Model Question Paper

Complex Numbers - Part IV

12th Standard

Maths

Reg.No. :

I.Answer all the Questions.

II.Use blue pen only.

Time : 02:00:00 Hrs Total Marks : 72

Section-A

6 x 1 = 6

1) If P represents the variable complex number Z and if $|2z-1|=2\ |z|$ then the locus of P is

(a) the straight line $x=\frac{1}{4}$ (b) the straight line $y=\frac{1}{4}$ (c) the straight line $z=\frac{1}{2}$ (d) the circle $x^2+\ y^2-4x-1=0$

2) The value of $\frac{1+e^{-i\theta}}{1+e^{i\theta}}$ is

(a) $cos\theta+isin\theta$ (b) $cos\theta-isin\theta$ (c) $sin\theta-icos\theta$ (d) $sin\theta+icos\theta$

3) If $z_n=cos\frac{n\pi}{3}+isin\frac{n\pi}{3}$ then $z_1z_2z_3......z_6$ is

(a) 1 (b) -1 (c) i (d) -i

4) If $-\bar{z}$ lies in the third quadrant then z lies in the

(a) first quadrant (b) second quadrant (c) third quadrant (d) fourth quadrant

5) If $x=cos\theta+isin\theta$ then the value of $x^n+\frac{1}{x^n}$ is

(a) $2cosn\theta$ (b) $2isinn\theta$ (c) $2sinn\theta$ (d) $2icosn\theta$

6) If $a=cos\alpha-i\sin\alpha$, $b=cos\beta-i\sin\beta$ and $c=cos\gamma-i\sin\gamma$ the $(a^2c^2-b^2)/abc$ is

(a) $cos\ 2(\alpha-\beta+\gamma)+i\sin2(\alpha-\beta+\gamma)$ (b) $-2cos(\alpha-\beta+\gamma)$ (c) $-2i\sin(\alpha-\beta+\gamma)$ (d) $2\cos(\alpha-\beta+\gamma)$

Section-B

4 x 3 = 12

7) If $cos\alpha+cos\beta+cos\gamma=0=sin\alpha+sin\beta+sin\gamma$, prove that $cos3\alpha+cos3\beta+cos3\gamma=3cos(\alpha+\beta+\gamma)$

8) If $(1+i)\quad(1+2i)\quad(1+3i)....(1+ni)=x+iy$ shows that 2.5.10.....$(1+n^2)=x^2+y^2$

9) Simplify : $\frac{(cos2\theta-i\sin2\theta)^7(cos3\theta+i\sin3\theta)^{-5}}{(cos4\theta+i\sin4\theta)^{12}(cos5\theta-i\sin5\theta)^{-6}}$

10) Express the following in the standard form of $a+ib$, $\frac{5+5i}{3-4i}$

Section-C

4 x 6 = 24

11) Prove that the complex numbers $3+3i$, $-3-3i$,$-3\sqrt{3}+3\sqrt{3}i$ are the vertices of an equilateral triangle in the complex plane.

12) Prove that the points representing the complex numbers $2i, 1+i, 4+4i$ and $3+5i$ on the Argand plane are the vertices of a rectangle.

13) Show that the points representing the complex numbers $7+9i, -3+7i, 3+3i$ form a right angled triangle on the Argand diagram.

14) Find the square root of $(-7+24i)$

Section-D

3 x 10 = 30

15) If $\alpha\quad and\quad\beta$ are the roots of the equation $x^2-2px+(p^2+q^2)=0$ and $tan\theta=\frac{q}{y+p}$ Show that $\frac{(y+\alpha)^n-(y+\beta)^n}{\alpha-\beta}=q^{n-1}\frac{sinn\theta}{sin^n\theta}$;

16) If $\alpha\quad and\quad\beta$ are the roots of $x^2-2x+4=0$ Prove that $\alpha^n-\beta^n=i\quad2^{n+1}\quad sin\frac{n\pi}{3}$; and deduct $\alpha^9-\beta^9$.

17) a) If $x+\frac{1}{x}=2cos\theta$ and $y+\frac{1}{y}=2cos\phi$ show that $\frac{x^m}{y^n}+\frac{y^n}{x^m}=2cos(m\theta-n\phi)$

(OR)

b) If $a=cos2\alpha+isin2\alpha$, $b=cos2\beta+isin2\beta$ and $c=cos2\gamma+isin2\gamma$ Prove that (i) $\sqrt{abc}+\frac{1}{\sqrt{abc}}=2cos(\alpha+\beta+\gamma)$ (ii) $\frac{a^2b^2+c^2}{abc}=2cos\quad2(\alpha+\beta-\gamma)$.

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