x = tan x + C

y = sin x + C cos x

Conclusion: Therefore, y = sin x + C cos x is the solution of $\frac{dy}{dx} + y\tan x = \sec x$

26. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} + y\cot x = 2\cos x$ is

A. $(y$ + sin $x)$sin x = C

B. $(y$ + cos x) sin x = C

C. $(y$ – sin x) sin x = C

D. None of these

Answer

Given $\frac{dy}{dx} + y\cot x = 2\cos x$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \cot x dx} = e^{\log \sin x} = \sin x$

General solution is $y\sin x = \int 2\cos x \sin x \, dx + C$

$$y\sin x = \int \sin 2x \, dx + C$$

$$y\sin x = -\frac{\cos 2x}{2} + C$$

$$y\sin x = \sin^2 x + C$$

(y-sin x)sin x = C

Conclusion: Therefore, (y-sin x)sin x = C is the solution of $\frac{dy}{dx} + y\cot x = 2\cos x$

27. Question

Mark (√) against the correct answer in the following:

The general solution of the DE $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

A. $xy = x^4 + C$

B. $4xy = x^4 + C$

C. $3xy = x^3 + C$

D. None of these

Answer

Given $\frac{dy}{dx} + \frac{y}{x} = x^2$

It is in the form $\frac{dy}{dx} + py = Qx$

Integrating factor $= e^{\int \frac{1}{x} dx} = e^{\log x} = x$

General solution is $yx = \int x^2 . x dx + C$

$$yx = \frac{x^4}{4} + C$$

Conclusion: Therefore, $yx = \frac{x^4}{4} + C$ is the solution of $\frac{dy}{dx} + \frac{y}{x} = x^2$