ST.ANNE'S MAT.HR.SEC.SCHOOL, THATTANCHAVADI

slip test

12th Standard 2019 EM

Date : 23-Jul-19

MATHEMATICS

Reg.No. : | | | | | |

HARDWORK NEVER FAILS...

Time : 00:45:00 Hrs

Total Marks : 30

PART-A

3 x 2 = 6

1) Find the sum of squares of roots of the equation $2x^4-8x+6x^2-3=0$.

Given equation is $2x^4-8x+6x^2-3=0$

Here a=2, b =-8, c= 6, d= 0, e=-3

Let $\propto$, β,ɤ yand δ be the roots of eqn (1)

Then by Vieta's formula,

$\sum_1 = \alpha + \beta + \gamma + \delta = \frac{-b}{a} = \frac{-(-8)}{2} = 4$

$\sum_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a} = \frac{6}{2} = 3$

$\sum_3 = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \frac{-d}{a} = \frac{0}{a}$

$\sum_4 = \alpha\beta\gamma\delta = \frac{e}{a} = \frac{-3}{2}$

Now, $(a+b+c+d)^2=a^2+b^2+c^2+d^2+2(ab+ac+ad+bc+cd)$

$\Rightarrow n\propto^2+\beta^2+\gamma^2+\delta^2=(\propto+\beta+\gamma+\delta)^2-2($

$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$

$\propto^2+\beta^2+\gamma^2=4^2-2(3)=16-6=10$

2) If the equations $x^2+px+q= 0$ and $x^2+p'x+q'= 0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$.

Given equation are $x^2px + q = 0 \quad ... \quad (1)$

and $x^2 + p'x + q' = 0$ ...(2)

Let ∝ be the common root for (1) and (2)

∴ $\propto^2$+p∝+q=0 ...(3)

and $\propto^2$+p'∝+q'=0 ...(4)

Solving (3) and (4) by cross multiplication method we get

p q 1 p

p' q' 1 p'

$\Rightarrow \frac{\alpha^2}{pq' - p'q} = \frac{\alpha}{q - q'} = \frac{1}{p' - p}$

consider $\frac{\alpha^2}{pq' - p'q} = \frac{\alpha}{q - q'}$

$\Rightarrow \frac{\alpha^2}{\alpha} = \frac{pq' - p'q}{q - q'}$

$\alpha = \frac{pq' - p'q}{q - q}$

Consider $\frac{\alpha}{q - q'} = \frac{1}{p' - p} \Rightarrow \alpha = \frac{q - q'}{p' - p}$

Hence its roots are $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$

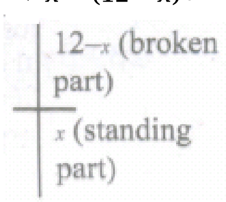
3) A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.

Given that the height of the tree is 12m.

Let x m be the standing part and (12 -x)m be the broken part.

Given $x = \sqrt[3]{12 - x}$

$\Rightarrow x = (12 - x)^{\frac{1}{3}}$



Taking power 3 both sides, we get

⇒ x^3=12-x

⇒ x^3+x-12=0 which is the required mathematical problem.

PART-B 3 x 3 = 9

4) If α and β are the roots of the quadratic equation $17x^2+43x-73 = 0$, construct a quadratic equation whose roots are α + 2 and β + 2.

Since α and β are the roots of $17x^2+43x-73 = 0$, we have $\alpha + \beta = \frac{-43}{17}$ and $\alpha\beta = \frac{-73}{17}$.

We wish to construct a quadratic equation with roots are α + 2 and β + 2 .Thus, to construct such a quadratic equation, calculate,

the sum of the roots = $\alpha + \beta + 4 = \frac{-4}{17} + 4 = \frac{25}{17}$ and

the product of the roots $= \alpha\beta + 2(\alpha+\beta) + 4 = \frac{-73}{17} + 2\left(\frac{-43}{17}\right) + 4 = \frac{-91}{17}$

Hence a quadratic equation with required roots is $x^2 - \frac{25}{17}x - \frac{91}{17} = 0$

Multiplying this equation by 17, gives $17x^2-25x-91 = 0$

which is also a quadratic equation having roots α + 2 and β + 2

5) If α and β are the roots of the quadratic equation $2x^2-7x+13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

Since α and β are the roots of the quadratic equation, we have $\alpha+\beta = \frac{7}{2}$ and $\alpha\beta = \frac{13}{2}$.

Thus, to construct a new quadratic equation,

Sum of the roots $= \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{-3}{4}$

Product of the roots $= \alpha^2\beta^2 = (\alpha\beta)^2 = \frac{169}{4}$

Thus a required quadratic equation is $x^2 + \frac{3}{4}x + \frac{169}{4} = 0$. From this we see that $4x^2+3x+169=0$ is a quadratic equation with roots α^2 and β^2.

6) If α, β, and γ are the roots of the equatio $x^3+pz^2+qx+r=0$, find the value of $\Sigma\frac{1}{\beta\gamma}$ in terms of the coefficients.

Since α, β, and γ are the roots of the equation $x^3+px^2+qx+r=0$, we have

$\Sigma_1\ \alpha + \beta + \gamma = -p$ amd $\Sigma_3\ \alpha\beta\gamma = -r$

$\Sigma\frac{1}{\beta\gamma} = \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} + \frac{1}{\alpha\beta} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{-p}{-r} = \frac{p}{r}.$

PART-C

3 x 5 = 15

7) Find the monic polynomial equation of minimum degree with real coefficients having $2-\sqrt{3}i$ as a root.

Since $2-\sqrt{3}i$ is a root of the required polynomial equation with real coefficients, $2+\sqrt{3}i$ is also a root. Hence the sum of the roots is 4and the product of the roots is 7. Thus $x^2-4x+7=0$ is the required monic polynomial equation.

8) Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

Since $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a root, x- $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a factor. To remove the outermost square root, we take x+ $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as another factor and find their product.

$$\left(x+\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(x-\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)=x^2-\frac{\sqrt{2}}{\sqrt{3}}$$

Still we didn't achieve our goal. So we include another factor $x_2+\frac{\sqrt{2}}{\sqrt{3}}$ and get the product.

$$\left(x^2-\frac{\sqrt{2}}{\sqrt{3}}\right)\left(x^2+\frac{\sqrt{2}}{\sqrt{3}}\right)=x^4-\frac{2}{3}$$

So, $3x^4-2=0$ is a required polynomial equation with the integer coefficients.

Now we identify the nature of roots of the given equation without solving the equation. The idea comes from the negativity, equality to 0, positivity of $\Delta=b^2-4ac$.

9) Prove that a line cannot intersect a circle at more than two points.

By choosing the coordinate axes suitably, we take the equation of the circle as $x^2+y^2=r^2$ and the equation of the straight line as $y=mx+c$. We know that the points of intersections of the circle and the straight line are the points which satisfy the simultaneous equation

$x^2+y^2=r^2$

$y = mx + c$ ... (2)

If we substitute mx + c for y in (1), we get

$x^2+(mx+c)^2-r^2=0$

which is same as the quadratic equation

$(1+m^2)x^2+2mcx+(c^2-r^2)=0$ ....(3)

This equation cannot have more than two solutions, and hence a line and a circle cannot intersect at more than two points. It is interesting to note that a substitution makes the problem of solving a system of two equations in two variables into a problem of solving a quadratic equation. Further we note that as the coefficients of the reduced quadratic polynomial are real, either both roots are real or both imaginary. If both roots are imaginary numbers, we conclude that the circle and the straight line do not intersect. In the case of real roots, either they are distinct or multiple roots of the polynomial. If they are distinct, substituting in (2), we get two values for y and hence two points of intersection. If we have equal roots, we say the straight line touches the circle as a tangent. As the polynomial (3) cannot have only one simple real root, a line cannot cut a circle at only one point.