ST.ANNE'S MAT.HR.SEC SCHOOL,THATTANCHAVADI

Weekly test-1:JUNE2019

12th Standard 2019 EM

Date : 23-Jul-19

Maths

Reg.No. :

Time : 01:15:00 Hrs

Total Marks : 50

PART-A

5 x 1 = 5

1) If A $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ = $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A =

(a) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ **(c)** $\boldsymbol{\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}}$ (d) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

2) If A is a non-singular matrix such that A^{-1} = $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1}$ =

(a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ **(d)** $\boldsymbol{\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}}$

3) If A = $\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and AB = I , then B =

(a) $\left(\cos^2\frac{\theta}{2}\right)A$ **(b)** $\boldsymbol{\left(\cos^2\frac{\theta}{2}\right)A^T}$ (c) $\left(\cos^2\theta\right)I$ (d) $(\text{Sin}^2\frac{\theta}{2})$A

4) If A = $\begin{pmatrix} cosx & sinx \\ -sinx & cosx \end{pmatrix}$ and A(adj A) =λ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then λ is

(a) sinx cosx **(b) 1** (c) 2 (d) none

5) In a square matrix the minor M_{ij} and the' co-factor A_{ij} of and element a_{ij} are related by ______

(a) $A_{ij} = -M_{ij}$ (b) $A_{ij} = M_{ij}$ **(c)** $\boldsymbol{A_{ij} = (-1)^{i+j} M_{ij}}$ (d) $A_{ij} = (-1)^{i-j} M_{ij}$

PART-B

7 x 2 = 14

6) If A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations x + y + 2z = 1, 3x + 2y + z = 7, 2x + y + 3z = 2.

Given A= $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$, B= $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

AB=

AB=
$$\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -5+3+6 & -5+2+3 & -10+1+9 \\ 7+3-10 & 7+2-3 & 14+1-15 \\ 1-3+2 & 1-2+1 & 2-1+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4.\ I_3$$

BA=
$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5+7+2 & 1+1-2 & 3-5+2 \\ -15+14+1 & 3+2-1 & 9-10+1 \\ -10+7+3 & 2+1-3 & 6-5+3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 4.\ I_3.$$

So, we get AB=BA=4. I_3

$$\Rightarrow \left(\frac{1}{4}A\right)B = B\left(\frac{1}{4}A\right) = 1$$

$\Rightarrow B^{-1} = \frac{1}{4} = 1$

Writing the given set of equations in matrix form we get,

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = \left[\frac{1}{4}A\right]\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} -5+7+6 \\ 7+7-10 \\ 1-7+2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$\therefore$ x=2, y=1, z=-1

Hence. the solution set is {2. 1. - l}.

7) 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

Let the time by one man alone be x days and one woman alone be y days

∴ By the given data,

$\frac{4}{x} + \frac{4}{y} = \frac{1}{3}$

and $\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$

put $\frac{1}{x}$=s and $\frac{1}{y}$=t

∴ 4s+4t=$\frac{1}{3}$

and 2x+5t=$\frac{1}{4}$

The matrix form of the system of equation is

⇒ AX=B where

$$\begin{bmatrix} 4 & 4 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{4} \end{bmatrix}$$

A= $\begin{bmatrix} 4 & 4 \\ 2 & 5 \end{bmatrix}$ and B= $\begin{bmatrix} \frac{1}{3} \\ \frac{2}{4} \end{bmatrix}$

X=A^{-1}B

Now |A|= $\begin{vmatrix} 4 & 4 \\ 2 & 5 \end{vmatrix}$ =20-8 =12≠0

∴ A^{-1} = $\frac{1}{|A|} adjA = \frac{1}{12}\begin{bmatrix} 5 & -4 \\ -2 & 4 \end{bmatrix}$

∴ X=A^{-1} B= $\frac{1}{12}\begin{bmatrix} 5 & -4 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} \frac{1}{3} \\ \frac{1}{4} \end{bmatrix}$

= $\frac{1}{12}\begin{bmatrix} \frac{5}{3} & -1 \\ \frac{-2}{3} & +1 \end{bmatrix}$

= $\frac{1}{12}\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \times \frac{1}{12} \\ \frac{1}{3} \times \frac{1}{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{18} \\ \frac{1}{36} \end{bmatrix}$

∴ $\frac{1}{18} \Rightarrow \frac{1}{x} = \frac{1}{18} \Rightarrow$ x=18

t=$\frac{1}{36} \Rightarrow \frac{1}{y} = \frac{1}{36} \Rightarrow$ y=36

8) In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

Let x represent the number of question with correct answer and y represent the number of questions with wrong answers.

By the given data, x + y 100 and ... (1)

$1.x-\frac{1}{4}y=80$

Multiplying by 4 we get we get

4x-y=320 ....(2)

From (1) and (2)

$\Delta=\begin{vmatrix} 1 & 1 \\ 4 & -1 \end{vmatrix}=-1-4=-5$

$\Delta_1=\begin{vmatrix} 100 & 1 \\ 320 & -1 \end{vmatrix}=-100-320=-420$

$\Delta_2=\begin{vmatrix} 1 & 100 \\ 4 & 320 \end{vmatrix}=320-400=-80$

$\therefore x=\frac{\triangle_1}{\triangle}=\frac{-720}{-5}=+84$

and $y=\frac{\triangle_2}{\triangle}=\frac{-80}{-5}=16$

Hence, the number of questions with correct answer is 84.

9) A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is Rs.150. The cost of the two dosai, two idlies and four vadais is Rs.200. The cost of five dosai, four idlies and two vadais is Rs.250. The family has Rs.350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

Let the cost of one dosa be Rs.x

The cost of one idli be Rs.y

and the cost of one vadai be Rs.z

By the given data,

2x+ 3y+ 2z =150

2x + 2y+ 4z=200

5x+4y+ 2z=250

$\therefore \Delta=\begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix}$

$=2\begin{vmatrix} 2 & 4 \\ 4 & 2 \end{vmatrix}-3\begin{vmatrix} 2 & 4 \\ 5 & 2 \end{vmatrix}+2\begin{vmatrix} 2 & 2 \\ 5 & 4 \end{vmatrix}$

=2(4 - 16) - 3(4 - 20) + 2(8 - 10)

= 2(- 12) - 3(- 16) + 2(- 2)

= - 24 + 48 - 4 = 20

$\Delta_1=\begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 250 & 4 & 2 \end{vmatrix}$

Taking 50 common from C3 we get,

=100

$$=100 \begin{vmatrix} 3 & 3 & 1 \\ 4 & 2 & 2 \\ 5 & 4 & 1 \end{vmatrix}$$

$$= 100\left[3 \begin{vmatrix} 2 & 2 \\ 4 & 1 \end{vmatrix} - 3 \begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 2 \\ 5 & 4 \end{vmatrix}\right]$$

= 100[3(2 - 8) - 3(4 - 10) + 1(16 - 10)]

= 100[3(-6) - 3(- 6) + 6]

= 100[- 18 + 18 + 6] = 600

$$\Delta_2 = \begin{vmatrix} 2 & 150 & 2 \\ 2 & 200 & 4 \\ 5 & 250 & 2 \end{vmatrix} = 100 \begin{vmatrix} 2 & 3 & 1 \\ 2 & 4 & 2 \\ 5 & 5 & 1 \end{vmatrix}$$

$$= 100\left[3 \begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix} - 3 \begin{vmatrix} 2 & 2 \\ 5 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 4 \\ 5 & 5 \end{vmatrix}\right]$$

= 100[2(4 - 10) - 3(2 - 10) + 1(10 - 20)]

= 100[2(- 6) - 3(- 8) + 1(- 10)]

= 100[- 12 + 24 - 10] = 100 [2] = 200.

$$\Delta_3 = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 5 & 4 & 250 \end{vmatrix} = 50 \begin{vmatrix} 2 & 3 & 3 \\ 2 & 2 & 4 \\ 5 & 4 & 5 \end{vmatrix}$$

$$= 50\left[2 \begin{vmatrix} 2 & 4 \\ 4 & 5 \end{vmatrix} - 3 \begin{vmatrix} 2 & 4 \\ 5 & 5 \end{vmatrix} + 3 \begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix}\right]$$

= 50 [2(10 - 16) - 3(10 - 20) + 3(8 - 10)]

= 50[2(- 6) - 3(- 10) +3(- 2)]

= 50 [- 12 + 30 - 6] = 50 [12] = 600.

$\therefore x=\frac{\triangle_1}{\triangle} = \frac{600}{20}=30$

$y=\frac{\triangle_2}{\triangle} = \frac{200}{20}=10$

$z=\frac{\triangle_3}{\triangle} = \frac{600}{20}=30$

Hence, the price of one dosa be Rs.30,one idli be Rs.10 and the price of 1 vadai be Rs.30.

Also the cost on dosa, six idlies and six vadai is

= 3x + 6y + 6z = 3(30) + 6(10) + 6(30)

= 90 + 60 + 180 = Rs.330

Since the family had Rs.350 in hand, they will be able to manage to pay the bill.

10) If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$, and $x - 1$, the remainders are 21,61 and 9 respectively. Find a,b and c. (Use Gaussian elimination method.)

Let $P(x) = ax^2+bx+c$

Given P(-3) = 21

[$\because$ P(x) ÷x+3, the remainder is 21]

⇒ a(-3)2+b(-3)+c=21

⇒ 9a-3b+c=21

Also, P(5)=61

⇒ a(5)2+b(5)+c=61

[using remainder theorem]

$\Rightarrow$ 25x+5b+c=61 ....(2)

and P(1)=9

$\Rightarrow$ a(1)2+b(1)+c=9

$\Rightarrow$ a+b+c=9 ....(3)

Reducing the augment matrix to an equivalent row-echelon form using elementary row operations, we get

$$\left[\begin{array}{ccc|c} 9 & - & 1 & 21 \\ 25 & 5 & 1 & 61 \\ -1 & 1 & 1 & 9 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 25 & 5 & 1 & 61 \\ 9 & -3 & 1 & 21 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 9R_1 R_2 \to R_2 - 25R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -20 & -24 & -164 \\ 0 & -12 & -8 & -60 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 \div R_3 \to R_3 \div 4} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & -6 & -41 \\ 0 & -3 & -2 & -15 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - \frac{3}{5}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & -6 & -41 \\ 0 & 0 & \frac{8}{5} & \frac{48}{5} \end{array}\right]$$

$$\xrightarrow{R_3 \to 5R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & -6 & -41 \\ 0 & 0 & 8 & 48 \end{array}\right]$$

Writing the equivalent equations from the row-echelon matrix we get,

a+b+c = 9 ....(1)

- 5b-6c = -41 .....(2)

8c=48

$\Rightarrow$ c=$\frac{48}{8}$=6

Substituting c=6 in (2) we get,

$\Rightarrow$ -5b-6(6)=-41

$\Rightarrow$ -5b=-41+36=-5

$\Rightarrow$ -5b=-41+36=-5

$\Rightarrow$ b=$\frac{-5}{-5}$=1

Substituting b = 1, c = 6 in (1) we get,

a+1+6=9

$\Rightarrow$ a+7=9

$\Rightarrow$ a=9-7

$\Rightarrow$ a=2

$\therefore$ a=2, b=1, and c=6

11) An amount of Rs.65,000 is invested in three bonds at the rates of 6%,8% and 9% per annum respectively. The total annual income is Rs.4,800. The income from the third bond is Rs.600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

Let the price of bond invested in 6%, 8% and 9% rates be let Rs.x, Rs.y and Rs.z respectively

$\therefore$ By the given data, x + y + z = 65000 ....(1)

$\frac{6\times x\times 1}{100}+\frac{8\times y\times 1}{100}+\frac{9\times z\times 1}{100}$=4800

[$\because$ Intrest=$\frac{PNR}{100}$]

$\Rightarrow \frac{6x}{100}+\frac{8y}{100}+\frac{9z}{100}$=4800

$\Rightarrow$ 6x+8y+9z=480000 ......(2)

Also, $\frac{9z}{100}=600+\frac{8y}{100}$

$\Rightarrow \frac{-8y}{100}+\frac{9y}{100}$=600

$\Rightarrow$ -8y+9z=60000 ....(3)

Reducing the augmented matrix to an equivalent row-echelon form by using elementary row operation, we get

$$\left[\begin{array}{ccc|c}1 & 1 & 1 & 65000\\ 6 & 8 & 9 & 480000\\ 0 & -8 & 9 & 60000\end{array}\right]$$

$$\xrightarrow{R_2\to R_2-6R_1}\left[\begin{array}{ccc|c}1 & 1 & 1 & 65000\\ 0 & 2 & 3 & 90000\\ 0 & -8 & 9 & 60000\end{array}\right]$$

$$\xrightarrow{R_3\to R_3+4R_2}\left[\begin{array}{ccc|c}1 & 1 & 1 & 65000\\ 0 & 2 & 3 & 90000\\ 0 & 0 & 21 & 420000\end{array}\right]$$

Writing the equivalent from the row echelon matrix we get,

x+y+z=65000 ....(1)

2y+z=90000 .....(2)

21z=42000

$\Rightarrow$ z=$\frac{420000}{21}$=20000

Substituting z = 20,000 in (2),

2y + 3(20,000) = -90000

$\Rightarrow$ 2y+60,000 =90,000

$\Rightarrow$ 2y=90,000-60,000

=30,000

$\Rightarrow$ y=$\frac{30,000}{2}$=15,000

Substitutingy = 15,000 and z = 20,000 in (1) we get,

x+15,000+20,000=65000

$\Rightarrow$ x+35,000=65000

$\Rightarrow$ x=65,000-35,000

$\Rightarrow$ 30,000

Thus the price of 6% bond is f 30,000 the price of 8% bond is f 15,000 and the price of 9% bond is f 20,000 is Rs.20,000.

12) A boy is walking along the path y = ax^2 + bx + c through the points (−6, 8),(−2, −12) , and (3, 8) . He wants to meet his friend at P(7,60) . Will he meet his friend? (Use Gaussian elimination method.)

Giveny = ax^2 + bx + c ...(1)

(-6,8) lies on (1)

$\Rightarrow 8=a(-6)^2+b(-6)+c$

⇒ 8=36a-6b+c ....(2)

(-2,12) lies on (1)

⇒ -12=a(-2)^2+b(-2)+c

⇒ -12=4a-2b+c ......(3)

Also (3,8) lies on (1)

⇒ 8=a(3)^2+b(3)+c

⇒ 8=9a+3b+c ....(4)

Reducing the augment matrix to an equivalent row-echelon form by using elementary. row operations, we get,

$$\left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 4 & -2 & 1 & -12 \\ 0 & 3 & 1 & 8 \end{array}\right] \xrightarrow{R_2 \to 9R_2 - R_1 R_3 \to 4R_3 - R_1} \left[\begin{array}{ccc|c} 36 & -6 & 1 & 8 \\ 0 & -12 & 8 & -116 \\ 0 & 18 & 3 & 24 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 \div 4 R_3 \to R_3 \div 3} \left[\begin{array}{ccc|c} 36 & -6 & 1 & -8 \\ 0 & -3 & 2 & -29 \\ 0 & 0 & 5 & -8 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 2R_2} \left[\begin{array}{ccc|c} 36 & -6 & 1 & -8 \\ 0 & -3 & 2 & -29 \\ 0 & 0 & 5 & -50 \end{array}\right]$$

Writing the equivalent equation from the row echelon matrix, we get 36a - 6b + c = 8 ....(1)

-3b+2c=-29 ....(2)

5c=-50

⇒ c=$\frac{-50}{5}$=-10

Substituting c = -10 in (2) we get,

-3b+2(-10)=-29

⇒ -3b+2(-10)=-29

⇒ -3b-20=-29

⇒ -3b=-9

⇒ b=$\frac{-9}{-3}$=3

Substituting b = 3 and c = -10 in (1) we get,

36a-6(3)-10=8

⇒ 36a-18-10=8

⇒ 36a-28=8

⇒ 6a=8+28=36

⇒ a=$\frac{36}{36}$=1

∴ a=1, b=3, c=-10

Hence the path of the boy is

y=1(x^2)+3(x)-10

⇒ y=x^2+3x-10

Since his friend is at P(7, 60),

60=(7)^2+3(7)-10

⇒ 60=49+21-10

⇒ 60=70-10=60

⇒ 60=60

Since (7, 60) satisfies his path, he can meet his friend who is at P(7, 60)

PART-C

3 x 3 = 9

13) Solve the following system of equations, using matrix inversion method:

$2x_1 + 3x_2 + 3x_3 = 5$, $x_1 - 2x_2 + x_3 = -4$, $3x_1 - x_2 - 2x_3 = 3$.

The matrix form of the system is AX = B, where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}.$$

We find $|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix} = 2(4 + 1) - 3(-2 - 3) + 3(-1 + 6) = 10 + 15 + 15 = 40 \neq 0$.

So, A^{-1} exists and

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{40}\begin{bmatrix} +(4+1) & -(-2-3) & +(-1+6) \\ -(-6+3) & +(-4-9) & -(-2-9) \\ +(3+6) & -(2-3) & +(-4-3) \end{bmatrix}^T = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

Then, applying $X = A^{-1}B$, we get

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

So, the solution is ($x_1 = 1$, $x_2 = 2$, $x_3 = -1$).

14) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the productsAB and BAand hence solve the system of equations $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$.

We find AB = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3$

and BA = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} -4+7+5 & 4-1-3 & 4-3-1 \\ -4+14-10 & 4-2+6 & 4-6+2 \\ -8-7+15 & 8+1-9 & 8+3-3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3$

So we get AB = BA = $8I_3$. That is, $(\frac{1}{8}A)B = B(\frac{1}{8}A) = I_3$. Hence, $B^{-1} = \frac{1}{8}A$.

Writing the given system of equations in matrix form, we get

$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$. That is B $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$.

So, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \left(\frac{1}{8}A\right)\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$

Hence, the solution is (x = 3, y = -2, z = -1).

15) Solve the following system of linear equations, by Gaussian elimination method :
4x + 3y + 6z = 25, x + 5y + 7z = 13, 2x + 9y + z = 1.

Transforming the augmented matrix to echelon form, we get

$$\left[\begin{array}{ccc|c} 4 & 3 & 6 & 25 \\ 1 & 5 & 7 & 13 \\ 2 & 9 & 1 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 4 & 3 & 6 & 25 \\ 2 & 9 & 1 & 1 \end{array}\right] \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 4R_1} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -27 \\ 0 & -1 & -13 & -25 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 \div (-1)]{R_2 \to R_2 \div (-1)} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & 17 & 22 & 27 \\ 0 & 1 & 13 & 25 \end{array}\right] \xrightarrow{R_3 \to 17R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & 17 & 22 & 27 \\ 0 & 0 & 199 & 398 \end{array}\right].$$

The equivalent system is written by using the echelon form:

x + 5y + 7 = 13, ... (1)

17y + 22z = 27, ... (2)

199z = 398. ...... (3)

From (3), we get $z = \frac{398}{199} = 2$.

Substituting z = 2 in (2), we get $y = \frac{27 - 22 \times 2}{17} = \frac{-17}{17} = -1$

Substituting z = 2, y = -1, in (1), we get x = 13 - 5 x (-1) - 7 x 2 = 4.

So, the solution is (x = 4, y = -1, z = 2).

PART-D

3 x 5 = 15

16) In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (30, 18) can you conclude that Chennai Super Kings won the match?
Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)

The path $y = ax^2 + bx + c$ passes through the points (10, 8), (20, 16), (40, 22). So, we get the system of equations $100a + 10b + c = 8$, $400a + 20b + c = 16$, $1600a + 40b + c = 22$. To apply Cramer's rule, we find

$$\Delta = \begin{vmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 1600 & 40 & 1 \end{vmatrix} = 1000 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 16 & 4 & 1 \end{vmatrix} = 1000\,[-2 + 12 - 6] = -6000,$$

$$\Delta_1 = \begin{vmatrix} 8 & 10 & 1 \\ 16 & 20 & 1 \\ 22 & 40 & 1 \end{vmatrix} = 20 \begin{vmatrix} 4 & 1 & 1 \\ 8 & 2 & 1 \\ 11 & 4 & 1 \end{vmatrix} = 20[-8 + 3 + 10] = 100,$$

$$\Delta_2 = \begin{vmatrix} 100 & 8 & 1 \\ 400 & 16 & 1 \\ 1600 & 22 & 1 \end{vmatrix} = 200 \begin{vmatrix} 1 & 4 & 1 \\ 4 & 8 & 1 \\ 16 & 11 & 1 \end{vmatrix} = 200[-3 + 48 - 84] = -7800,$$

$$\Delta_3 = \begin{vmatrix} 100 & 10 & 8 \\ 400 & 20 & 16 \\ 1600 & 40 & 22 \end{vmatrix} = 2000 \begin{vmatrix} 1 & 1 & 4 \\ 4 & 2 & 8 \\ 16 & 4 & 11 \end{vmatrix} = 2000[-10 + 84 - 64] = 20000.$$

By Cramer's rule, we get $a = \frac{\Delta_1}{\Delta} = -\frac{1}{60}$, $b = \frac{\Delta_2}{\Delta} = \frac{7800}{6000} = \frac{78}{60} = \frac{13}{10}$, $c = \frac{\Delta_3}{\Delta} = \frac{20000}{6000} = -\frac{20}{6} = -\frac{10}{3}$.

So, the equation of the path is $y = \frac{1}{60}x^2 + \frac{13}{10}x - \frac{10}{3}$.

When x = 70, we get y = 6. So, the ball went by 6 metres high over the boundary line and it is impossible for a fielder standing even just before the boundary line to jump and catch the ball. Hence the ball went for a super six and the Chennai Super Kings won the match.

17) Using determinants; find the quadratic defined by $f(x) = ax^2 + bx + c$, if $f(1) = 0$, $f(2) = -2$ and $f(3) = -6$.

Given $f(x) = ax^2 + bx + c$

$f(1) = 0$

$\Rightarrow a(1)^2 + b(1) + c = 0$

$\Rightarrow a + b + c = 0$ ....(1)

$f(2) = -2$

$\Rightarrow a(2^2) + b(2) + c = -2$

$\Rightarrow 4a + 2b + c = -2$ ....(2)

$f(3) = -6$

$\Rightarrow a(3)^2 + b(3) + c = -6$

$\Rightarrow 9a + 3b + c = -6$ ....(3)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 1 \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} - 1 \begin{vmatrix} 4 & 1 \\ 9 & 1 \end{vmatrix} + 1 \begin{vmatrix} 4 & 2 \\ 9 & 3 \end{vmatrix}$$

$= 1\,(2 - 3) - 1\,(4 - 9) + 1\,(12 - 18)$

$= -1 + 5 - 6 = -2 \neq 0$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 1 \\ -2 & 2 & 1 \\ -6 & 3 & 1 \end{vmatrix} = 0 - 1 \begin{vmatrix} -2 & 1 \\ -6 & 1 \end{vmatrix} + 1 \begin{vmatrix} -2 & 2 \\ -6 & 1 \end{vmatrix}$$

$= -1\,(-2 + 6) + 1(-6 + 12) = -1\,(4) + 1\,(6) = 2$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 1 \\ 4 & -2 & 1 \\ 9 & -6 & 1 \end{vmatrix} = 1 \begin{vmatrix} -2 & 1 \\ -6 & 1 \end{vmatrix} + 0 + 1 \begin{vmatrix} 4 & -2 \\ 9 & -6 \end{vmatrix}$$

$-1\,(-2 + 6) + 1(-24 + 18) = 4 - 6 = -2$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 0 \\ 4 & 2 & -2 \\ 9 & 3 & -6 \end{vmatrix} = 1 \begin{vmatrix} 2 & -2 \\ 3 & -6 \end{vmatrix} - 1 \begin{vmatrix} 4 & -2 \\ 9 & -6 \end{vmatrix}$$

$= 1\,(-12 + 6) - 1\,(-24 + 18) = -6 + 6 = 0$

$a = \frac{\triangle_1}{\triangle} = \frac{2}{-2} = -1$

$b = \frac{\triangle_2}{\triangle} = \frac{-2}{-2} = 1$

$c = \frac{\triangle_3}{\triangle} = \frac{0}{-2}$

$\therefore f(x) = (-1)x^2 + 1x + 0$

$\Rightarrow f\{x) = x^2 + x.$

18) The sum of three numbers is 20. If we multiply the third number by 2 and add the first number to the result we get 23. By adding second and third numbers to 3 times the first number we get 46. Find the numbers using Cramer's rule.

Let the required numbers be x, y and z

By the given data,

$x + y + z = 20$ ....(1)

$2z + x = 23 \Rightarrow x + 2z = 23$ ....(2)

$y + z + 3x = 46 \Rightarrow 3x + y + z = 46$ ....(3)

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$= -2 + 5 + 1 = 4$

$$\Delta_1 = \begin{vmatrix} 20 & 1 & 1 \\ 23 & 0 & 2 \\ 46 & 1 & 1 \end{vmatrix} = 20\begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 23 & 2 \\ 46 & 1 \end{vmatrix} + 1\begin{vmatrix} 23 & 0 \\ 46 & 1 \end{vmatrix}$$

$= -40 + 69 + 23 = 52$

$$\Delta_2 = \begin{vmatrix} 1 & 20 & 1 \\ 1 & 23 & 2 \\ 3 & 46 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 23 & 2 \\ 46 & 1 \end{vmatrix} - 20\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & 23 \\ 3 & 46 \end{vmatrix}$$

$= -69 + 100 - 23 = 8$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 20 \\ 1 & 2 & 23 \\ 3 & 1 & 46 \end{vmatrix}$$

$$= 1\begin{vmatrix} 0 & 23 \\ 1 & 46 \end{vmatrix} - 1\begin{vmatrix} 1 & 23 \\ 3 & 46 \end{vmatrix} + 20\begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$$

$= -23 + 23 + 20 = 20$

$\therefore x = \frac{\triangle_1}{\triangle} = \frac{52}{4} = 13$

$y = \frac{\triangle_2}{\triangle} = \frac{8}{4} = 2$ and $z = \frac{\triangle_3}{\triangle} = \frac{20}{4} = 5$

Hence the required numbers are 13, 2 and 5.

3 x 1 = 3

19) |adj (adj A)|

$|A|^{n-2}.A$

20) $(\text{adj } A)^{-1}$

$\text{adj}(A^{-1})$

21) $(\lambda A)^{-1}$

$\frac{1}{\lambda}A^{-1}$

2 x 2 = 4

22) If A is symmetric then

(1) $A^T = A$

(2) adj A is symmetric

(3) adj (A^T)= (adj A)^T

(4) A is orthogonal

A is orthogonal

23) If A is a non-singular matrix of odd order them

1) Order of A is 2m + 1

(2) Order of A is 2m + 2

(3) |adj AI is positive

(4) IAI ≠ 0

Order of A is 2m + 2