ST.ANNE'S MAT.HR.SEC.SCHOOL, THATTANCHAVADI

Slip Test Unit 3 (A2)

12th Standard 2019 EM

Date : 23-Jul-19

MATHEMATICS

Reg.No. : | | | | | |

HARDWORK NEVER FAILS...

Time : 00:45:00 Hrs

Total Marks : 30

PART-A

9 x 2 = 18

1) Find the sum of squares of roots of the equation $2x^4-8x+6x^2-3=0$.

Given equation is $2x^4-8x+6x^2-3=0$

Here a=2, b =-8, c= 6, d= 0, e=-3

Let $\propto$, β,γ yand δ be the roots of eqn (1)

Then by Vieta's formula,

$\sum_1 = \alpha + \beta + \gamma + \delta = \frac{-b}{a} = \frac{-(-8)}{2} = 4$

$\sum_2 = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a} = \frac{6}{2} = 3$

$\sum_3 = \alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = \frac{-d}{a} = \frac{0}{a}$

$\sum_4 = \alpha\beta\gamma\delta = \frac{e}{a} = \frac{-3}{2}$

Now, $(a+b+c+d)^2=a^2+b^2+c^2+d^2+2(ab+ac+ad+bc+cd)$

$\Rightarrow n\propto^2+\beta^2+\gamma^2+\delta^2=(\propto+\beta+\gamma+\delta)^2-2($

$\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$

$\propto^2+\beta^2+\gamma^2=4^2-2(3)=16-6=10$

2) If α, β, γ and δ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8=0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

Given polynomial equation is

$2x^4+5x^3-7x^2+8=0$

Here a=2, b=5,c=-7,d=0,e=8

By vieta's formula,

$\alpha+\beta+\gamma+\delta = \frac{-b}{a} = \frac{-5}{2}$

$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = \frac{c}{a} = \frac{-7}{2}$

$\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta = \frac{-d}{a} = 0$

$\alpha\beta\gamma\delta = \frac{e}{a} = \frac{8}{2} = 4$

Given roots of quadratic equation are

$\propto$+β+ɤ+δ and $\propto$βɤδ

∴ sum of the roots=($\propto$+β+ɤ+δ)($\propto$βɤδ)

$= \left(\frac{-5}{2}+4\right) = \frac{-5+8}{2} = \frac{3}{2}$

$= (\alpha+\beta+\gamma+\delta)(\alpha\beta\gamma\delta)$

$= \left(\frac{-5}{2}\right)(4) = \frac{-20}{2} = -10$

∴ The sum required quadrate equation is x^2-x

(sum of the roots)+product of the roots=0

$\Rightarrow x^2 - x\left(\frac{3}{2}\right) - 10 = 0$

$\Rightarrow 2x^2 - 3x - 20 = 0$

3) Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

Given $(\sqrt{5}-\sqrt{3})$ is a root

$\Rightarrow \sqrt{5}+\sqrt{3}$

∴ Sum of the roots $= \sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3} = 2\sqrt{5}$

Product of the roots

$= (\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$

$= (\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$

∴ One of the factor is X^2 -x (sum of the roots) + product of the roots

$\Rightarrow x^2 - 2x\sqrt{5} + 2$

The other factor also will be $x^2 - 2x\sqrt{5} + 2$

$(x^2 - 2x\sqrt{5} + 2)(x^2 + 2x\sqrt{5} + 2) = 0$

$\Rightarrow (x^2 + 2 - 2\sqrt{5}x)(x^2 + 2 + 2\sqrt{5}x) = 0$

$\Rightarrow \left(x^2+2\right)^2 - (2\sqrt{5}x)^2 = 0$

$[\because (a+b)(a-b) = a^2 - b^2]$

$\Rightarrow x^4 + 4x^2 + 4(5)x^2 = 0$

$\Rightarrow x^4 + 4x^2 + 4 - 20x^2 = 0$

$\Rightarrow x^4 - 16x^2 + 4 = 0$

4) Solve: (2x-1)(x+3)(x-2)(2x+3)+20=0

Rearrange the terms as

$(2x-1)(2x+3)(x+3)(x-2)+20=0$

$\Rightarrow (4x^2+6x-2x-3)(x^2-2x+3x-6)+20=0$

$\Rightarrow (4x^2+4x-3)(x^2+X-6)+20=0$

put $x^2+x=y$

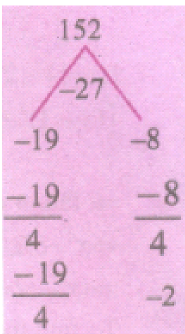
$\Rightarrow (4y-3)(y-6)+20=0$

$\Rightarrow 4y^2-24y-3y+18+20=0$

$\Rightarrow 4y^2-27y+38=0$

$\Rightarrow (y-2)(4y-19)=0$

$y = 2, \frac{19}{4}$



Case (i)

When y=2

$x^2+x=2$

$x^2+x-2.=0$

$\Rightarrow (x+2)(x-1)=0$

$\Rightarrow x=-2,1$

Case (ii)

When $y = \frac{19}{4}, x^2 + x = \frac{19}{4}$

$\Rightarrow 4x^2 + 4x = 19$

$\Rightarrow 4x^2 - 4x - 19 = 0$

$\Rightarrow x = \frac{-4 \pm \sqrt{16-4(4)(-19)}}{8}$

$\Rightarrow x = \frac{-4 \pm \sqrt{16+304}}{8}$

$\Rightarrow x = \frac{-4 \pm \sqrt{320}}{8}$

$\Rightarrow x = \frac{4 \pm 8\sqrt{5}}{8}$

$\Rightarrow x = \frac{-4(-1 \pm 2\sqrt{5})}{8}$

$\Rightarrow x = -1 \pm 2\sqrt{5}$

Hence the roots are $-2, 1, -1 \pm 2\sqrt{5}$

5) Solve the equation $3x^3-26x^2+52x-24=0$ if its roots form a geometric progression.

Given cubic equation is $3x^3$-$26x^2$+52x-24=0

Here, a = 3, b = -2b, c = 52,d = -24.

Since the roots form an geometric progression,

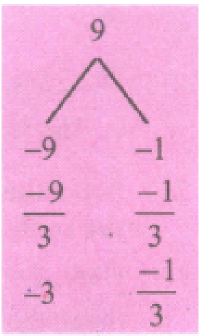
The roots are $\frac{a}{r}$ a, ar, sum of the roots=$\frac{-b}{a}$

$$\Rightarrow \frac{a}{r} + a + ar = \frac{26}{3} \qquad ...(1)$$

and product of the roots $\frac{-d}{a}$

$$\Rightarrow \frac{a}{\cancel{r}} + a + \cancel{ar} = \frac{24}{3} = 8$$
$$a^3 = 8 = 2^3$$

a=2



$\therefore$ (1) becomes $\frac{2}{r}$+2+2r=$\frac{26}{3}$

$$\Rightarrow \frac{2+2r+2r^2}{r} = \frac{26}{3} \Rightarrow \frac{1+r+r^2}{r} = \frac{13}{3}$$

$$\Rightarrow 3 + 3r + 3r^2 = 13r \Rightarrow 3r^2 - 10r + 3 = 0$$

$$\Rightarrow (r-3)(3r-1) = 0 \Rightarrow r = 3$$

$\therefore$ The roots are $\frac{a}{r}$,a, and ar

$$\Rightarrow \frac{2}{3}, 2 \quad and \quad 2(3) \Rightarrow \frac{2}{3}m2 \quad and \quad 6$$

6) Determine k and solve the equation $2x^3$-$6x^2$+3x+k=0 if one of its roots is twice the sum of the other two roots.

Given cubic equation is $2x^3-6x^2+3x+k=0$

Here, a = 2, b = -6, c = 3, d= k

Let ∝,β,ɤ be the roots

Given ∝=2(β+ɤ)⇒$\frac{\alpha}{2}$=β+ɤ...(1)

Now, $\alpha+\beta+\gamma=\frac{-b}{a}=-\frac{(-6)}{2}=3$

$\frac{\alpha}{2}+\alpha=3\Rightarrow\frac{\alpha+2\alpha}{2}=3\Rightarrow\frac{3\alpha}{2}=3$

$\Rightarrow\alpha=2$

$\alpha\beta\gamma=\frac{-d}{a}=\frac{-k}{2}\Rightarrow 2.\beta\gamma=\frac{-k}{2}$

$\beta\gamma=\frac{-k}{4}\qquad ...(2)$

Also, $\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}$

$2\beta+\beta\gamma+2\gamma=\frac{3}{2}$

$2(\beta+\gamma)+\beta\gamma=\frac{3}{2}$

$\alpha\frac{-k}{4}=\frac{3}{2}\quad[from(1)\&(2)]$

Also, $2-\frac{k}{4}=\frac{3}{2}[\because\alpha=2]$

$2-\frac{3}{2}=\frac{k}{4}\Rightarrow\frac{1}{2}=\frac{k}{4}$

$k=\frac{4}{2}\Rightarrow k=2$

From(2), $\beta\gamma=\frac{-k}{4}=\frac{-2}{4}=\frac{-1}{2}\Rightarrow\gamma=\frac{-1}{2\beta}$

From $(1),\beta+\gamma=\frac{\alpha}{2}=\frac{2}{2}=1$

Substituting $\gamma=\frac{-1}{2\beta}$ We get

$\beta-\frac{1}{2\beta}=1\Rightarrow 2\beta^2-1=2\beta\Rightarrow 2\beta-2\beta-1=0$

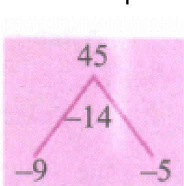
$\beta=\frac{2\pm\sqrt{4-4(2)(-1)}}{4}=\frac{2\pm\sqrt{4+8}}{4}$

$=\frac{2\pm\sqrt{12}}{4}=\frac{2\pm2\sqrt{3}}{4}$

$\beta=\frac{1\pm\sqrt{3}}{2}$

Hence the roots are $2,\frac{1+\sqrt{3}}{2},\frac{1-\sqrt{3}}{2}$

7) Solve the equation : $x^4-14x^2+45=0$



Put $x^2=y$

$\Rightarrow y^2-14y+45=0$

$\Rightarrow (y-9)(y-5)=0$

$\Rightarrow$ y= 9 or y = 5

$\Rightarrow x^2=9$ or $x^2=5$

$\Rightarrow x=\pm3$ or $x=\pm\sqrt{5}$

Hence the roots are 3, -3, √5 and -√5.

8) Solve the cubic equations:

$8x^3-2x^2-7x+3=0$

Let.f(x) = $8x^3- 2x^2 - 7x +3 = 0$

Here sum of the co-efficients of odd terms

= 8-7=1

and sum of the co-efficients of even terms

= -2 +3 = 1

Hence, x = - 1 is a root of f(x)

Let us divide.f{x) by (x + 1)

$$\begin{array}{r|rrrr} -1 & 8 & -2 & -7 & 3 \\ & \downarrow & -8 & 10 & -3 \\ \hline & 8 & -10 & 3 & \boxed{0} \end{array}$$

∴ The other factor is $8x^2 - 10x + 3$

$$\Rightarrow x = \frac{10 \pm \sqrt{100-4(8)(3)}}{2\times 8}$$

$$\Rightarrow x = \frac{10 \pm \sqrt{100-96}}{16} \Rightarrow x = \frac{10 \pm 2}{16}$$

$$\Rightarrow x = \frac{12}{16} or$$

$$x = \frac{8}{16} \Rightarrow x = \frac{3}{4}, \frac{1}{2}.$$

∴ The roots are -1, $\frac{1}{2}, \frac{3}{4}$

9) If sin ∝, cos ∝ are the roots of the equation $ax^2 + bx + c$-0 ($c \neq 0$), then prove that $(n + c)^2 - b^2 + c^2$

Sum of the roots = sin ∝ + cos ∝ = $\frac{-b}{a}$

Product of the roots = sin ∝ cos ∝ = $\frac{c}{a}$

Now1= $\cos^2\propto+\sin^2\propto$

= $(\sin\propto+\cos\propto)^2$ -2 sin ∝ cos ∝

$$1 = \frac{b^2}{a^2} - \frac{2c}{a} \Rightarrow 1 = \frac{b^2-2ac}{a^2}$$

$\Rightarrow a^2 = b^2 - 2ac \Rightarrow a^2 + 2ac = b^2$

Addingcsboth sides, $a^2 +2ac+c^2 = b^2+c^2$

$\Rightarrow (a+c)^2 = b^2 + c^2$

PART-B

4 x 3 = 12

10) Find the condition that the roots of $x^3+ax^2+bx+c = 0$ are in the ratio p:q:r.

Since two roots are in the ratio p:q:r, we can assume the roots as pλ,qλ and rλ .

Then, we get

Σ_1 = pλ+qλ+rλ = -a ....(1)

Σ_2 = (pλ)(qλ)+(qλ)(rλ)+(rλ)(pλ) ....(2)

Σ_3 = (pλ)(qλ)(rλ)=-c ....(3)

Now, we get

(1) ⇒ $\lambda = -\frac{a}{p+q+r}$ ......(4)

(3) ⇒ $\lambda^3 = \frac{c}{pqr}$ ....(5)

Substituting (4) in (5), we get

$\left(\frac{a}{p+q+r}\right)^3 = -\frac{c}{pqr}$ ⇒ $pqra^3 = c(p+q+r)^3$.

11) If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$ in terms of p.

The discriminant Δ =$((4p)^2 - 4(4)(p+2) = 16(p^2-p-2) = 16(p+1)(p-2)$. So we get

Δ < 0 if -1< p< 2

Δ=0 if p=-1 or p=2

Δ > 0 if -∞< p< -1 or 2< p< ∞

Thus the given polynomial has

imaginary roots if -1< p< 2;

equal real roots if p=–1 or p=2;

distinct real roots if -∞< p< -1 o 2< p< ∞.

12) If 2+i and 3-$\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$, find all roots.

Since the coefficient of the equations are all rational numbers, 2+i and 3-$\sqrt{2}$ are roots, we get 2-i and 3+$\sqrt{2}$ are also roots of the given equation. Thus (x-(2+i)), (x-(2-i)), (x-(3-$\sqrt{2}$)) and (x-(3+$\sqrt{2}$)) are factors. Thus their product.

((x-(2+i))(x-(2-i))(x-(3-$\sqrt{2}$))(x-(3+$\sqrt{2}$)) is a factor of the given polynomial equation. That is, $(x^2-4x+5)(x^2-6x+7)$ is a factor. Dividing the given polynomial equation by this factor, we get the other factor as (x^2-3x-4) which implies that 4 and −1 are the other two roots. Thus

2+i,2-i,3+$\sqrt{2}$,-$\sqrt{2}$,-1, and 4 are the roots of the given polynomial equation.

13) Find the condition that the roots of $ax^3+bx^2+cx+d=0$ are in geometric progression. Assume a,b,c,d ≠0.

Let the roots be in G.P.

Then, we can assume them in the from $\frac{\alpha}{\lambda}$,α,αλ.

Applying the Vieta's formula, we get

$\Sigma_1 = \alpha\left(\frac{1}{\lambda} + 1 + \lambda\right) = \frac{b}{a}$ .....(1)

$\Sigma_2 = \alpha^2\left(\frac{1}{\lambda} + 1 + \lambda\right) = \frac{c}{a}$ ......(2)

$\Sigma_3 = \alpha^3 = \frac{d}{a}$ ....(3)

Dividing (2) by (1), we get

$\alpha = -\frac{c}{b}$ ....(4)

Substituting (4) in (3), we get $\left(-\frac{c}{b}\right)^3 = \frac{d}{a}$ ⇒ $ac^3 =$

db^3.